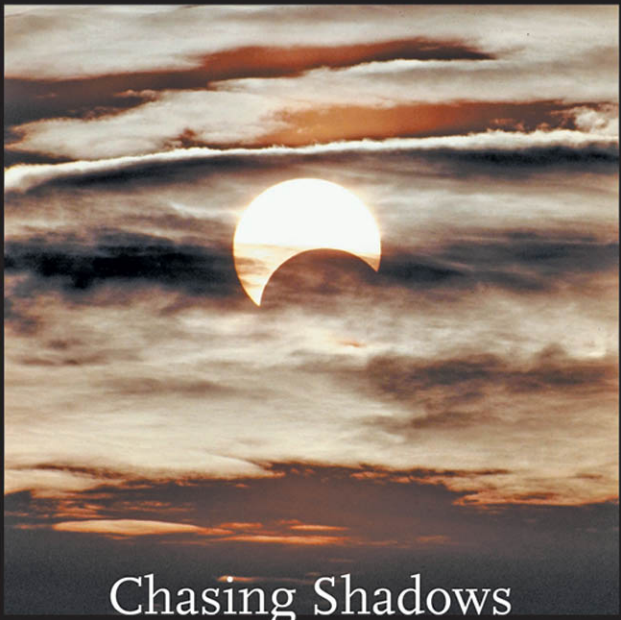

Chasing Shadows

Mathematics, Astronomy, and the Early History of Eclipse Reckoning

CLEMENCY MONTELLE

VISIT…

Chasing Shadows

Johns Hopkins Studies in the History of Mathematics
Ronald Calinger, Series Editor

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Mathematics, Astronomy, and the Early History of Eclipse Reckoning

CLEMENCY MONTELLE

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To my three boys

Well, after this, said Socrates, when I was worn out with my physical investigations, it occurred to me that I must guard against the same sort of risk which people run when they watch and study an eclipse of the sun; they really do sometimes injure their eyes, unless they study its reflection in water or some other medium. I conceived of something like this happening to myself, and I was afraid that by observing objects with my eyes and trying to comprehend them with each of my other senses I might blind my soul altogether. So I decided that I must have recourse to theories, and use them in trying to discover the truth about things.

—Plato, Phaedo 99d–e; ed. Hamilton

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Preface

"In order to understand the text," Professor David Pingree—chair and the only permanent faculty member of the Department of the History of Mathematics at Brown University—used to insist with his eyes sparkling, "you have to know what it says *before* you read it." I don't know if I could ever reproduce the incredulous look that came over me the first time he told me this, but gradually I began to appreciate this sentiment to be utterly true when it came to working with sources in the history of the exact sciences. Indeed, this field has many challenges, which are compounded when one starts one's inquiry with the primary sources themselves; it is a demanding but immensely rewarding field. I hardly knew my luck when I sat there on that first day of graduate school, he and I face-to-face across a desk, in a room piled high with facsimiles of manuscripts, a dusty blackboard, and rickety wooden chairs—the very ones other greats such as Neugebauer and Aaboe had sat in for hundreds of hours. Professor Pingree and I read manuscripts of all sorts in the exact sciences and tried to figure out what they said—this was a veritable apprenticeship of an extraordinary kind.

From such singular training I present this work which, I hope, captures in every way the spirit of his tutelage. It is intended to be a product of the skills he nurtured: reading original sources in multiple dead languages, complete with careful textual study, mathematical analysis, and broader reflection of the relationship between early scientific societies and the instances of transmission of scientific ideas between them. I hope that I have met his expectations, if only in some small measure.

A survey of eclipse science through four different cultures over several millennia is vast. As well as synthesizing and building upon the work of other scholars, I present new material to enrich the analysis. There are, however, aspects outstanding that I have been unable to solve or adequately comment on. It is my hope that presenting them here in a larger context will but encourage them to be reconsidered further.

Many people have helped with the project in various respects: John Steele, John Britton, Alex Jones, Alice Slotksy, Toke Knudsen, Joan Richardson, John Hannah, Eliza Collins, Alex James, and Professor Silex (you know who you are) have all been academically supportive and inspirational. I am sincerely grateful to Trevor Lipscombe, Greg Nicholl, Jennifer Slater, and Michele Callaghan for their professional advice, patience, and encouragement. In broader respects, this book has been carried along with the love and care of my husband Yann and the continual distraction of my two energetic sons Pierre and Gabriel; my parents Felicity and Grant, who see me safely through the world, my brother Samuel and my Japanese sister Ayako, my French mother-in-law Chantal, my paternal grandparents, Dawn and Alan, and my grandfather, who was the first to show me the stars.

I owe a huge debt of gratitude to two people in particular for the final presentation, content, and completion of this book. Philip Catton carefully read the first draft and challenged me to dig deeper, showing me the way forward. His encouragement, insight, and support are ever present in this book. Kim Plofker, who read the second draft, is to be thanked for her fastidiousness, scholarly acumen, and indefatigability, as well as her sharp sense of humor which has kept me smiling throughout the process. Both have great faith in me, which I find humbling. Nevertheless, the final presentation is my responsibility.

Chasing Shadows

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CHAPTER ONE

Heavenly Hide and Seek

An Introduction to Eclipse Reckoning in Early Times

"Was there an eclipse?" demanded the peripatetic Aristotle. "What is an eclipse? Why is there an eclipse, or rather, why is the moon eclipsed?"[1] From earliest times, lunar and solar eclipses have provided persistent and conspicuous inspiration for thinkers to study celestial phenomena. But Aristotle's discussion about eclipses in this context proved profoundly significant in ways that extended far beyond the purview of astronomy. Besides the direct effect on astronomical research, early inquirers were ineluctably drawn to broader forms of inquiry by eclipse phenomena in ways that are emblematic of the vast yet gradual unfolding of scientific thought-forms throughout human history.

It should come as no surprise that eclipses are inspirational. They are arguably the most spectacular events in the heavens. Throughout history, the celestial drama of these occasions has terrified citizens, inspired poets and myth-makers, shaped social conventions, provided means to foretell the future, guided decision making, and immobilized entire armies, as well as fascinated and challenged professional astronomers and amateur sky-watchers alike. And for those cultures with little understanding of the mechanisms behind their occurrence, it must have been petrifying to watch as either of the two most permanent fixtures in the sky became obscured with little warning. Whether these occasions inspired awe or instigated dread, their regularity along with their theatrical impact have earned eclipse reckoning a vital role in the astral researches of various societies, from ancient to modern.

Eclipse phenomena, which are so simply enjoyed by even the most untrained eye, are some of the most complex occurrences in the sky to account for scientifically. In fact, in the premodern world, of all celestial events, eclipses presented the astronomer with the most challenges. Problems that were easy to formulate, such as frequency, duration, and amount of obscuration, rested upon more advanced astronomical and mathematical considerations than

were generally available. The successful theoretical consideration of eclipses required a solid foundation in all the basic elements of astronomy (excluding planetary theory) and advanced mathematical methods. Accurate techniques for computing solar and lunar longitudes, lunar latitude, lunar anomaly, and parallax, and appropriate geographical coordinates were indispensable. In an environment without the benefits of telescopes, sensitive timing devices, and accurate measuring equipment—and, for the earliest societies, without a significant astronomical legacy—the task of establishing these parameters was full of obstacles and challenges.

More broadly, eclipse science, bound inextricably to observation in its nascent stages, ultimately directs the mind to a reality behind what is observed. Sky-watchers do not see the geometry of the astronomical alignments as directly as they can register their observational effects. Still less tangible are the trajectories of these celestial bodies, the tracing of great circles, the intersections of paths, the pivots or rotations that produce these alignments. It would take inquirers millennia to reach the point of reckoning geometrically with a reality behind the noticeable facts, and even longer to harmonize them with the physical actuality.

Archeological researches amply testify to the fact that even prehistoric societies considered the movements of celestial phenomena. To varying extents, these investigations drew some of them on to consider the more complicated recurrence of eclipses, lunar as well as solar.[2] Such events not only awed these prehistoric peoples but also urged some among them to new forms of intellection. Without the arts of literacy, early prehistoric cultures were entirely reliant on oral forms of expression and thus on the memory arts. In such circumstances, investment in knowledge is restricted by the pressures and practical considerations surrounding its storage in memory. Certainly, it is attested that nonliterate peoples possessed exceptional facility in the memory arts, but the organization of their knowledge was bound to be structured largely with respect to memorability, overriding a more unified rationalistic approach. Thus, the fact that the earliest known testimonies which reflect a considered reckoning of recurring events were physical structures devoted to the sun, the moon, and the stars is profound. Their existence reflects an emergent commitment to rationally apprehensible order inspired by the movements of the heavens. The significance of this as a development of culture is not to be underestimated. Vital to the emergence of human rationality was an irresistible wonder and scrutiny of the heavens overhead.

The appreciation of recurrence grounded more specific questions about these patterns. For those early peoples with no astronomical legacy, the search

for such answers had to begin with record keeping. It was no coincidence that the very first literate civilization was in a position to address these aspirations. They had at their disposal the recently created systems of writing to keep track of the celestial bodies in ways that oral cultures could never maintain. Sustained observation and subsequent documentation ensured the gradual recognition of patterns and periodicities of various celestial phenomena and more importantly revealed a potential to predict (initially very roughly, but soon with successive refinement) astronomical events, including the timing of eclipses. Furthermore, the subsequent grouping, excerpting, and rearrangement of related recorded data brought to light relationships and patterns that could not have been detected otherwise. An investment in numerical patterns to capture the events in the sky proved exceedingly robust. In fact, inadequacies of this sky-arithmetic (if they were registered at all) were not deemed significant enough to induce the developers of such schemes to become discontented with their numerical approach.

In utterly contrasting cultural conditions, the appreciation of recurrence inspired a radically different approach. Because of a variety of factors, among which historical, societal, and, arguably, linguistic circumstances are relevant, some early peoples nurtured a more abstract, theoretical tendency of mind. Initially encouraged by inchoate philosophical ruminations, inquirers were increasingly confronted by the distinction between facts as they might present themselves *apparently* and what they might be *really*. This theoretical tendency of mind was further endorsed after increased cultural investment in symbolic forms of reasoning. Mathematical apparatus and constructions, diagrams which embodied relations, all lent themselves to more abstract considerations. Inquiry then was based more and more on a reality conceived of in a literal-minded way. Space itself could be imagined as if viewed from some higher vantage point. Ultimately, inquirers felt their way to a sense of discovery of a reality behind the facts. No longer was prediction on its own adequate; thinkers determinedly set their ambitions on the deep-seated search for explanation. The heavens were revered by these peoples not only for their regularity and predictability, but more deeply because of the rationally apprehensible order and beauty to which they pointed.

In the context of astronomy, sky-geometry emerged as an ambition. The first speculations intended to capture the apparent celestial motions were wildly wrong, but telling. Encouraged by various successes with respect to more theoretically oriented astronomical reflections, such as the demonstration of the sphericity of earth and the relative positions of the luminaries, and guided by certain abstract stipulations, such as concentricity, circular motion, uniformity,

and regularity, inquirers proposed models that were increasingly better able to capture the qualitative patterns of the celestial motions. An awareness of the limitations of observation grew, which further reinforced the advantages of the theoretically minded approach. Interestingly, practitioners were slow to imbue these qualitative models with quantitative significance. Indeed, it is ironic that simultaneously in time, but in different astronomical traditions, there were those thinkers that considered patterns behind the facts, and others who focused on reality behind the facts, and these two approaches continued without contact. Of course, the implications for astronomy by the merger of these two approaches were huge and it was to be their eventual connection and synthesis that produced the first astronomical model that had predictive capacity as well as explanatory power. So potent was this model that it was not overturned for more than a millennium after its inception.

In terms of eclipse phenomena, it was to be the independently arising, more mechanistic conceptions of the celestial realm, together with the discovery of the spherical nature of the earth, the sizes and shapes of the luminaries, and their relative positions that, when combined with heavy investment in numerical patterning, meant that the aspirations to predict both lunar and solar eclipses with confidence and describe their basic features could be realized for the first time. As a result, to trace the journey toward successful eclipse prediction is to uncover many profound discoveries, creative problem solving, and remarkable insights. An understanding of ancient eclipse theory is essential to our understanding of ancient astronomy as a whole.

No accounting for the long history of this overall development can fail to grasp its prodigious difficulty. It was protracted, discontinuous, nonlinear, and the work of countless individuals. Transmission of ideas between cultures was vital, yet complex. Skills would develop and later be lost. Critical insights were obtained and then languished. Manifold solutions to mathematical and astronomical problems would be maintained with no way of establishing priority. Precedence could be given to those suggestions that satisfied criteria wholly outside of astronomy. The notion that the human command of eclipse reckoning has been simply cumulative is far from the historical truth. Yet the necessary, detailed historical analysis nonetheless reveals a sense of overall change. It is, among other things, a change from observational to theoretical, from an accountancy of the appearances to a considered illumination of the reality behind the facts. Eclipses have served human beings by inspiring broader forms of intellection. Science itself is an eventual upshot of these developments.

1.1 Development and Cross-Cultural Transmission

The ability to successfully predict lunar and solar eclipses and fully account for their details was a long time coming; efforts were spread both diachronically and geographically. Naturally then, any meaningful account of the development of eclipse theory from antiquity on can be achieved only by a thorough examination of the transmission of ideas from various scientifically active societies in the ancient and medieval world. Such an accomplishment was not achieved by any one scientific culture; it was a cumulative effort. It was not a collaboration simultaneous in time; rather, the achievements of one culture were transmitted to an inheritor culture, which might assimilate some of these ideas, elaborate on some of them, reject some of them, misunderstand some of them, develop some of them. Indeed, it was a given that these transmitted ideas would need some adjustments so that they would fit into the scientific culture that received them. In many instances, scientific cultures may have shared models and parameters, but, in the case of those selected in this study, they did not transmit the ways in which each functioned as a scientific culture.

The circumstances under which science and its practitioners worked were quite distinct from culture to culture. These differences were manifested in the ways in which science was recorded, taught, practiced, and patronized, and elements such as the role of observations, methods of time keeping, mathematical techniques, and so on, were all handled differently. Most literate cultures in the ancient world took an interest in eclipses, and responded to the phenomenon with various degrees of "technicality." There are too many to adequately cover here; the societies studied in this work have been selected because of the role they play in the shaping of eclipse theory within the broader tradition of "western" astronomy. Those traditions considered significant in their contribution are the ancient Near East, ancient Greece, India, and the Islamic Near East. No one of these cultures on its own can claim complete credit for the legacy that was to be passed on to the modern world. Rather the tradition was shaped through collective and overlapping incremental insights, both small and large, and by the preservation and spread of knowledge. In addition, the far reach of these civilizations, geographically, temporally, politically, and culturally, is a strong argument for singling them out.[3]

Other cultures, with equally valuable and in many cases sophisticated insights into eclipses, such as China and Mesoamerica, are indeed important. However they are traditions that remained largely isolated and independent in the premodern world—traditions that impacted the broader western tradition less directly and less immediately.[4]

1.1.1 The Ancient Near East

The enormous intellectual debt owed to the ancient Near East for incubating the conditions which saw the emergence of writing and the arts of literacy is well known. Less recognized, but equally as vital, was the significant impact their industry in the astral sciences had on subsequent researches in celestial phenomena for millennia to come. Researches into the celestial sciences spread both west and east to contiguous civilizations (including Egypt, the Hittites, Syria, Palestine, Greece, Iran, and India) but also had significant impact on the ensuing generations of these societies in successive periods—impressively far beyond the temporal reach of the original instigators.

Ancient Near Eastern conventions and practices provided a technical backdrop against which all subsequent astronomical researches were played out. Mesopotamian mathematical practices were adopted and conventionalized by their successors. Computation using the sexagesimal (base-60) number system became standard practice for the astral sciences (and still persists today in many instances). Metrological conventions and the ways in which these practitioners sought to measure and quantify celestial phenomena became the basis of astronomical reckoning and recording. These included expressions for units of length and magnitude (such as fingers [0;5°], cubits [2°]), and time and arc (such as degrees [UŠ], the division of the circle into "360°," the lunar day or *tithi*—a thirtieth of a mean synodic month, the division of the hours of daylight into 12 [*bēru*]). Calendrical conventions (such as division and arrangement of the year, intercalary conventions, *parapegma*) and chronological systems (such as the use of the Seleucid era) were assimilated. Tools such as the gnomon, the water clock, and the hemispherical sundial became essential accessories for astronomers.

Babylonian parameters and period relations proved inspirational and fundamental for all subsequent efforts at more involved celestial models. Among them were the simple lunar period relation (nine anomalistic months equals 248 days), the later-named Metonic cycle (19 years equals 235 synodic months) and Saros cycle for eclipses (18 years $\approx$ 223 synodic months $\approx$ 242 draconitic months $\approx$ 239 anomalistic months), length of daylight ratios (notably 3:2 for Babylon), the length of the mean synodic month (29;31,50,8,20), lunar latitude (1 year = 12;22,8 synodic months), and so on. Techniques used to model celestial phenomena were adopted, including linear zigzag "functions" and arithmetical sequences to model phenomena such as rising times. Furthermore, the celestial phenomena themselves and their manifest features which were deemed worthy of recording were transmitted. For example, in the context of eclipses, features such as magnitude, direction of impact, angle of

impact, duration, and color were among the aspects included in subsequent accounts.

Conventions that helped the practitioners in the ancient Near East demarcate the sky so as to plot and follow the celestial patterns and relationships as they unfolded in the heavens became central: features such as the "ecliptic," the zodiacal divisions, and the ecliptic coordinate system, as well as the arrangements of stars into constellations, all have their origins with this civilization. However, these aspects were not always the preserve of "mainstream" astronomical traditions, but were also transmitted and sustained via other related traditions, such as celestial divination and broader astrological practices. The extent of such incorporation of all aspects of the astral sciences in the ancient Near East by later generations guaranteed them an unquestionably influential role in the history of science. Astonishingly, the extent of their influence has only come to light in the last hundred years, before which time almost all relevant evidence concerning their achievements lay buried.

1.1.2 Ancient Greece

The dependence on ancient Near Eastern investigations of the formation of mathematical astronomy in ancient Greece can be seen as early as the fourth century BCE, and more general traces of astronomical transmission can be detected as far back as the eighth century BCE; the extent of the relationship is still being clarified as new evidence comes to light. In turn, the powerful resulting models developed by Greek practitioners spread to ancient and medieval societies; the subsequent impact on Indian, Arabic, and medieval European astronomical traditions was profound.

Details of transmission of ancient Near Eastern ideas to Greece can be identified primarily through texts in the exact sciences proper; however, there is an undeniable presence of Mesopotamian-inspired references in the earliest Greek literary texts, including Homer and Hesiod, and historical texts, such as the reports from historians like Herodotus about various techniques that the Greeks learned from the Babylonians.[5] As early as the Achaemenid period (538–331 BCE), Babylonian material was already having an impact on Greek astronomical efforts, through scholars such as Meton, Kleostratus, and Euktemon.[6] The turn of the fourth century saw the appearance in Greek sources of further aspects of astronomy as practiced by ancient Near Eastern contemporaries. Eudoxus is reported to have introduced the "Assyrian sphere," probably referring not to a celestial-sphere type model, but to the description in his work of many constellations, the arrangement of which is originally Mesopotamian. Eudoxus assigns equinoxes and solstices in the tradition of MUL.APIN, one of the earliest Mesopotamian astronomical compendiums,

composed, it is supposed, sometime in the late second millennium BCE.[7] Familiar period relations and the relative velocities of the planets are recounted by Plato,[8] and by the third century BCE Hellenistic scholars adopted conventions in sexagesimal arithmetic in their astronomical calculations and integrated the Babylonian division of the zodiacal circle into 360° in their reckonings.

By about 300 BCE, a new type of mathematical astronomy from Mesopotamia penetrated the Greek scene. Greek commentators of the time, such as Pliny, Strabo, Posidonios, and Vettius Valens, name individual Babylonian astronomers—Kidenas, Naburianos, and Sudines—and attribute to them the more mathematically oriented astronomical models that were introduced to Greece. Indeed, Babylonian ephemerides, in particular, what is now called the system B and its underlying parameters, formed the basis for individuals such as Hipparchus (second century BCE) to develop his mean lunar motions, and the lunar apogee and node. Hipparchus' work was synthesized and developed by Ptolemy (second century CE). Ptolemy also drew at least ten records of lunar eclipses from Babylonian sources, observed between 722 BCE and 383 BCE, probably extracted from the astronomical diaries or some intermediary text. The named Babylonian Goal-Year periods were known and used by manifold later authors, including John of Lydia (mid sixth century), Rhetorius of Egypt (early seventh century), Theophilis of Edessa (eighth century), and Michael Psellos (eleventh century). Techniques for computing planetary positions that were original to Mesopotamia and integrated by Greek and Egyptian astronomers in Roman Egypt are preserved in Demotic and Greek papyri. Greek astrology was inspired and shaped by traditional texts from the ancient Near East. Evidence confirms that authors such as Hephaestio of Thebes, John of Lydia, and Proclus used sources based on Mesopotamian divination texts as a basis for their own systems. These are but a few of the known links.

Of significance for the transmission of astral sciences beyond Greece is the massive influence of Ptolemy's seminal work, the *Almagest*, as it was to be later renamed by Arabic scholars, as well as his *Handy Tables*. Babylonian aspects as they were adopted into the Greek model were passed on to other cultures largely through the medium of the *Almagest*, which spread like wildfire throughout Eurasia. Apart from largely technical details and corrections, his work remained the primary source of inspiration for more than a millennium, being replaced only by a massive upheaval in the astronomical paradigm. Cultures that remained ignorant of this work directly found that their traditions floundered. India, for example, remained unaware of Ptolemaic theories in their direct form, as a translation of the *Almagest* was not made directly into Sanskrit until the eighteenth century.[9]

1.1.3 India

In Sanskrit literature, works on astronomy, astrology, and mathematics, as well as divination, were grouped together in a class of texts called *jyotiḥśāstra* (literally "science of the stars").[10] Traces of astronomy from the ancient Near East can be detected as early as the Vedic texts—around 1000 and 500 BCE.[11] The descriptions of constellations, intercalation rules, the ideal calendar, as well as other features contained in these texts, bear a striking resemblance to those set out in the early Mesopotamian astronomical manual MUL.APIN. Babylonian omen literature reached India by at least the fifth and fourth centuries BCE, supported by the historical fact that the Achaemenid Empire had a presence in Gandhāra in Northwestern India and the Indus Valley. A collection of Pali texts datable to the fifth century BCE contains unmistakable features from Babylonian omen texts such as *Enūma Anu Enlil* and *Šumma ālu*. The earliest surviving Sanskrit omen literature, the *Saṃhitā* texts, such as the *Gargasaṃhitā* (ca. 100 CE) and beyond, was ultimately Mesopotamian.[12] Furthermore, the organization and content of these Sanskrit texts reveal that the originals reached India independently of Greece.

The very first work on mathematical astronomy in India, the *Jyotiṣavedāṅga*, was composed by Lagadha in the late fifth century BCE. This work describes times for sacrifices and has been shown to be almost completely derived from Mesopotamian sources.[13] Additional astronomical and astrological material from Greece was translated into Sanskrit in the following centuries, notably the second, third, and fourth centuries CE, each of which embodied a distinctly different system. The first was a Greek synthesis of originally Mesopotamian theories from the Seleucid and Parthian periods which are summarized in the compendium by Varāhamihira called the *Pañcasiddhāntikā*, written in the sixth century CE. The second imported pre-Ptolemaic Hellenistic astronomy. This preservation of astronomical endeavors prior to Ptolemy in Indian sources is critical and is perhaps one of India's most significant contributions to the history of science.[14] Because of the overwhelming success of the Ptolemaic model, all researches prior to its publication became obsolete in the Greek scene and thus were no longer copied and preserved. India seems to be the only repository that continued to maintain the non-Ptolemaic tradition despite its losing favor with the very culture in which it flourished. The third system was a planetary model designed with the prerequisite of Aristotelian concentricity.[15]

Also of consequence was the distinctly different intellectual climate in India which shaped the ways in which the astral sciences were carried out. An emphasis on oral tradition placed a stringent set of prerequisites on practitioners which influenced the ways in which they developed and maintained

their tradition. Furthermore, there is an absence of a tradition of observational astronomy in India.[16] The first accurate star catalog does not appear in the literature until 1370 with the translation from the Persian by Mahendra Sūri of Ptolemy's star catalog, and the first extant recording of a *set* of astronomical observations within the context of mathematical astronomy is found as late as the fourteenth century, in Parameśvara's *Siddhāntadīpikā*, where a number of lunar and solar eclipse observations and some of their details are reported. Even then, details as to how these observations have affected Parameśvara's parameters are nowhere made explicit.

The absence of this observational tradition in Indian endeavors is no doubt a result of many factors. One large influence must be the fact that their base parameters were ultimately imported from foreign sources. This, combined with the fundamental disparity in approach of the original sources of their mathematical models (that is, the predominantly arithmetical models from the ancient Near East and the geometric models from the Greeks), is no doubt the overwhelming reason for their seeming indifference to consistency and coherent methodology in their models as well as their "lack of concern with observational astronomy."[17]

Despite this, critical mathematical insights were accomplished in India that were to have great impact on the history of astronomy. The cumbersome chord function used by Ptolemy and others was updated by Indian practitioners, who developed the sine, cosine, and versine functions. Analemma techniques original to Greece to tackle problems in spherical trigonometry were expanded. The Hindu system of numerals and the emergence of the place-value system, and various iterative and approximative techniques were to prove fundamental for further generations to improve and develop their astronomical models. Many of these mathematical innovations were adopted by Sassanian and early Islamic astronomical practitioners, and, through these Arabic channels, to Byzantium and the Latin West.

1.1.4 The Islamic Near East

The transmission and integration of foreign astronomical ideas into the newly formed Islamic Empire is intensely complicated,[18] given the multifarious astronomical systems that were actively being maintained throughout Eurasia up until this time. As early as the eighth century CE, ideas were introduced from India into the Arabic tradition through translations of Pahlavī and Sanskrit sources. Most of these ideas were not original to India or Iran, but were ultimately Greek in origin. However, these theories had undergone significant modifications at the hands of their recipients, so that they could be thoroughly amalgamated into the existing tradition to the extent that in many cases they

took on a distinctly new form. More importantly though, the type of Greek astronomy received by Indian scholars was non-Ptolemaic.

As a result of passage through India, there arose a system boosted by superior mathematical techniques such as trigonometry and methods of approximation; a double epicyclic system synthesized out of original disagreements between Greek mathematical astronomers and peripatetic philosophers from the first and second centuries; and an array of new parameters based on critical conjunctions separated by massive lengths of time from which mean motions could be extracted. In addition, as a result of passage through Iran, Greek ideas became distinctly Sassanian. The resulting adaptation was a blend of aspects from Greco-Iranian astronomy with Ptolemaic concepts that were introduced and developed by scholars such as al-Fazāri and Ya'qūb ibn Ṭāriq.[19]

As well as Greco-Indian and Greco-Iranian influences on Islamic scholars, they also received Greek theories from additional channels and in different forms, notably in the late eighth century (the Greco-Syrian tradition with a Ptolemaic orientation) and then finally again during the ninth (the Byzantium tradition of Ptolemaic theories). The motivation for these further influxes was the presence of these seemingly incompatible Greco-Indian and Greco-Iranian traditions. The inherent inconsistencies of these many proposals presented the Islamic scholars in the 'Abbāsid period with some immediate challenges. The fundamentally irreconcilable models, with internal discrepancies and undeniable inaccuracies, caused them to search for a fix. In light of this situation, 'Abbāsid rulers instigated an ambitious plan, commissioning translations of Greek and Syriac texts to modernize and reform astronomy with ancient ideas. This phenomenon, known as the translation movement, saw texts not limited to the exact sciences, but indeed all accessible nonhistorical, nonliterary texts being translated from Greek into Arabic.[20] Of prime importance to astronomy was the translation of Ptolemy's *Almagest* and of the *Handy Tables*. The careful study and mastery of these two works allowed scholars by the late ninth century to thoroughly and completely overhaul all existing incompatible traditions, by subscribing to the Ptolemaic model from Ptolemy directly.

These efforts and the embracing of the Ptolemaic system as central led to the emergence of a new type of mathematical astronomy, one that was in essence based on Ptolemy, but with inconsistencies identified and ironed out, new features added, and the latest mathematical techniques incorporated. Al-Battānī (868–929) represents the culmination of this project; his work epitomizes a grasp of Ptolemaic theory improved by more powerful means of computation, with the trigonometry begun by Indians and continued by Islamic mathematicians such as Ḥabash al-Ḥāsib (d. 864/74), and updated via

contemporary and increasingly precise observations. Other systems simply deteriorated faced with the potency of this model. Mathematical astronomy at this point represents a facility and a command over theory and observation that was to set the standard for modern western astronomy.

The spread of the Islamic empire meant that these ideas had an impact on a significant number of scholars. Further challenges to the astronomical model by Islamic scholars came not from technical objections, but rather from broad philosophical, physical, and theological concerns. Of particular contention was the clash between Ptolemaic planetary theory and Aristotelian physics in the context of which was closer to Islamic theological doctrine. The principal concern was the desire to restore the concentricity of planetary orbits. In particular, Islamic scholars harbored serious reservations about the metaphysical consistency of the deferent. In response, practitioners such as al-Biṭrūji (fl. ca. 1190) experimented with mathematical models to restore concentricity of planetary motions, and others in the school of Marāgha (such as al-Ṭūsī [b. 1201], Ibn al-Shāṭir [b. ca. 1305]) devoted themselves to the project of devising ingenious equivalent geometrical mechanisms so that the Ptolemaic deferent, so objectionable indeed, could be discarded entirely.

Altogether, the early history of mathematical astronomy is a complex interplay between cultural contact and exchange, and the transmission, integration, and subsequent synthesis of ideas. Thus, it is essential to an understanding of the development of eclipse theory to trace the transmission of scientific ideas from one culture to another. It seems a daunting, if not near impossible, task to follow this complex web of interactions, given the state, condition, and in some cases scarcity of the extant sources, as well as the varying standards of attribution, acknowledgment, and referencing in early cultures. However, by virtue simply of their being mathematical, one is able, to a large extent, to detect and trace these scientific exchanges; as one of the foremost champions of such work, David Pingree, gently encouraged, "fortunately the nature of an advanced astronomical theory is such that it cannot be altered beyond indisputable recognition,"[21] and in most cases, despite the lack of formal acknowledgment or explicit connection, parameters and coefficients, theories, and models can be traced right back to their originators.

The injection of foreign ideas into a scientific climate distinct from that of their inception had some dramatic effects. The Greeks were considering the very concepts that the Babylonians lacked in order to improve their understanding of eclipses. In turn, the scholars of the Islamic Near East were able to stand back and critically analyze the details and weaknesses of the Greek model, in a way beyond the ability of the fathers of these concepts, given the sheer enormity of establishing the first cohesive theoretical account. In

India, the attention to iterative and other computational techniques was of significant importance to accuracy in establishing key instances and intervals of eclipses. In this context, then, this study will attempt to trace the ways in which knowledge about eclipses was originated, transmitted, and developed by these early cultures.

1.2 Eclipses and Their Human Significance

Most cultures endow whatever phenomena they investigate with human significance. From earliest times, eclipses seem to have been considered ominous. In the ancient Near East, they were a significant feature in divination literature. Such literature was generated as a result of the conviction that all natural phenomena were ominous—"signs" sent as messages from the Gods to humankind, foretelling future events. While these signs could foretell good outcomes as well as bad, eclipses were almost always considered to portend evil. They might foretell the downfall of an empire, the spread of disease, danger to the king and his subjects. The various features of each particular eclipse, such as direction of impact and clearing, duration, and color, could generate further details as to who was to be affected and who was to be spared these effects. However, the effects of these signs were not considered unavoidable, but, with the necessary rituals and preparations, could be averted. Given this fact, the motivation to be able to predict eclipses was strong. If one could predict the occurrence of an eclipse, one could prepare in advance the necessary counter to the ill effects. Thus, there was a strong incentive to predict eclipses, as with all celestial events.

The aspirations to predict these events along with the fundamental assumptions underlying the practice of divination resulted in an interesting juxtaposition of conviction. The belief that a pantheon of seemingly impetuous Gods sent signs that revealed the future seems to rest rather uncomfortably with the requirement that these events are regular and recurring, and hence predictable. Fascinatingly, the peoples in the ancient Near East, and those beyond, maintained the conviction that these events were ominous, despite the fact that they had an evident appreciation of the patterns of their occurrences.

The effect of the awareness of these eclipse periods and the success of predictive mathematical models on this conviction was a long time coming in the history of the astral sciences. A serious reconsideration of whether eclipses truly are ominous was certainly neither a natural nor obvious way for intellection to progress. In fact, the ancients continued to deem these events ominous long after they had a clear understanding of their periodicities. Perhaps this explains why the ancient Near Eastern thinkers did not attempt to infer the

actual celestial mechanisms or develop the cosmological model beyond a very primitive, mythologically inspired conception.

However, even those cultures with sophisticated cosmological pictures of the heavens continued to maintain the ominous significance of celestial events. In ancient Greece, Ptolemy's seminal work the *Almagest* often describes the computations of features with little application beyond astrology. In addition, Ptolemy wrote an entire work, the *Tetrabiblos*, dedicated to horoscopic astrology. In India, notable practitioners, such as Brahmagupta and Bhāskara, carefully balance the need to maintain the traditional ominous significance of celestial phenomena within a developing, more empirically rigorous, cosmological picture. This tension between tradition and innovation also manifests itself in India in the *virodhaparihāra* (literally "removal of contradiction") tradition which appears most directly in the second half of the second millenium.[22] Increasingly, a subtle transfer of interest did manifest itself in those practitioners who observed and predicted eclipses and speculated on their causes, despite the pressure on them to maintain the original incentives for these investigations. It was this tension which was never to be resolved in the ancient Near East, and troubled many practitioners for generations to come.

1.3 Observation and Theory

> The sciences and especially the natural sciences and mathematics seem to be theoretical subjects in and of themselves. They arose when Greek theoretical traditions replaced the empirical traditions of the Babylonians and the Egyptians.
>
> —Feyerabend, *Problems of Empiricism* (1981)

> Mathematical theory played a major role in Babylonian astronomy as compared with the very modest role of observations.
>
> —Neugebauer, *The Exact Sciences in Antiquity* (1957)

One obstacle common to all early cultures that sought to improve their reckoning of eclipses was that brought about by observational limitations. Eclipses are visually remarkable, but their particular features can be difficult to record to detailed levels of precision. Using methods of modern computation, historians can "retrocast" the positions of the luminaries and thus the dates and circumstances of eclipses reported in early sources, and from that can gain a solid notion about those features which early peoples noticed and how they interpreted them.[23]

There were many challenges for the earliest cultures seeking to establish patterns and predictions about the frequency and circumstances of eclipses. Their first and only source of information was to make and catalog observations themselves, as they occurred in the calendar, and take note of their particular features. Arguably one of the oldest empirical realizations was the fact that lunar eclipses generally occurred six months apart, with the occasional five-month interval. This insight itself is remarkable, given that other circumstances disturb this pattern, such as visibility conditions, luminaries being below the horizon, and so on. After the identification of their frequency, different aspects of an eclipse such as time, duration, and amount of obscuration were recognized which required a higher level of inspection.

1.3.1 Mathematical Astronomy

From a modern perspective, the practice of astronomy is grounded in observations of celestial phenomena that are methodical, systematic, and continuous. Tentative conclusions are drawn from empirical data, and then are tested directly and under modified circumstances. Efforts are directed toward increasingly successful predictions and a more detailed, harmonious, and complete understanding of celestial objects. History, however, causes us to reevaluate this point of view. From their earliest beginnings, the astral sciences built themselves upon a staggeringly small number of observations. From this small set, almost immediately, a mathematical structure was established which was to be little revised as a result of further observations.

The security the mathematical models brought and the reluctance to tamper with them are understandable in these early societies. Not only did the intrinsic difficulties related to observation encourage this, but also the the very type of predictive scheme maintained somewhat of a false sense of success and underplayed some of the inherent cumulative inaccuracies in these mathematical extrapolations. These earliest forms of predictive scheme were based on period relations, in which various repeating cycles of phenomena were completed (or approximately completed) across known intervals of time. In fact, workable period relations can be established remarkably quickly—in a matter of decades for many celestial phenomena. These period relations seem to be fairly accurate because the inherent error pertaining to particular observations is distributed over the entire interval. Aberrations in individual instances of the phenomena that the model predicted would have been deemed trivial, when the mathematical model could accurately predict periodic recurrence. Indeed, for the ancient observers, it was impossible to establish with confidence whether the difference was a theoretical inadequacy or an observational fault.

It is little surprise then that those cultures whose fundamental period relations spanned massive amounts of time (India, for example, with its system based on the *Mahayuga* cycle of 4,320,000 years) had the poorest record of observational astronomy in their tradition. Indeed, the predisposition to mathematical modeling over continuous systematic observational programs in the history of astronomy can be understood in this light. Mathematics provided a welcome relief from the exigencies of observation and the lack of technology. Early astronomy therefore is rightly called *mathematical astronomy*. It was not until the introduction of the telescope in the seventeenth century that astronomers would commit themselves to systematic continuous observational programs that would have them reevaluating their theories.

Nonetheless, a large number of astronomers made observations and the extant records are full of them.[24] In the ancient Near East, eclipse predictions did not get any more accurate, but were given with greater precision, which suggests that revisions to the models were based on mathematical relations and computational concerns.[25] In ancient Greece, one is immediately impressed by the fact that the resulting models, such as those expounded in Ptolemy's *Almagest*, proved to be very powerful, but the numerical parameters for these models were too much based on a small number of carefully and thoughtfully selected observations. Ptolemy and others realized that there were multiple causes for the apparent motions and positions of celestial bodies and their proposed models reflected this; indeed, to distinguish these effects could only be achieved theoretically. In the case where there were two primary variables responsible for the perturbation in a position, it was well understood by Ptolemy that in order to be able to get a useful observational datum to apply to the model, one needed to observe the position of the body when one of the two variables was zero, hence eliminating it.

The detection of the effects of parallax and the subsequent attempts to account for it represent a fascinating example of the complex relationship between observation and nascent theorizing in early astronomy. The effects of parallax are only just large enough to be noticeable, although parallax's theoretical elucidation is remarkably simple when considered from a geometrical perspective. However, the angles involved are simply too small to be measured reliably and can only be determined with a marginal degree of accuracy. Only lunar parallax could be gauged with any accuracy by early inquirers. The practical challenges to measuring the effects of parallax directly are substantial, and in place of suitable measuring equipment, theoretical considerations were the basis for its determination. Here again, there is the crude detection of an aberration, but the precise effects are beyond the astronomer's abilities to measure directly. In place of reliable empirical input, the effects are instead determined

and refined by recourse to theory. Horizontal (or maximum) parallax was derived theoretically. Indeed, the importance of this slight deviation is most dramatic in eclipses. The effects of parallax can influence the heavenly alignments that bring about eclipses. They are significant enough that a predicted solar eclipse can instead pass, and one can occur that had not been predicted.

1.3.2 Observational Astronomy

The practical challenges for the early astronomers were enormous, particularly when it came to the technicalities of observation, and they were well aware of these obstacles. These challenges included those related to the method of observation and the equipment available for it and those related to the conditions under which the phenomenon is to be observed. Among the first category are included limitations of the human eye, nonstandardized forms of measuring equipment, the imprecision of timing devices such as water clocks, the fuzziness of the tip of a shadow cast from a gnomon, the finite diameters of slits and pins at instruments, and the difficulty of estimating very small angles,[26] some effects being so small that they are impossible to measure directly or with precision, such as parallax, solstices, and stations and first and last visibilities of planets. Examples in the second category include weather, refraction, geographical challenges, and the reliability of testimony.

It was common enough for the ancient astronomers to acknowledge certain inaccuracies or imprecision in observation, but the ways in which they differentiated (if at all) between the various types of error differed enormously. Measures to address these inaccuracies (if any) might be discussed on a broad scale, but the active determination to differentiate and address these in practice was much less frequent. The astronomers were not in a position to process such often subtle discrepancies. To a large extent, where challenges to observation presented themselves, practitioners sought to overcome the obstacle by means of recourse to theory, that is, they compensated for observational deficiencies by theoretical inferences. So, ironically, observational inadequacies promoted reliance on theory. After the initial connections of astronomy to observation, its developments and modifications very quickly became theoretically driven. Astronomy did not significantly reconnect to its observational basis in any profound way until the rise of modern science, millennia later.

Measures to minimize the inaccuracy of observations were taken in several ways. Most frequently advocated by ancient authors is the repetition of observations. However, this is of little practical advantage in overcoming observational imprecision, and, furthermore, it was not a practice that was actually embraced by those very individuals advocating it. As argued in the previous section, prominent practitioners based their parameters on a very small

number of carefully selected observations. Another approach was to take observations that were significantly distanced in time. For example, Ptolemy frequently advocates the importance of taking observations separated by a large interval.[27] This is ingenious: the accuracy of observations becomes less important because imprecision will have a negligible effect when spread over a period of many centuries. In many cases, observations separated by so much time could be tolerated with up to an entire 1° of imprecision; for the most part, this degree would have been detected only in the fourth or fifth sexagesimal place when divided out. Ptolemy is not unaware of this as he frequently chooses to express parameters to the number of sexagesimal places to which these variations go unregistered. Several practitioners may be responding to the false sense of accuracy mathematical results can produce when they give upper and lower estimates on parameters, rather than an exact figure. For example, when Aristarchus gives his reckoning of distances of the luminaries and the earth, he calculates that the distance of the sun from the earth is greater than 18 but less than 20 times the distance of the moon, and he gives the diameter of the sun to earth as a ratio greater than 19/3 and less than 43/6.

Another consequence of an astronomical system that relies heavily on mathematical-astronomical techniques is the subtle yet profound distinction between accuracy and precision.[28] The extent to which this was recognized in the ancient world is still largely undetermined, although, to be sure, practitioners such as Ptolemy certainly invoked the distinction. Terminology suggests that the difference was maintained, although not strictly. Ptolemy frequently refers to his results being *engista* or "very nearly" (*Alm.* X, i iii 299.2) to emphasize the approximate nature of his parameters. It seems that a distinction was made occasionally in Greece with respect to results that were *akribeia* (accurate) versus those that were *orthos* (correct) or *alētheia* (true).[29]

As well as a recognition of the limitations of observations, there were also reservations regarding the dependability of data collected from observations, and such worries shaped the practice of astronomers in different ways. Given the infancy of the astronomical tradition and the extent of the technical challenges to be overcome, there were times when practitioners grappled with somewhat extraordinary reports, that is, accounts that were wildly out of kilter with expectations. The primary difficulty lay in whether these accounts should be accepted and thus cause the renovation of an otherwise satisfactory model or whether they should be dismissed as the result of a "technical" failure.

For example, in a telling excerpt, the late second-century CE Hellenistic astronomer Cleomedes grapples with the possibility of observational error versus a bona fide observation. In his work *Caelestia*[30] he recounts testimony of a so-called paradoxical eclipse—to accept this account as accurate would

require much readjustment to established principles. This paradoxical event involved a lunar eclipse which was said to have been observed when both luminaries were *above* the horizon—a seemingly impossible event with respect to the understandings of Cleomedes' day, because a lunar eclipse is caused by the moon entering into the shadow cast by the earth from the sun, requiring them to be strictly at opposition. He confesses:

> Lunar eclipses are such as we have demonstrated. But statements made about paradoxical eclipses seem to contradict the theory that establishes that the Moon is eclipsed by falling into the shadow of the earth. For some say that a lunar eclipse occurs even when both luminaries are observed above the horizon . . . If this theory is correct, we shall have to lay claim to a different cause for the Moon's eclipse. (*de Caelestia*, as translated by Bowen and Todd [2004] pp. 159ff)

Many issues with respect to observation are raised in the course of Cleomedes' lengthy discussion of this issue. He reflects upon various proposals made by his predecessors to deal with this paradoxical eclipse, and, before discussing other possible resolutions, he notes that one of his options is to completely reject the validity of this claim:

> We must confront [proponents of paradoxical eclipses] initially by claiming that this theory is fabricated by certain people who wish to impose a problem on the astronomers and philosophers who are engaged with these matters. For numerous lunar eclipses have occurred both total and partial, and have all been recorded, yet nobody is reported as having recorded this type of eclipse, at least up to our lifetime—no Chaldean, Egyptian, or other scientist or philosopher. The claim is just a fabrication. (*de Caelestia*, as translated by Bowen and Todd [2004] pp. 160–61)

He then explains the impossibility of this type of eclipse occurring, given the accepted understanding of the relative positions of the luminaries. However, he does not discount other factors, and with uncanny foresight speculates that this account may be the result of an atmospheric effect. By analogy, Cleomedes discusses the recognized effects of refraction and elementary laws of optics. He relates, as an example, how the image of a gold ring being placed in a cup is distorted and suggests similar effects in this case. He wraps up his discussion with the decisive: "Still, that the Moon is eclipsed *only* by falling into the shadow of the Earth is a cognitively reliable (*enarges*) [conclusion] derived from the phenomena" (*de Caelestia*, as translated by Bowen and Todd [2004] p. 163). The Greek term *enarges* is significant in revealing a dependency on theorizing; its use here appeals to a conclusion based on the priority and reliability of cognitive inference over *visual* evidence.[31]

Further insight into the relative priorities of observation and theoretical insight among ancient inquirers is given by Ptolemy:

> It is necessary to give priority to the [astronomical] phenomena over [data] from records of travel . . . most intervals, however, and especially those to the east or west, have been reported in the crude manner, not because those who undertook the researches were careless, but perhaps because it was not yet understood how useful the more mathematical mode of investigation is, and because no one bothered to record more lunar eclipses that were observed simultaneously at different localities. (Ptolemy's *Geography* I, 4, as translated by Jones [1990] pp. 62–63)

Ptolemy's tolerance for a discrepancy between data and theory with respect to the lunar diameter is often raised by modern scholarship. The lunar model of Ptolemy is often subject to criticism because, although it predicts very well the moon's motion in longitude and latitude, it brings the position of the moon dramatically too close to the earth, to the extent that the moon, by his reckoning, should appear twice the size that it does at quadrature. This fact Ptolemy remains silent on, as do later practitioners in this tradition. Perhaps a great dependence on theory over empirically derived information can explain this discrepancy. Ptolemy's drive to maintain his theory was very strong and persisted in the face of some compelling evidence. Theory, it seems, would take precedence over observation.

1.4 Aims of This Study

This study intends to confront the interconnected set of developments across various early societies and diverse research cultures, to reveal the historical complexity of human inquiry into eclipse phenomena. Despite knowledge transaction between cultures, and despite in limited respects an accumulation of skills (in fact, on many occasions various cultures reversed rather than furthered significant elements), different cultures furthered intellectual change in distinctive ways. This is a hallmark of early scientific undertakings. Therefore, in order to fully account for such trajectories, this investigation will be carried out through a critical and detailed examination of the primary sources themselves.

The original texts for each selected early society will be analyzed so as to gain insight into the ways in which individuals themselves practiced, and then the efforts of these individuals will be set against those of their contemporaries within that society to give a broader portrayal of general cultural trends. Further, primary source material will be subject to deliberate cross-cultural

comparison—differences and similarities will be noted cross-culturally where appropriate, to produce a picture of the overall industry in early times. This course of study by necessity devotes careful attention to linguistic details, to the various symbolic forms of reasoning available and how they affect discourse, to interpretative and critical aspects of the text, and even at times to relevant paleographical, philological, and artifactual particulars, and thus can capture in an essential sense the sorts of pressures, motivations, and reasons behind the practices of early peoples. This sort of primary-source based study is crucial and must precede more general reflection.

Ultimately, a study such as this presents an opportunity to step back from all the particulars and complexity of detail, and to reflect upon a cultural unfolding of a more profound nature. Organized deliberation concerning eclipses is emblematic in its development of the emergence of an ever more theoretical reckoning about heavenly phenomena. Beyond this, astronomy had the effect of drawing inquirers further to consider a reality behind the empirical facts. This sentiment was not lost on ancient thinkers—astronomy's capacity to serve as a directive force was noticed by many of them. Plato was not the first to insist that astronomy draws the mind upward to deeper contemptation.[32] How this attitude manifested itself practically through history will now be considered in the context of eclipse phenomena, arguably the best known heavenly occurrence to compel peoples to indeed set their sights aloft.

CHAPTER TWO

The Rudiments of Alignment
Basic Eclipse Theory

2.1 The Moon and the Sun

Perhaps the single most prominent feature of the night sky is the disk of the moon. With its waxing and waning at such short recurring intervals, even the most casual observer cannot but notice and appreciate its periodicities. The circuit of the moon can be envisioned as (approximately) a great circle on the celestial sphere, a huge spherical surface enclosing the earth onto which all heavenly bodies are projected. The earth is centrally placed in this sphere and can be imagined as stationary (fig. 2.1). With the earth thus fixed, the celestial sphere makes one giant rotation in a *westward* direction, that is from east to west, every 24 hours, carrying with it all its contents. In terms of how the sky *appears*, it is of no consequence whether the earth is considered to rotate on its axis or whether the celestial sphere revolves in the opposite direction daily, as astronomers believed in early times. In fact, there is a certain convenience to the latter. Observers reckoning themselves situated on the earth gazing outward to an infinitely large enclosing dome capture their situation more readily and naturally. Indeed, even today those who use astronomy for practical means (such as navigation, surveying, orienteering, and so on) routinely use this perspective. A further advantage of such a conception is that it can capture directions and relative positions of celestial bodies, without the need to consider their distances from the earth. For an observer on the earth, only half of this sphere and its contents is visible at any instant, and to demarcate this sphere, the terrestrial equator can be projected onto the celestial sphere, to give the celestial equator, and the terrestrial poles extended to give the celestial poles.

In addition to the apparent daily motion of the celestial sphere, the sun, the moon, and the planets have additional motions (fig. 2.2). The sun travels in the celestial sphere on an orbit slightly inclined to the celestial equator in an

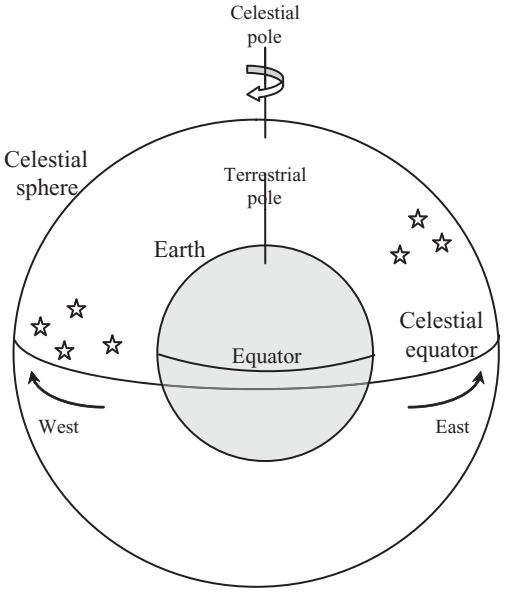


Figure 2.1 The celestial sphere.

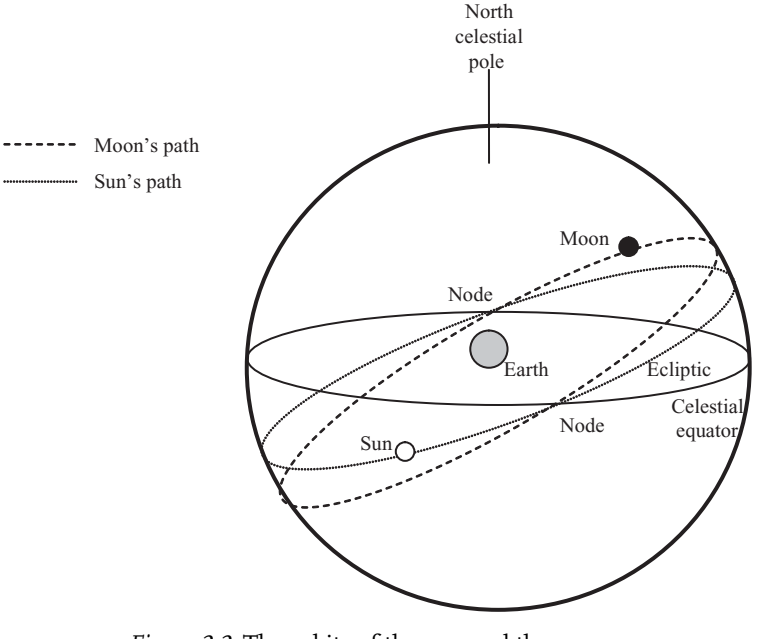


Figure 2.2 The orbits of the sun and the moon.

eastward direction, that is from west to east, with the fixed stars as its backdrop. This path is commonly known as the ecliptic and is inclined at a little under 24° with respect to the celestial equator. This circuit takes the sun about a year to complete, producing an apparent daily motion of about 1° per day. The moon travels around its orbit which is inclined to the ecliptic a further 5° from the celestial equator. It moves more swiftly than the sun, at roughly 13° per day, and will return to approximately the same position on its circuit after approximately 27⅓ days. This interval is known as a *sidereal* month.

Systematic understanding of the periods and phases of the moon is intimately connected with the sun. Due to this fact, and because of the inclinations and varying intersections of the individual orbits of the luminaries, there are, in addition to the sidereal month, three further significant intervals related to the moon. As the moon travels around its orbit, it changes its position with respect to the sun, which causes different parts of its orb to be illuminated (figs. 2.3 and 2.4). This is the cause of the familiar waxing and waning effects or the growth and decay of the crescent.

When the moon is at *conjunction*, it is exactly between the earth and the sun with the nonilluminated side facing the earth. When it is in this phase, also called the *new moon*, it cannot be seen. Next, as the moon gradually creeps around its orbit, the crescent appears. It is the appearance of this first crescent that for so many early cultures indicated the first day of the month. Gradually the thin crescent swells until it has the appearance of a half circle. This phase is called the *first quarter*. It occurs when the moon is very nearly 90° away from

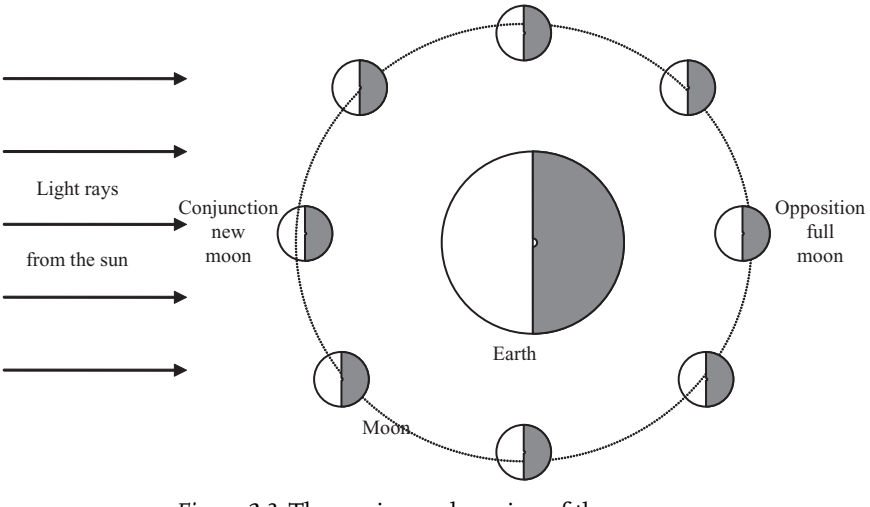


Figure 2.3 The waxing and waning of the moon.

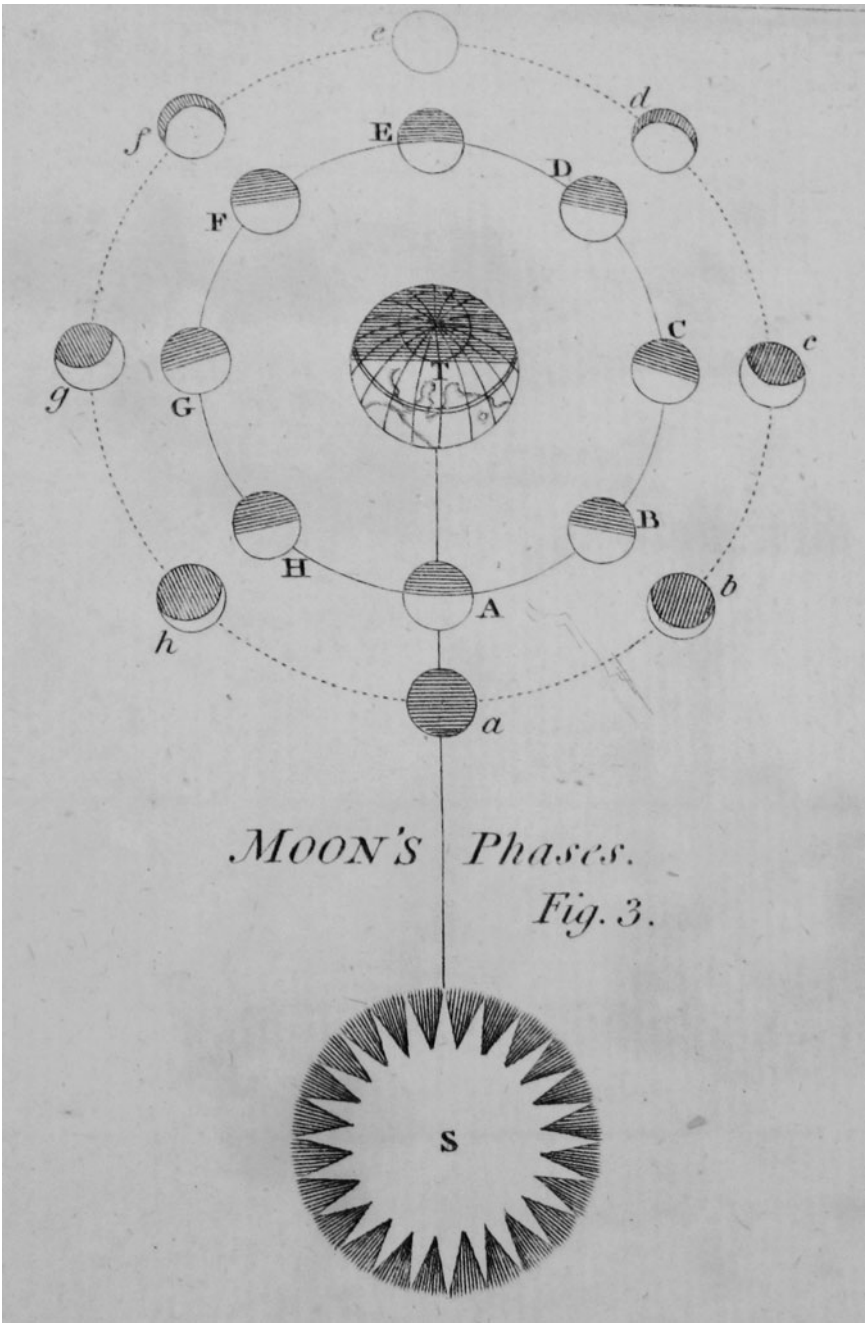


Figure 2.4 The moon's phases from Charles Hutton's *A Mathematical and Philosophical Dictionary* (London, 1795). Photo used with the kind permission of the University of Canterbury Library, New Zealand

the sun. At 90° it is in its first or *eastern quadrature*. It continues to swell, going through its *gibbous* phase until it reaches a point diametrically opposite the sun so that its illuminated half is facing the earth. This is *opposition*, also called the *full moon*, when the moon appears as a full round circle. It then continues, reentering its gibbous phase, reaching its *last quarter*. Again it becomes a half circle at *western quadrature* and then continues to diminish until the crescent disappears entirely and it is at new moon once again. This whole process is known as the *synodic cycle*.

The interval it takes the moon to travel from conjunction to conjunction is known as the *synodic* month or one lunation. This interval is on average a little over 29½ days. The terms conjunction and opposition can also be referred to by the common term *syzygy* from the Greek word *súzugos* meaning "yoked together."[1] At conjunction the moon is always invisible, except for the one situation in which the moon is eclipsing the sun.

Phases of the moon
Conjunction or new moon
First visibility
First quarter
Opposition or full moon
Last quarter
Last visibility
Conjunction

Because the moon's orbit is inclined to the plane of the ecliptic by about 5°, there are two distinct points of intersection. These are called the (lunar) nodes and are differentiated by the orientation of the progress of the moon through them; when the motion of the moon is from south to north, this node is the *ascending* node and when the motion is from the north to south, *descending*. The nodes however do not remain fixed against the backdrop of the stars in the celestial sphere; they too rotate, in a retrograde (or backward) fashion, by a little under 2° per month. This gives rise to another important interval, the *nodical* month (also known as the *draconitic* month), that is, the time it takes for the moon to travel from one node to the same node, a little under 27⅓ days.

The fourth and final significant lunar interval is the *anomalistic* month. The speed at which the moon moves around against the backdrop of fixed stars is variable (or, in other words, its orbit around the earth is really elliptical). It moves most slowly when it is farthest away from the earth (its *apogee*, literally "away from the earth") and fastest at its closest point (its *perigee*, literally "near the earth"). However, the moon's perigee is not fixed with respect

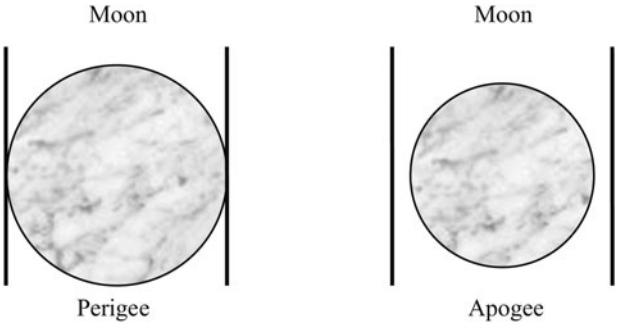


Figure 2.5 The variation in angular diameter of the moon.

to the celestial backdrop, but rotates eastward a little over 3° each month (it takes about nine years to complete a full rotation). The interval from perigee back to perigee, that is from fastest speed to fastest speed, is the anomalistic month.

The four different intervals with their modern day lengths are as follows:

Intervals Related to the Moon

Draconitic month	27.2122 days
Sidereal month	27.3216 days
Anomalistic month	27.5546 days
Synodic month	29.53059 days (average)

The diameter of the sun is 1.4 million kilometers; its angular diameter ranges from 32.7 to 31.6 minutes of arc. The moon has an actual diameter of 3,476 kilometers and its distance from the earth varies between 356,000 kilometers (perigee) and 406,000 kilometers (apogee). Because of this variation, the angular diameter of the moon will appear greater at perigee and smaller at apogee (fig. 2.5). Its angular size ranges from 33.5 to 29.43 minutes of arc.

2.2 Eclipses

The eclipse of a celestial body[2] happens whenever that body becomes obscured, whether totally or partially, for one of the following two reasons:

- the celestial body travels into the shadow of the earth (or some other celestial body);
- the rays that normally reach the earth's surface from the sun (or some other celestial body) are obscured by an additional celestial body.

Indeed, many celestial bodies in the sky can be eclipsed or eclipsers (for example, eclipses of the satellites of Jupiter, or one planet eclipsing another); however, the most dramatic eclipses in the visible celestial sphere are always those that involve the sun, the moon, and the earth. Because of the proximity to the earth of the moon and the sun, the intensity of the rays of the sun, and the familiarity of the orbs of the sun and moon to even those without an interest in the heavens, eclipses have always been events that have had ominous significance and impacted various cultural and social practices.

Lunar and solar eclipses occur as a result of the alignment of the three bodies of the sun, the moon, and the earth. Lunar eclipses are possible only at full moon, and solar at new moon (fig. 2.6). If there were no inclination between the lunar and solar orbits, eclipses would occur at every syzygy. However, because of it, at the majority of conjunctions and oppositions, the moon is at a sufficiently removed distance from the plane of the ecliptic that it continues its motion without obscuring or being obscured. It is when the moon is close to a node at a syzygy that an eclipse is possible. This is the distance that determines the eclipse limits (fig. 2.7), the outer limits at which an eclipse can occur.

In the case of a lunar eclipse, these circumstances occur only every six (or sometimes five) months. This is a direct consequence of the differing lengths of the draconitic and synodic months. The six-month reccurrence for lunar

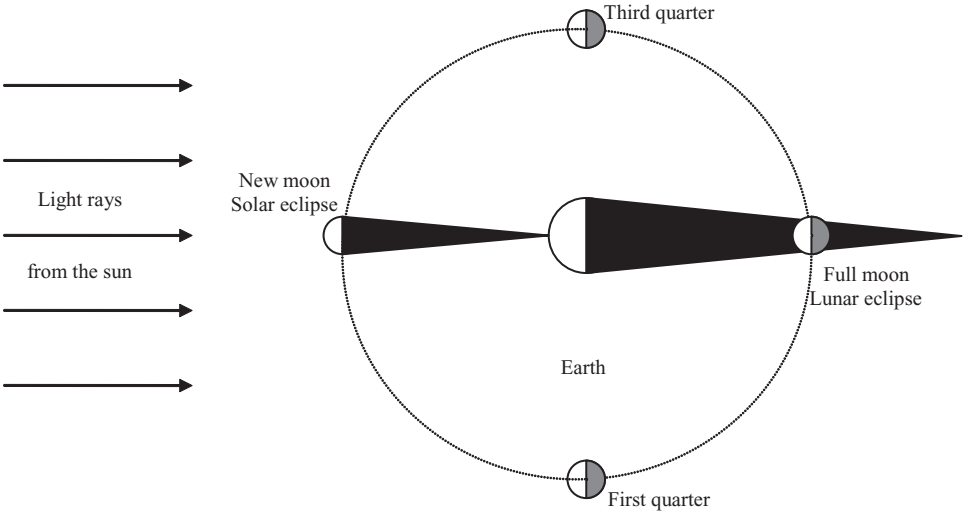


Figure 2.6 The alignments of the sun, moon, and earth to produce a solar and lunar eclipse.

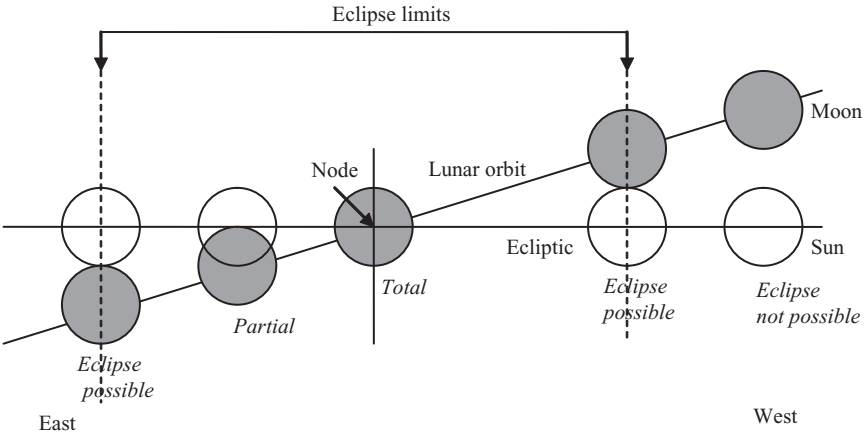


Figure 2.7 Eclipse limits.

eclipses is an interval small and regular enough to be ascertained empirically. As a rule-of-thumb method, one could look for an eclipse six months after the last eclipse possibility (EP). This simple technique could have been used by many early cultures, and indeed there is evidence it was used frequently in the ancient Near East, for example.

For fully successful modeling of both lunar and solar eclipses, there must be a consideration of the following basic prerequisites:

- basic parameters for the (mean) motions of the sun and the moon
- rates of nodal elongation
- information about the inclinations of the orbits
- sensible parameters for the diameters of the bodies in question
- a recognition of the effects of parallax (for solar eclipses)

Due to the low accuracy of measuring equipment in the ancient world, it was not possible to obtain parameters such as the mean velocities of celestial bodies in a short amount of time. Instead, large time intervals, in which a whole number of complete repetitions of the phenomena in question occurred, would be divided up to produce parameters for the (mean) motion over smaller intervals of time, such as daily motion.[3]

2.2.1 Lunar Eclipses

As we have seen there are two types of eclipse that can take place, one when the moon is at conjunction, the other when it is at opposition. At the latter,

a lunar eclipse occurs. When the moon is 180° away from the sun and its latitude is very small, it, the earth, and the sun will all be on (or close to) the same straight radial line. The earth will block the rays of the sun that normally reach the moon, depriving it of light, or, in other words, the earth creates a shadow which the moon enters, hence creating the eclipsing effect (figs. 2.8 and 2.9). There are two distinct parts to a shadow caused by any nonpointlike light source like the sun. The first is the *umbra*, which is the area of total shadow, that is, a dark cone-shaped zone in which no part of the light source is visible. However, there is also a secondary fanlike region that is less dark, called the *penumbra*, where the light is not completely cut off by the eclipsing body.

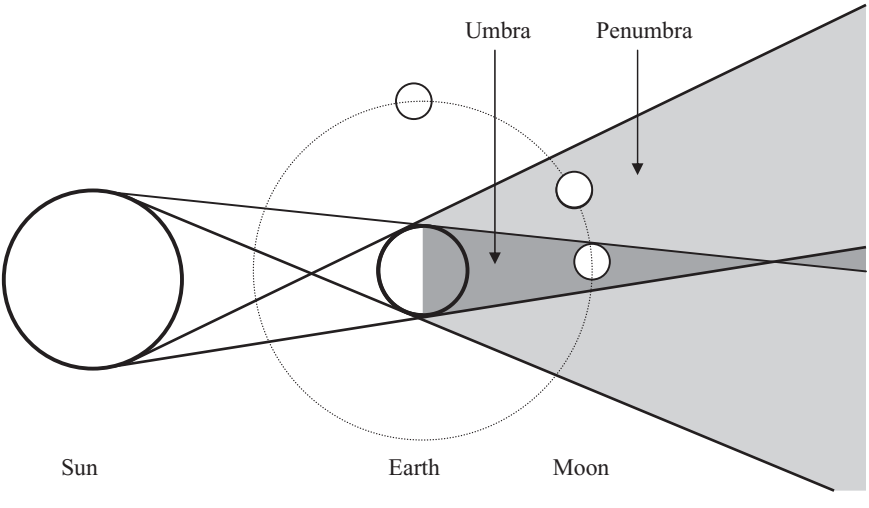


Figure 2.8 A lunar eclipse.

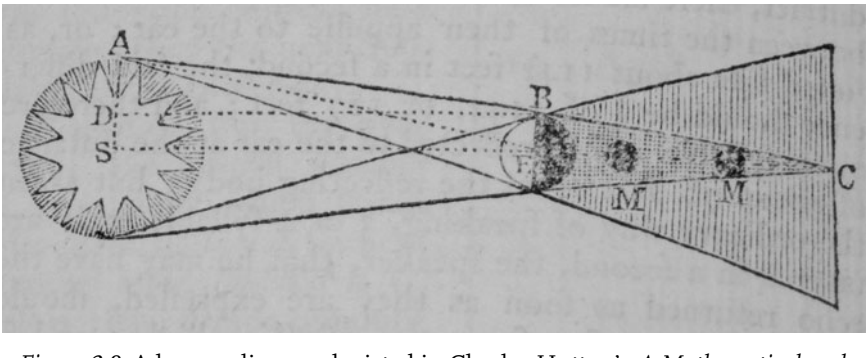


Figure 2.9 A lunar eclipse as depicted in Charles Hutton's *A Mathematical and Philosophical Dictionary* (London, 1795). Photo used with the kind permission of the University of Canterbury Library, New Zealand

A lunar eclipse is visible from all vantage points on the earth where the moon can be seen (i.e., is above the horizon), and is not restricted to particular latitudes on the earth as in the case of a solar eclipse. For an observer in a single locality, roughly half of all lunar eclipses can be observed, weather permitting. These features greatly enhanced the success in predicting lunar eclipses in ancient times.

Because of the eastward direction of the moon's motion, a lunar eclipse will always begin from the western edge of the shadow (the eastern edge of the lunar disk will be eclipsed first) and clear in the east (the western edge of the lunar disk will be the last to be immersed in the shadow). However, depending on the orientation of the node at which the eclipse occurs (i.e., ascending or descending) it can be at the southern edge of the shadow or the northern. As will be discussed, this angle of impact (or associated angles) was considered significant for astrological reasons and was commonly calculated in astronomical treatises.

In a lunar eclipse, the moon can become fully immersed in the earth's shadow. This is called a total eclipse, and the time during which the moon is fully immersed in the earth's shadow is known as the duration of totality. During a total eclipse, the disk of the moon never disappears completely, for a small amount of sunlight is bent and flows into the umbral shadow, catching the disk, albeit dimly. Owing to the refractive nature of the atmosphere, the red tones of the visible spectrum are more easily passed and the blue tones are scattered. This causes the disk of the moon to appear ruddy red. Colors associated with lunar eclipses have been described variously—from bloodred to brownish-ash and all hues in between. The change in color from white to red (and perhaps the shades in between) of the disk of the moon motivated many ancient cultures to attribute astrological meaning to a variety of colors of the disk, many of which are impossible.

A second type of eclipse is when the moon is close to but a sufficient distance away from the node at opposition so that it simply grazes the earth's umbral shadow and is only partially obscured. This is known as a partial eclipse. A third type of lunar eclipse occurs when the moon travels only through the faint penumbral shadow cast by the earth, missing the darker umbra. This type of eclipse is less noticeable—at least half of the moon must enter the penumbra in order for the slight dimming of the disk of the moon to be detectable. For this reason, these eclipses were not recognized in early times. Figure 2.10 illustrates the three different types of lunar eclipse.

There are five significant phases of a total eclipse of the moon: first contact, beginning of totality, mid-eclipse, end of totality, and release (figs. 2.11 and 2.12). First contact is the instant the moon enters the dark shadow cast by the

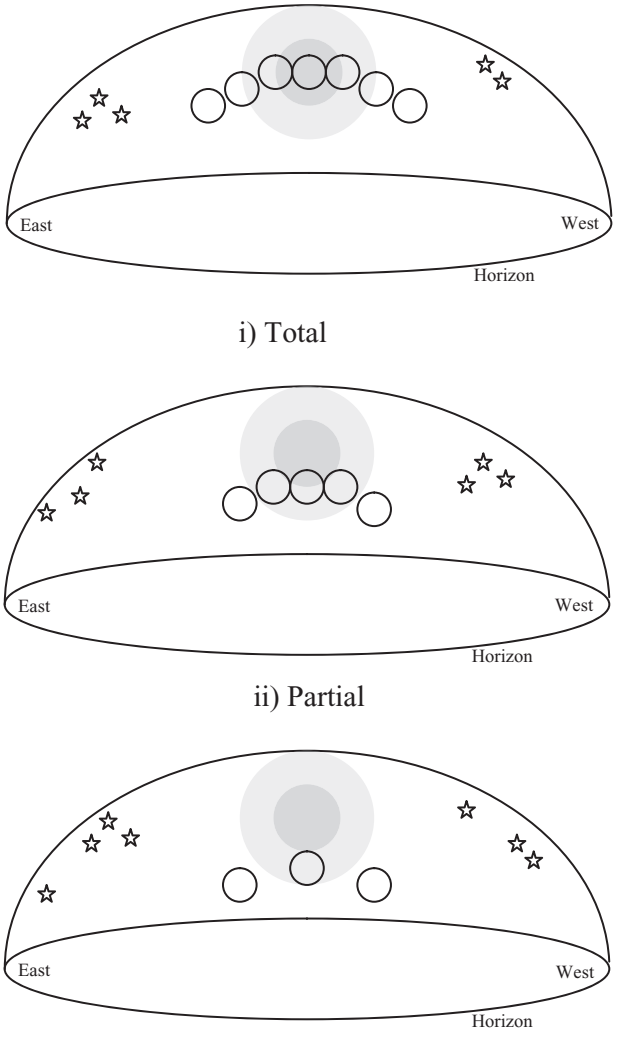


Figure 2.10 The celestial sphere.

sun. This is almost impossible to observe precisely with the naked eye because of the visible imprecision of the rim of the shadow. The moon continues along its orbit until it is fully immersed in the earth's shadow. This is known as the beginning of totality, or second contact. The moon, totally obscured, continues on its path through the shadow; the time it takes to do this is different for each individual eclipse. Mid-eclipse is generally taken as the time exactly halfway

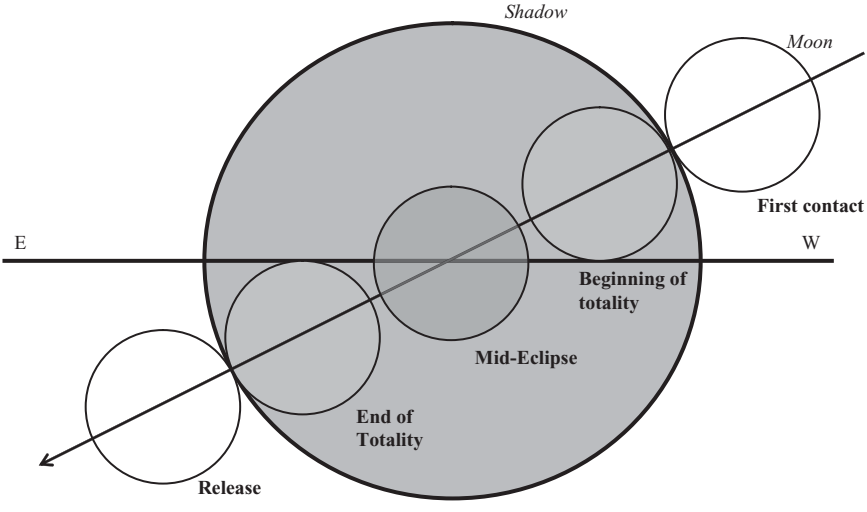


Figure 2.11 The five phases of an eclipse.

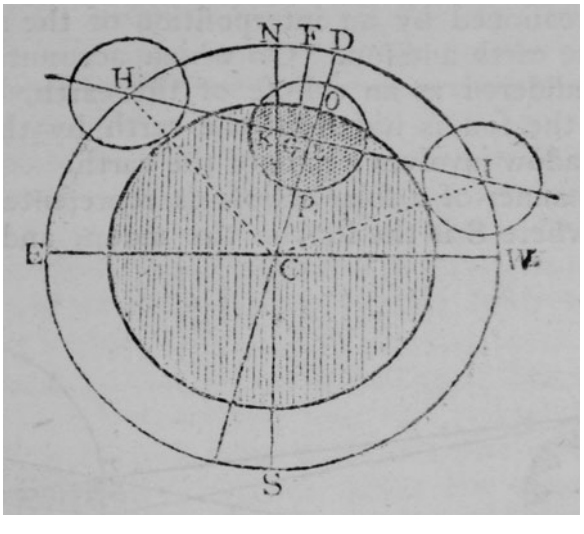


Figure 2.12 A partial lunar eclipse showing the path of the moon through the shadow as depicted in Charles Hutton's *A Mathematical and Philosophical Dictionary* (London, 1795). Photo used with the kind permission of the University of Canterbury Library, New Zealand

between the beginning of totality and the end, but is not a phenomenon that is observable. The last instant that the moon is totally eclipsed and is touching the inside edge of the shadow is called the end of totality, or third contact. Next the moon begins to become illuminated again until it leaves the shadow completely. The instant at which it is last touching the shadow, being fully illuminated, is known as the release, or last contact, and then the eclipse ends.

Partial eclipses, for self-evident reasons, are described simply in terms of first contact, mid-eclipse, and release; they do not have a period of totality. The most significant feature of a partial eclipse, however, is the *magnitude* of the eclipse, that is, the portion of the moon that is eclipsed at the maximum amount of obscuration. There are different ways to measure the magnitude of a partial eclipse. The most common is to measure the amount of obscuration along the diameter, rather than the actual area covered (fig. 2.13). In early times, the diameter of the disk of the moon was frequently divided into twelve equal parts; the magnitude would be gauged using this as a scale (see, for example, Ptolemy, *Almagest* VI, 8). Less commonly, early thinkers would represent the magnitude by the area of the lunar disk that was obscured (see also Ptolemy, *Almagest* VI, 8). The magnitude of a lunar eclipse is not simply dependent on the latitude of the moon, but also, because the moon's path around the earth

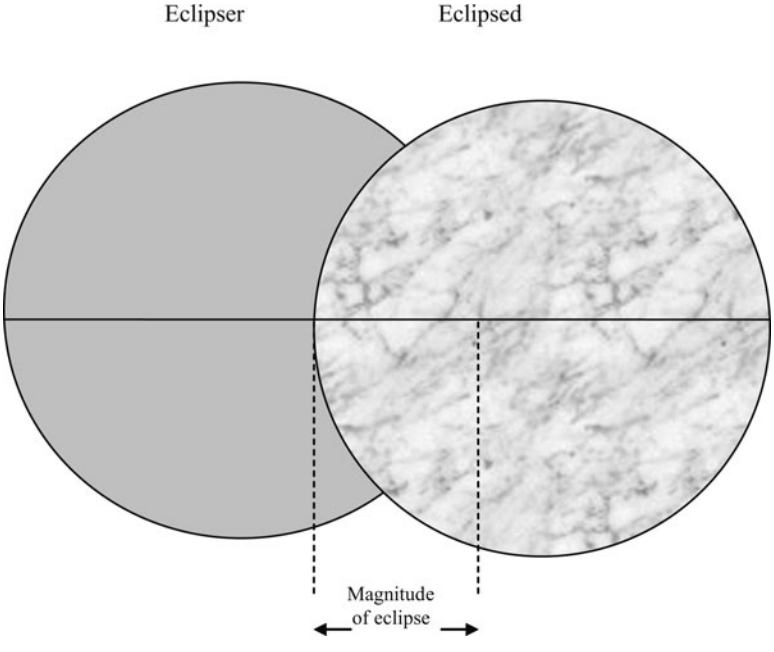


Figure 2.13 The magnitude of an eclipse.

is not strictly circular, upon its distance from the earth (the apparent size of its disk). Modern astronomers use the decimal format to record magnitude. A total eclipse has a magnitude of 1.0; partial eclipses are between 0.0 and 1.0, and those eclipses with a magnitude greater than 1.0 indicate a period of totality.

Lunar eclipses can last for, at maximum, a little under four hours, with at the most one hour and forty minutes of this being total.

2.2.2 Solar Eclipses

The other possibility for an eclipse is when the moon is a sufficiently small distance away from the node at *conjunction*, a situation in which a solar eclipse can occur. In this situation again the moon, the earth, and the sun are all in a straight line; however, it is the moon that is now obscuring the light rays of the sun. The moon passes between the earth and the sun, creating a shadow on a portion of the earth's surface (figs. 2.14 and 2.15). At minimum, two solar eclipses must appear somewhere on earth each year; up to five may appear in any given year. These are not observable in the same locality.

Not all solar eclipses are alike. In fact there are three visually distinct types of solar eclipse due to the circumstances of the sun-moon-earth alignment and the distance between the moon and the earth. The first, a total solar eclipse, occurs when some point on the earth's surface falls within the moon's umbral cone and the sun is completely obscured.

The second is a partial solar eclipse, in which some part of the earth's surface travels into the moon's penumbra, but not into the umbra. The third,

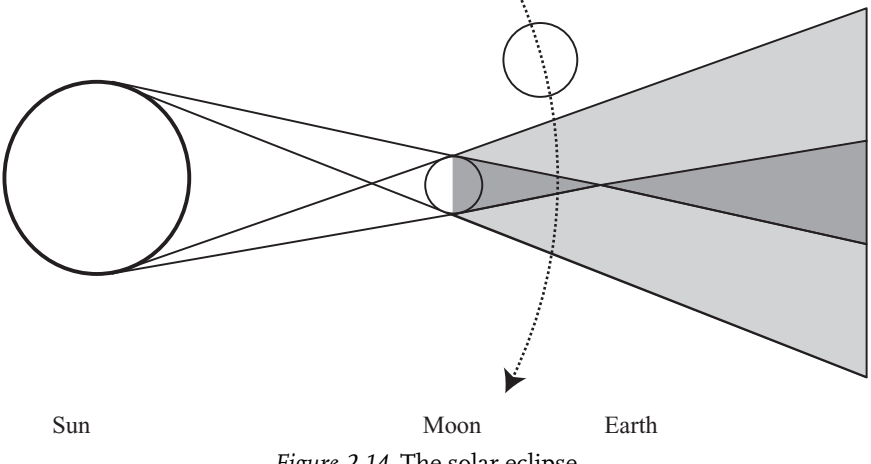


Figure 2.14 The solar eclipse.

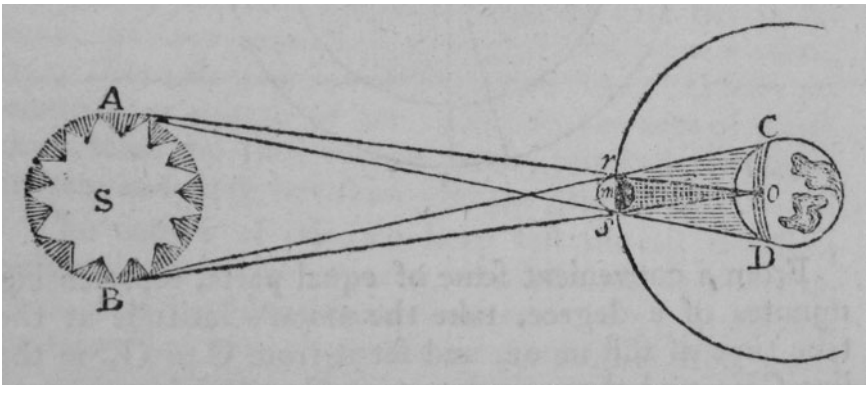


Figure 2.15 A solar eclipse as depicted in Charles Hutton's *A Mathematical and Philosophical Dictionary* (London, 1795). Photo used with the kind permission of the University of Canterbury Library, New Zealand

and perhaps most spectacular, is the annular eclipse. In this type of eclipse, the moon is aligned in front of the sun, but does not fully obscure the disk of the sun because the umbral cone does not quite reach the earth, which remains a distance away from the focal point. The rays of the sun stream out from a ring shape around the moon's outer periphery.

The apparent diameters of the moon and the sun are very similar, but they are variable, which can affect eclipse duration. The variation of the sun's apparent diameter is a lot less than the moon's variation, but it is nonetheless a significant factor in determining which total solar eclipses will be annular and which will not. The eclipse shadow sweeps from west to east because of the relative motion of the sun and the moon.

In the course of any solar eclipse, the shadow created by the moon falls on only a small area of the earth, so that the appearance of a solar eclipse is highly dependent on where the observer is situated. Observers at one latitude may view the eclipse, while others at different latitudes may not; and, of course, the sun, eclipsed or not, is visible at any one time in only those regions for which it is above the horizon. In fact, total solar eclipses are only visible in a path about 250 kilometers wide. At the most, totality can last up to 7½ minutes, but is usually much shorter. Furthermore, most partial solar eclipses are not detectable until the sun is at least $^{9}/_{10}$ covered by the moon—the dimming effect of sunlight is slight for any smaller amount. These factors make it very difficult to predict the circumstances of a solar eclipse without very advanced techniques that take into consideration many different details.

As noted, timing and location are critical for solar eclipses. Accordingly, a factor essential to any successful prediction model for solar eclipses is the

effect of parallax. Parallax, from the Greek *parallaxis* (literally "change" or "deviation"), is an effect accounting for the fact that the observer is not situated in the center of the earth, for which the calculations are made, but rather on the earth's surface. The effect of parallax is to make objects seem *lower* (that is, closer to the horizon) in the sky than they would seem to be as seen from the center of the earth. Theoretically, the concept of parallax is straightforward. For bodies that are relatively close to the earth, one must differentiate between the center of the earth *O* for which all the parameters have been calculated and the positions of the observers *A* and *B*, which are displaced an amount equal to the radius of the earth (figs. 2.16 and 2.17). Against the backdrop of fixed stars, celestial objects close to the earth, such as the moon, appear in different places. When a celestial object is at the zenith (the point directly above the observer in the sky), there is no parallax. As the body comes closer to the horizon the parallax increases, pushing its position downward so that it is seen lower in the sky than it really is (fig. 2.18).

The parallax π corrects the celestial coordinates for the geographical position of the observer. Suppose *O* is the center of the earth, *A* the position of an observer on the surface of the earth, *Z* their zenith, and *M* the moon (fig. 2.19). Then the angle ζ' is the apparent lunar zenith distance (with reference to the observer *A*) and ζ is the true zenith distance (with reference to *O*). Their difference is the lunar parallax:

$$\pi_m = \zeta' - \zeta .$$

The solar parallax is found similarly. For the occurrence of a solar eclipse, the sun, the moon, and the point on the earth's surface must be in alignment

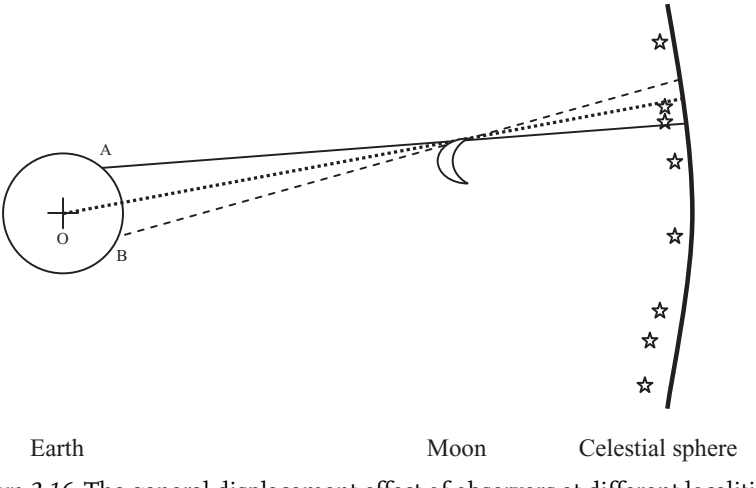


Figure 2.16 The general displacement effect of observers at different localities.

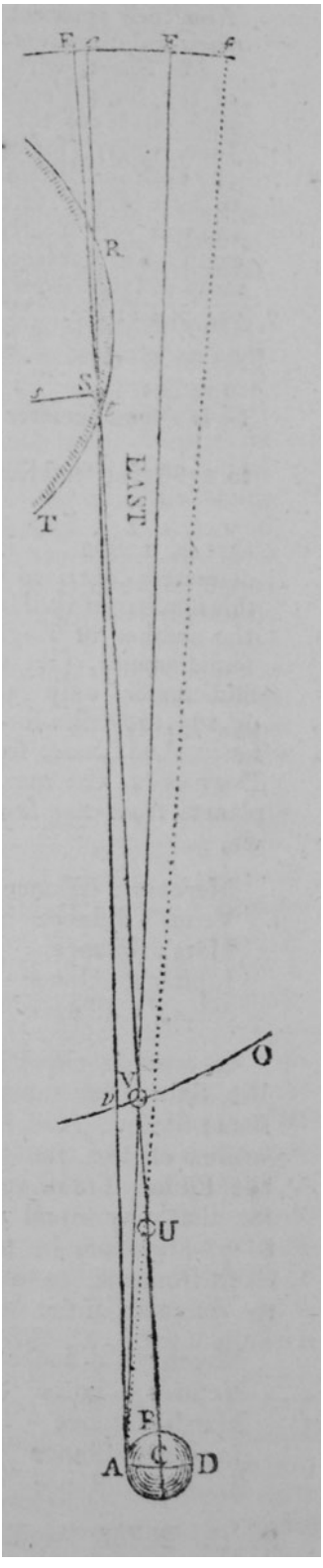


Figure 2.17 The effects of observing from different places on the earth as depicted in Charles Hutton's *A Mathematical and Philosophical Dictionary* (London, 1795). Photo used with the kind permission of the University of Canterbury Library, New Zealand

(or very close it). The total effects of both solar and lunar parallax must be considered. Parallax is a vector measured along any one of the zenith distance circles through the celestial body that are perpendicular to the horizon. For practical purposes, it can be divided into two vectors of longitude (π_λ) and latitude (π_β), which can be applied to its ecliptic co-ordinates (fig. 2.20) where M' is the position of the moon with the effects of parallax applied. Various mathematical techniques can be invoked to determine the two components of parallax. It was common in early times to treat the spherical triangle formed by the components as plane. More sophisticated techniques were later developed, which used appropriate relations from spherical trigonometry.

As is intuitively obvious, parallax plays a large role in the determination of the elements in solar eclipses, because the position of the moon can be changed enough by parallax to cause an eclipse that would not have been predicted, or the opposite. The moon's horizontal (maximum) parallax has been computed to be about 0;57° and the solar parallax a little less than 0;0,9°.[4]

Essential to parallax computations is the establishment of the terrestrial latitude. There were many ways in which this was done in early times. Furthermore, the respective motions of the moon and the sun have a large impact on the circumstances of a solar eclipse, causing

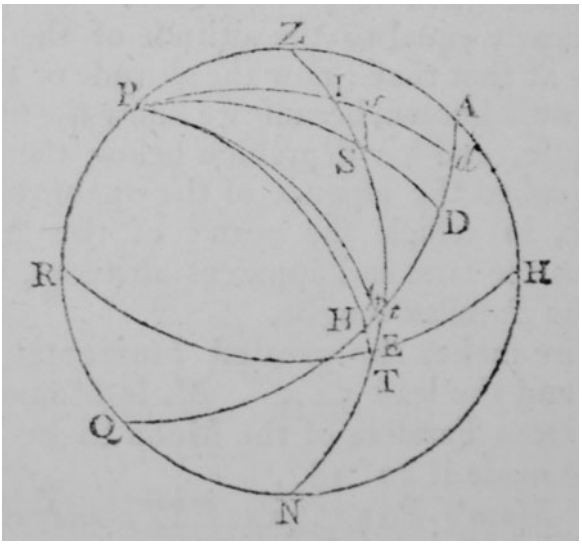


Figure 2.18 The effects of parallax on the positions of the luminaries as depicted in Charles Hutton's *A Mathematical and Philosophical Dictionary* (London, 1795). Photo used with the kind permission of the University of Canterbury Library, New Zealand

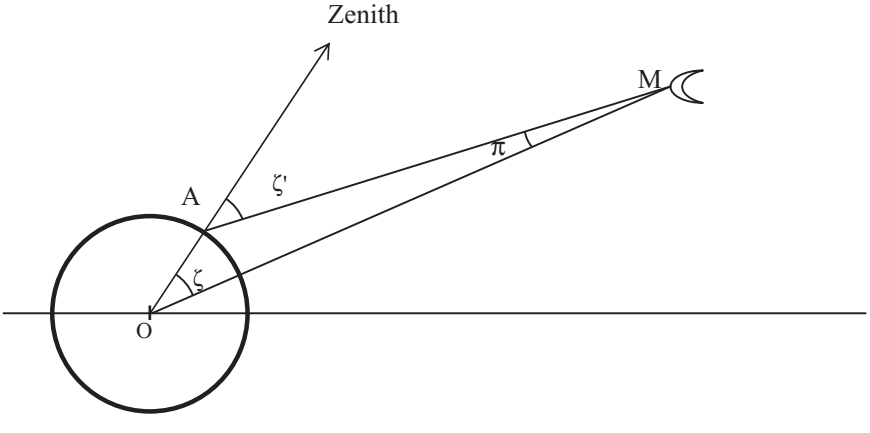


Figure 2.19 Parallax.

the eclipse shadow to travel eastward. In addition to their relative motions, the earth itself is rotating westward, which can have a slowing effect on the shadow. The extent to which the earth's rotation slows the shadow, however, is highly dependent on a variety of factors, the most prominent being the position of the eclipsed sun in the sky.[5]

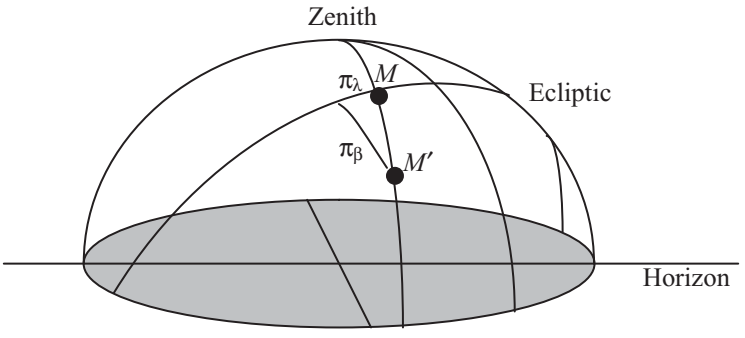


Figure 2.20 The longitudinal and latitudinal components of parallax.

2.3 Prediction and Patterns of Eclipses

The necessary conditions for both a lunar and a solar eclipse are that: the moon be (a) at syzygy and (b) near a node. In the case of a lunar eclipse, the intervals between successive eclipses can be derived adequately as follows. Roughly speaking,[6] the moon will return to the same node again in one full draconitic month (≈ 27.2 days) which will satisfy the first condition. However, the moon will not be in opposition with the sun again until after one full synodic month (≈ 29.5 days), so that at the next opposition the moon will be $29.5 - 27.2 = 2.3$ days worth of traveling past the node. At the opposition after this, it will be twice that distance (4.6 days worth) from the node, at the next opposition, three times (8.9 days), and so forth. After six months, the difference will be 13.8 days worth of traveling past the node. This figure, compared with the 13.6 days ($= 27.2 \div 2$) difference between the two nodes, means that the moon will satisfy both conditions (a) and (b) and an eclipse, with slightly different circumstances, will occur. Therefore, eclipses occur roughly six months apart.

By a similar logic, one can also quickly demonstrate that, after several six-month intervals, there will be a five-month interval between successive eclipses. This rough outline of intervals between eclipses is demonstrated more rigorously by Ptolemy (*Almagest* VI, 6), for example, for both lunar and solar eclipses. One further factor to take into account is the position of the moon with respect to the observers on the earth. If the moon is above the horizon the eclipse will be visible, but if the moon is below the horizon, the eclipse will *not* be visible. This cuts the number of eclipses visible to an observer at any one location roughly in half. As Ptolemy observes, solar eclipses can occur at an interval of

only one month apart, but never to observers in the same locations. For the most part, one of the pair will be observable only in the regions around the north pole and the other around the south pole.

The above method calculates the successive lunar eclipses, but larger intervals for eclipse sequences which give details about recurring eclipses with similar features were found and recorded. After any particular eclipse, it is clear that another eclipse will occur when identical conditions recur, that is, first, when the moon is in the same position with respect to the node, and, second, when the moon is at the same type of syzygy (opposition) again. These conditions are achieved after a complete number of synodic months (satisfying the first condition) and after a complete number of draconitic months (satisfying the second condition). In other words, the lowest approximate common multiple of the length of a synodic month and the length of a draconitic month is sought. There do exist several approximations which are close enough to give satisfactory periods. The best known of these is what is now called the *Saros* cycle, first noticed in the ancient Near East. The Saros uses the relation that

$$223 \text{ synodic months} \approx 242 \text{ draconitic months, or}$$

$$6585.32157 \text{ days} \approx 6585.352724 \text{ days.}$$

Furthermore, conveniently, this interval is almost the same as 239 anomalistic months (= 6585.5494 days), which implies that after a period of 223 synodic months, the successive eclipses in each cycle will share duration, magnitude, and spacing. Relations such as the Saros were used in early times both to predict when there was a possibility of an eclipse, and to reveal when there was not.

2.4 The Celestial Sphere

Until now, the focus has been on the sun and the moon and the various features that account for the circumstances of their alignments. However, eclipses occur against a greater celestial backdrop, and some further conventions and concepts need to be briefly introduced. Any system of spherical coordinates can be uniquely described by means of three essential elements: a great circle, (one of) its poles, and a point on that great circle which is the zero point. There are three key coordinate systems useful for representing the positions of bodies in the celestial sphere which are used throughout this book: the horizon system, the equator system, and the ecliptic system.

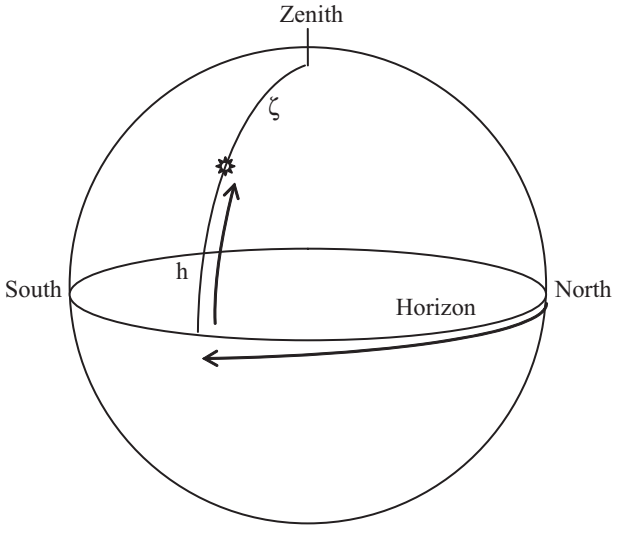


Figure 2.21 Horizon coordinates.

The Horizon Coordinate System

The first uses the horizon as its reference circle (fig. 2.21). For an observer at the local horizon, the point directly overhead is known as the zenith (Z). The great circle which passes through the north point on the horizon, the zenith, and the south point on the horizon is known as the (local) meridian. The celestial north pole (P) is the axis of rotation of the celestial sphere, which remains fixed in the sky, and the angle between this and the horizon is the local terrestrial latitude (ϕ). A pair of coordinates, known as horizon coordinates, can be used to describe the position of a body in the sky with respect to the horizon. The first, the azimuth, is a measurement of the position along the horizon from the north point, which is positive from the north in an eastwards direction. The second, the altitude (h), is the angular height above the horizon. The azimuth of an object is almost never invoked in early astronomy, but the altitude is frequently used and often can be measured by direct observation (for example, in the case of the sun). The complement of the altitude ($\zeta = 90^\circ - h$) is the zenith distance and is also used frequently.

The Equatorial Coordinate System

Equatorial coordinates use as their great circle the celestial equator (fig. 2.22), and the individual point, the vernal equinox (♈—the point at which the ecliptic and the celestial equator intersect), as their reference. The position of a body in

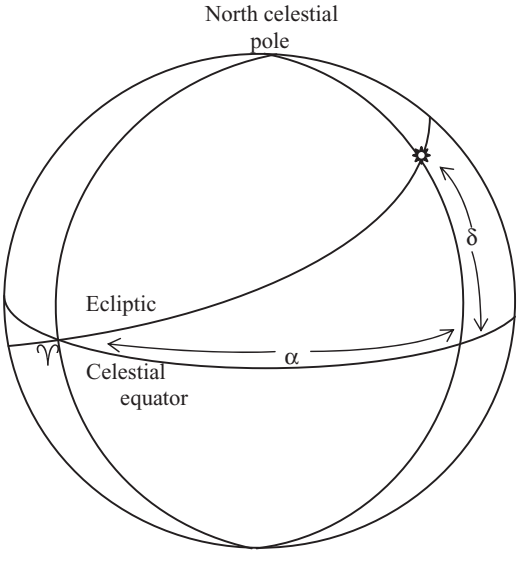


Figure 2.22 Equatorial coordinates.

the celestial sphere can be dropped perpendicularly onto the celestial equator. The distance from this point along the celestial equator to the vernal equinox is called the right ascension (α), increasing from west to east. The angular distance from the body to the celestial equator is called the declination (δ). Declination is measured in degrees, but right ascension is usually measured in hours. This is because it is measured along the celestial equator. One of the main advantages of the equatorial system is that it connects time to the change in right ascension: 24 hours corresponds to 360°, and accordingly 1 hour = 15° and 1° = 4 minutes.

The Ecliptic Coordinate System

Lastly, ecliptic coordinates use the ecliptic as their reference circle (fig. 2.23), the vernal point ♈ as the zero point, and the pole of the ecliptic. The position of a celestial body can have longitude (λ), increasing from west to east, which is measured from the vernal equinox along the ecliptic, and latitude (β), which is the angular distance of the arc dropped perpendicularly from the body to the ecliptic. Frequently, celestial longitudes are given with respect to zodiacal signs, 30° divisions of the ecliptic, which derive their names from the configurations of stars or constellations which roughly occupy the same space. The modern names and symbols for the twelve zodiacal signs are as follows.

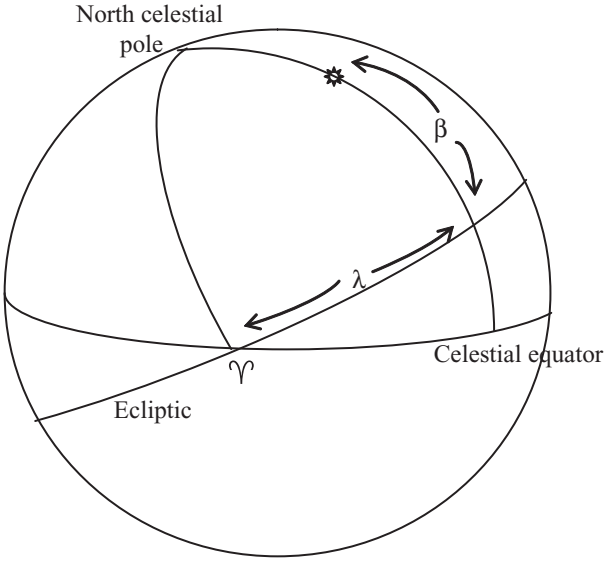


Figure 2.23 Ecliptic coordinates.

Name	Sign	λ
Aries	♈	0 to 30°
Taurus	♉	30 to 60°
Gemini	♊	60 to 90°
Cancer	♋	90 to 120°
Leo	♌	120 to 150°
Virgo	♍	150 to 180°
Libra	♎	180 to 210°
Scorpio	♏	210 to 240°
Sagittarius	♐	240 to 270°
Capricorn	♑	270 to 300°
Aquarius	♒	300 to 330°
Pisces	♓	330 to 360°

2.4.1 The Climes

There were various conventions to section the surface of the earth as well as to divide up the sky. The climes, from the Greek *klima,* pl. *klimata* (region or inclination), are the geographical areas that resulted from the practice of dividing up the terrestrial sphere into zones, the limits of which were parallel to the equator. One of the first to do this was Aristotle,[7] who divided up the entire globe into five zones: two frigid (the antarctic and arctic), two temperate,

and one torrid (in the equatorial zone). In early Hellenistic times, the northern hemisphere was typically divided into seven climes, each delimited by a parallel of terrestrial latitude. These divisions were made on the basis of the length of the longest day (which occurred on the summer solstice) for each region. Ptolemy used this idea to divide the northern temperate zone into seven for use in his astronomical system, which he gave as follows.[8]

Clime	Region	Latitude	Longest daylight
I	Meroe	16;26°	13^h
II	Syene	23;51°	$13;30^h$
III	Lower Egypt	30;22°	14^h
IV	Rhodes	36;00°	$14;30^h$
V	Hellespont	40;56°	15^h
VI	Middle Pontos	45;01°	$15;30^h$
VII	Borythenes	48;[32]°	16^h

2.5 Mathematical Conventions

2.5.1 The Sexagesimal System

Early astronomical traditions preferred to express quantities using the *sexagesimal* system of enumeration, that is base 60. The sexagesimal system emerged in the ancient Near East and was adopted by Greek astronomers and hence became the standard convention in the Indian, Islamic and western traditions. Traces of this system still remain in common use today, such as the current system for measuring time in hours, minutes, and seconds: 1 hour is 60 minutes, 1 minute is 60 seconds, and so on.

Unlike the modern place-value base-10 system, which expresses a number according to multiples of ten, a number written in sexagesimal form operates in multiples of 60. The values 0 to 59 are expressed as combinations of units, but all multiples of 60 are represented in the place next to the units, and so on. Thus, for example, the numbers

$$35 \quad 86 \quad 411$$

are rendered sexagesimally as

$$35 \quad 1,26 \quad 6,51$$

since $86 = 1 \times 60 + 26$ and $411 = 6 \times 60 + 51$.

To represent numbers sexagesimally, scholars have developed conventional notation, which is now near-universal among historians. A simple space is left between successive sexagesimal places (although some scholars prefer a comma), units and/or tens that are absent are indicated by a place-holding zero,

and numbers are generally represented, faithful to their original articulation, without indication of their absolute value (it is the place in which a number is put that gives it its value). However, a scholar may choose to indicate (his or her reckoning of) a number's absolute value by the strategic placement of a semicolon, the sexagesimal equivalent of a decimal point; all numbers to the left of the semicolon are multiples of successive powers of 60, and all numbers to the right are multiples of successive reciprocal powers of 60. This is the convention adapted here. So for example, the numbers:

$$\tfrac{1}{2} \qquad 70\tfrac{1}{3} \qquad 360\tfrac{1}{4}$$

are rendered sexagesimally as

$$1;30 \qquad 1,10;20 \qquad 6,0;15$$

since $1\tfrac{1}{2} = 1 + {}^{30}\!/_{60}$, $70\tfrac{1}{3} = 1 \times 60 + 10 + {}^{20}\!/_{60}$, $360\tfrac{1}{4} = 6 \times 60 + 0 + {}^{15}\!/_{60}$.

This system is distinct from the various symbols used to represent the actual numerals, which varied from culture to culture.

2.5.2 *Early Trigonometry*

Trigonometry, that is, the ability to move between the measurements of angles and lengths, was of fundamental importance in astronomy.[9] Hipparchus of Rhodes in the second century BCE was the first to treat the relationship between angles and lengths systematically, proposing a system that concerned chords in a circle.[10] Hipparchus' works are lost, but the details of this chord function are reproduced by Ptolemy in the *Almagest*. The relation between the chord function described by him and the modern sine is as follows (fig. 2.24a):

$$\mathrm{Crd}2\theta = 2R\sin\theta,$$

or in other words half the chord of double the angle. Modern trigonometry is based on a unit circle with radius $R = 1$. Early practitioners used a variety of values—Ptolemy used $R = 60$.

The chord function was used until the introduction by Indian scholars of a function equivalent to the sine function. This proposal quickly replaced the Greek table of chords. Called *jyārdha* (literally "half of the chord"), this new function saved practitioners from the process of continually doubling arcs and halving the resulting chords. It emerged as early as the fifth century BCE,[11] along with related functions (fig. 2.24b): the cosine *koṭijyā* and the versine *šara* (literally "the arrow") or *utkramajyā* (literally "the reversed sine") which is $1 - \cos\theta$. Various values for the radius of the base circle were used, including 120, 150, and 3438, among others. The last value, $R = 3438$, can be derived

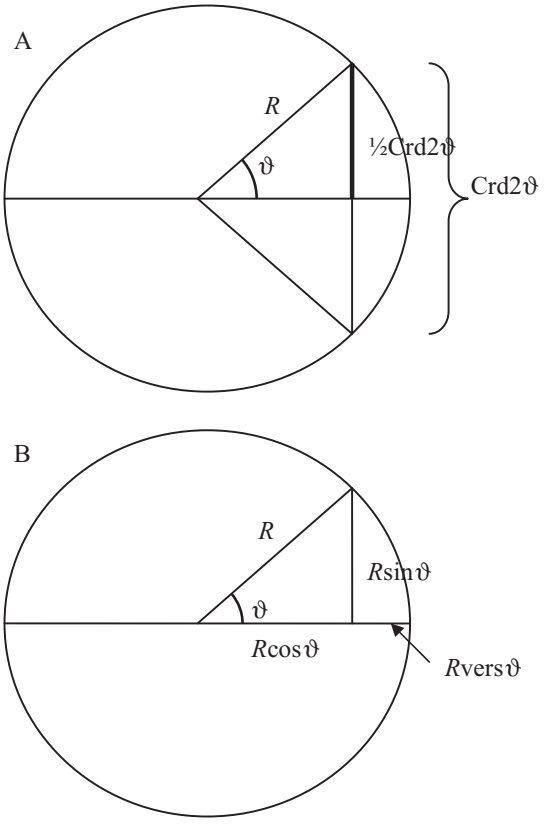


Figure 2.24 The chord function (*A*) and the sine function (*B*).

from consideration of the circumference of the circle measured in minutes and its relation to the diameter:

$$R = \frac{360 \cdot 60}{2\pi} = \frac{21,600 \text{ minutes}}{2\pi} \approx 3438.$$

This value has the mathematical benefit that the sines of small arcs with this radius are nearly equal to the arcs themselves. The relationship between the modern sine function and the ancient (with differing radii) is expressed by the following convention:

$$\text{Sin}\theta = R \sin\theta$$

where the modern sine function with $R = 1$ has a lower case "s" and the ancient, an upper case "S."

CHAPTER THREE

An Accountancy of Appearances

Mesopotamian Eclipse Reckoning

The night of the 13th day there was no observation;
let the son of the king sleep.
The night of the 14th day there was no observation;
let the son of the king sleep.
The night of the 15th day there was no observation;
let the son of the king sleep.
The night of the 16th day, a great observation: the moon made an eclipse; this eclipse of the moon . . . [tablet breaks off]

—SAA VIII:279, as translated by Hunger [1992]

AN.MI *du-luh-hu-u*
Eclipse (means) trouble

—SAA VIII:55 line 5, as translated by Hunger [1992]

3.1 Overview

The close of the fourth millennium BCE saw communities of agrarian peoples who inhabited the region between the rivers of the Euphrates and the Tigris, up to then largely autonomous, reach a point of sufficient density, interdependency, and complexity to be considered an emergent civilization. As well as increasingly tight social and political organization, extensive trade and circulation of commodities, deliberate large-scale agriculture and labor, and monumental art and architecture, the emergence of writing and with it the related arts of literacy were vital to the transformation of these prehistoric peoples. The technology of writing made possible new forms of intellectual activity, among them the desire to comprehend and classify the world, including the heavens. Writing ensured that efforts were lifted beyond the spontaneously observational. It encouraged record keeping and afforded a

greater level of detail in the reckoning of observable phenomena. However, the connection with preliterate cultural thought-forms was still strong, and manifested itself even in those pursuits made possible by the technology of writing.

For the first time, periodic celestial events were systematically observed and recorded by means of increasingly consistent principles and coherent systems of measurement. Concerning eclipses, ancient Near Eastern peoples produced ever more deliberate, careful, purposeful, and extensive observational reckonings. However, whether their consideration of eclipses ever did amount to the articulation of theory is questionable. Theoretical thinking prioritizes organizing conceptions and unifying principles over the accountancy of particular facts. The work of scholars in the ancient Near East concerning eclipses did propel them toward a more sophisticated accountancy of the appearances. However, the overwhelmingly predictive ambitions and the mathematical investment of their endeavors meant that they never decisively transitioned to a fully theoretical orientation for thought. Their efforts were directed toward accounting for a phenomenon, not its explanation.

Sources that make up a corpus illustrative of Mesopotamian celestial inquiry can be traced back to the second millennium BCE. They form just part of an already thriving and diverse intellectual environment maintained by professional scribes. These include pursuits in many different areas, such as literature and sacred texts, legal works, mathematical and economic texts, and lexicographic endeavors. As far as can be confirmed by the extant sources, early works in the astral sciences were not so much exclusively astronomical as they were part of the broader tradition of celestial divination, horoscopy, and astral magic. The earliest astronomical activity wove itself into massive collections of omens based on astral phenomena; this was to prove an enduring influence throughout inquiry in the ancient Near East. Most notable from this genre was the omen compendium *Enūma Anu Enlil* compiled some time in the Old Babylonian Period (ca. 2000–1600 BCE). A surge of interest in mathematical astronomical methods and other approaches appeared a few centuries into the first millennium BCE (the first extant diary dates from 652 BCE). A new and distinctive approach to astronomical phenomena, the mathematically grounded ephemerides of the Astronomical Cuneiform Texts, became prominent in the Seleucid period (311–125 BCE). Celestial inquiry that can be identified as distinctly Mesopotamian ceased to develop beyond the period of Parthian control, around the late first century CE.

In many respects, the understanding of eclipses as ominous reveals both what was old and what was new. Early treatment of eclipse phenomena retains remnants of mytho-poetic consciousness characteristic of earlier preliterate

cultures—most significantly the prosaic impulse mingled with the use of the poetic imagination to wrap human significance into phenomena that they observed. However, this long-standing anthropocentricity was given new form. The compilation of omens into huge compendia extended far beyond the limits of nonliterate societies, and the documentation in the form of protasis-apodosis statements[1] catalogued far more content and detail than could ever have been remembered. The technology of writing opened up potentialities for new thought-forms. But the contemplation of the heavens never lost its connection with human affairs. In particular, it never opened up more abstract questions in which human concerns were not present. The divination tradition demanded answers as to *when* and *where* a celestial event would happen. It did not encourage astronomers to probe more deeply into these events to consider the *why* and the *how*, nor to consider the heavenly bodies and their paths and timings, and how these bodies determine what celestial phenomena will be viewed, outside of this context. Astronomy stayed always connected to its divination heritage and because of this scholars always prioritized the arithmetical and predictive over the geometrical and explanatory.

To predict eclipses, and to better their abilities to do so, practitioners in the ancient Near East pursued patterns in the facts, as they could be captured arithmetically. Sources show that a geometrical orientation was never pursued beyond a rudimentary level. While they perfected and expanded an accountancy of the appearances, they did so quite indifferently to the physical reality behind the facts. In light of this, the extent to which they can be considered as theorizing is limited.

Unlike the other scientific cultures discussed in this study, Mesopotamia in the second millennium BCE does not appear to have inherited any significant astronomical legacy, though something may yet turn up in the earlier Sumerian records. Thus the need to establish fundamental parameters from scratch ensured thorough and comprehensive observations and record-keeping practices. From these observations arose the first tentative approximations to the periodicities of celestial events, and gradually from these were developed more intricate methods. However, the older schemes seem not to have been discarded in favor of the new proposals, but in many cases continued to be used alongside them.

In the entire extant Mesopotamian scientific corpus, there exist no sources that attempt to describe the fundamental mechanisms behind eclipses of the moon and the sun. From the most thorough scientific text to the anecdotal accounts of the royal diviners, not even a single proposal is ventured concerning the physical factors that cause eclipses to happen. The primary emphasis instead is *whether* an eclipse will occur, *when* the next will occur, and what

recurring arrangement can be established that can be used to secure the successful prediction of eclipses on into the future.

There exist a number of nonscientific Sumerian and Akkadian texts that offer explanations for the physical structure of the universe and its origins. These texts suggest that the cosmological conceptions in the ancient Near East remained largely unchanged from the very first extant account until the decline of cuneiform literature around two and a half millennia later.[2] These accounts[3] reveal a universe that is composed of a number of superimposed layers, variously human and divine. The layers themselves are separated from each other by open space and are fixed in place by cosmic bonds in the following arrangement:

The Heaven of Anu
The Middle Heavens
The Sky
The Surface of the Earth
The "Apsu"
The Underworld

The level in which humans dwell, the surface of the earth, is said to have "edges."[4] Sumerian myths conceive of the earth as a solid block of matter.[5] The underworld is on this block but directly beneath the surface. Akkadian texts describe a circular surface,[6] some adding that it is surrounded by a cosmic sea, others by very high mountains. The idea of a circular surface is further suggested by the ways to which it is referred in geographical terms, such as *kippat erṣeti* (literally "the circle of the earth").[7]

The relationship between these cosmological conceptions and the extent to which they influenced the actual practice of astronomy are somewhat unclear. Astronomers who worked under assumptions such as a layered universe and a flat earth would have no incentive to dwell on issues such as the size and shape of planetary bodies, nor have any motivation to give an account of the mechanisms of eclipses—their cosmography would have no place for a "shadow of the earth," nor would celestial positions be displaced because of the position of the observer on the surface of the earth relative to the center, as spherical-earth conceptions would entail. Concepts such as parallax, which seem to have been first noticed by Greek thinkers, are only meaningful to those who hold the belief that the earth is a sphere with an appreciable radius with respect to the distance of the moon. Šamaš and Sin, the Gods of the sun and the moon, respectively, had a definite anthropomorphic embodiment, and thus to consider them in terms of diameters and distances may be somewhat absurd.

Furthermore, celestial phenomena were believed to be direct communications[8] from the Gods in which future events were revealed. Indeed, eclipses were highly ominous events, impacting the lives of humans and their livelihoods, from entire populations to the very king himself. These communications, however, particularly those foretelling imminent evil, were not irreversible, but could be prevented by suitable responses and actions. Such responses included rites called NAM.BÚR.BI rituals[9] (one of which included the banging of a kettle drum[10]) or else the Substitute King Ritual, in which a king for whom danger was indicated by an eclipse was replaced by a commoner in order to escape the evil effects, which would befall the "substitute" king instead.[11] Such beliefs and practices in society attest to a very strong motivation to acquire the ability to predict eclipses. In many cases, it seemed not to be the eclipse as a phenomenon that was stimulating the astronomers, but rather its application and importance for the health and well-being of the king and his kingdom. The prospect of missing an eclipse occurrence and thus the ability to prepare for it was a serious concern. For this reason, an eclipse could never be missed. This detail encouraged scholars to be somewhat overgenerous in their forecasts, rather than conservative.

The whole notion that eclipses, and indeed all celestial events, were the messages of the Gods to humankind is seemingly at odds with the premise on which the astronomers were basing their efforts, namely that these were events that can be predicted ahead of time. However, this inherent contradiction neither impeded Mesopotamian astronomy as a whole from developing nor created the least apparent worry, for example, that the events would lose their meaning if systematized.

These sentiments, along with the deification of celestial bodies, attest to the persistence of mytho-poetic connections. Practitioners in the ancient Near East could not yet fully extract the phenomena from the overwhelming human significance that they had long since embodied. Their direct relevance to humans dominated, despite developing aspirations to treat celestial phenomena increasingly systematically. The maintaining of such connections precludes insights of a theoretical nature. The lack of concern in developing a cosmological model can be understood in this context. To search for an underlying geometry of arrangements and to demand the sort of unity of space and time required for a cosmological representation was not an ambition which found incentive in a society that had newly invested itself in the arts of literacy. The predominance of these connections entailed that aspirations never extended to a search for a reality behind the facts. Rather, practitioners sought to refine their control over the patterns behind the facts. Their engagement with the phenomena remained largely orientated in this way.

Extant sources form but a small percentage of the complete (hypothetical) cuneiform scientific corpus (estimates put it at as little as 5%). As a result, there exists the potential for a biased picture of the Mesopotamian astral sciences. Accidents of preservation may distort the evaluation of the achievements and development in this field. In many cases, how widely accepted or rejected a particular scheme was within the wider community can only be guessed at, especially in the cases in which there remain only one or two tablets on a particular topic. For example, in some aspects of eclipse computations, there remains at best a handful of texts from which to reconstruct the whole scheme along with its parameters. Techniques that embody "improvements" of eclipse parameters were not always adopted by practitioners in the extant texts. Furthermore, there exists a certain disregard for a thorough explanation of the schemes (except for perhaps the procedure texts). For the most part, there are only ephemerides, lists of calculated numbers, and dates from which to determine the underlying mathematical relations.

Not only is there a lack of surviving source material, but in the majority of cases, what does survive is fragmented. Although the consistency of numerical computations allows for heavy reconstruction once the underlying patterns have been established, there still remain many puzzles that cannot be solved because of the fragmentary state of a tablet. Because of the highly numerical nature of most of the extant records, it is virtually impossible to establish a technical vocabulary for many of the concepts. The few terms that are used in technical contexts are also used in nontechnical environments, thus making it difficult to establish the precise meaning of the term.

As a result, although the Mesopotamian astronomers developed some powerful predictive tools, our knowledge concerning the techniques they advanced related to eclipses is in most instances insecure. Methods can be detected directly or indirectly in a variety of contexts.[12] A shift in precision from 5° to 1° in the recording of observed eclipse times has been noticed by modern scholars,[13] which supports the idea that the mathematical schemes were, on the whole, being improved. But the accuracy of eclipse predictions does not change through the course of time, suggesting that any modifications that were being applied had little consequence.

The volume and detail of the Mesopotamian observational records, the elegance of their arithmetical patterns, the durability of their written records, and the far-reaching influence of their culture ensured that the astral achievements of the ancient Near East were to have a profound impact on the ensuing major scientifically literate societies (Greece, India, Iran, Egypt, Syria, among others) and their successor cultures.

3.2 The Earliest Accounting for Eclipses: *Enūma Anu Enlil*

Early systematic treatments of eclipses in the ancient Near East can be found in omen literature. Among the many omen series that were composed, the great omen compendium, *Enūma Anu Enlil*[14] (*EAE*) is most relevant in this context as it contains descriptions of many celestial events in the standard protasis-apodosis format, including protases that are possible, plausible,[15] or impossible, and apodoses that contain the vital connection of these events with their human significance. The celestial phenomena included in this seventy-tablet compendium catalog those events that can occur in the skies in the broadest sense; lunar, solar, and planetary phenomena, weather phenomena and optical effects relating to the atmosphere were also included. What this nexus of phenomenal references reveals is the overwhelming task of the earliest literate thinkers to sort out natural phenomena and to account for their details. This accounting had to be thoroughly established before any consideration of these events beyond their appearances could be achieved. While the primary intention of this engagement with the phenomena was to collect and ratify the connections between signs and their portentous outcomes, a secondary offshoot of these efforts was in fact an increasing appreciation of unanticipated regularities and orderliness in the very signs themselves.

Eclipses of the sun and the moon are a prominent theme in this large work; material relating to lunar eclipses is contained primarily in Tablets 15–22,[16] and to solar eclipses in Tablets 23–27.[17] Although the primary intention of the compilers was to link phenomena to their ominous significance and not to describe the phenomena in and of themselves, much can be extracted from the content and presentation of the protasis material: what features of eclipses were noticed and deemed significant, how they were described and interpreted, the emphasis on them of all the phenomena that appeared in the sky. Indeed, the initial inspiration for the inclusion and description of phenomena, eclipses and otherwise, in the protases must have originated from observation. But protases were certainly not limited to a description of what had been observed. (Eclipses that occurred on days other than the middle or end of the month, impossible configurations of planets, and bizarre atmospheric phenomena were included in the protases.)[18]

The development of writing opened up new potentialities of organization and scrutiny to scholars. The presentation and evaluation of data as it was being recorded offered scholars more abstract forms of appraisal and association, such as those suggested by symmetry of data, schematic completeness, and logical analogy. This encouraged scholars to include descriptions of phenomena that had not yet been observed, but, by such sympathies, were deemed

possible. The distinction between the observable and the unobservable was not imperative. In a similar sentiment, initially scholars did not distinguish between those periodically recurring phenomena that followed patterns that could be established in advance and those that did not (for example, the rising of the stars as opposed to weather conditions). In fact, an environment motivated by divination practices led the astronomers to prefer those events that were improbable or even actually impossible over those that were regular, commonplace, and/or predictable. Exceptional and astonishing events portend significant consequences, and thus are all the more commanding of attention.

In the context of eclipses, these practices are significant. Eclipse occurrences, which were eventually recognized as generally obedient to simple predictive schemes, were often cataloged with respect to events that were not, such as the apparent color of the disk, wind direction, and so on. This too very much restricted the scope of those who studied eclipse phenomena. Under such circumstances more general abstract considerations remained beyond the purview of their investigative efforts. The separation of those events that were periodically recurring and predictable and those that were not was a long time coming. The overriding assumption that these phenomena were signs sent from the Gods meant that every configuration and detail was significant and thus had to be accounted for. There was no impetus for the investigation of causal connection beyond the divine. This commitment entailed that scholars granted no epistemological priority to one set of phenomena over another.

Despite the implausibility of some of the material documented, many of the elements that were carefully recorded by the compilers were later useful in constructing eclipse theory. Furthermore, it was precisely these recorded features that established a framework—criteria of the aspects they deemed important associated with eclipses—that was adopted by astronomers for millennia to come. Aspects such as time and duration, direction of impact, and color were all features that were considered basic to any standard account of eclipse theory by the Greeks, the Indians, and the Arabs, among other scientific cultures.

Arguably the most noticeable element with respect to a lunar or solar eclipse is on what day it occurs in the lunar month.[19] This is the one feature, assuming that the calendar is correctly established, which is fixed, only varying by a day or two in the civil calendar. Even for the most casual observer, it does not take many eclipses to determine that lunar eclipses occur in the middle of the month and solar eclipses at the very end. In texts as early as Old Babylonian Eclipse Tablets and *EAE*, such a feature was noticed, albeit crudely. For example, many omen protases involving lunar eclipses have as their theme eclipses occurring on the 14th or 15th of the month:

DIŠ *ina* ITI.GU$_4$ UD.14.KAM AN.MI GAR-*ma* . . .
[If an eclipse occurs on the 14th of Aiaru . . .]
(*EAE* 16: 9–11 p. 87)

DIŠ *ina* ITI.GU$_4$ UD.15.KAM AN.MI GAR-*ma* . . .
[If an eclipse occurs on the 15th of Aiaru . . .]
(*EAE* 16: 12–17 p. 87)

ina la a-dan-ni-šú UD.12.KAM UD.13.KAM AN.MI GAR-*ma*
[an eclipse occurs not according to its period (i.e., on the 12th or 13th day).]
(*EAE* 15 Source a: Obv. 1 p. 81)

But even this fact was far from settled.[20] In the various redactions of this omen series, days 14–16 are the most prominent, but also 20 and 21 (and very occasionally 19) are mentioned. The latter dates are puzzling, although one must keep in mind when dealing with predictive phenomena that celestial omen texts are in no way required to adhere to any type of physical reality, although they often reflect it; exceptions to the rule are always of high significance in terms of their effect (largely, it seems, because they can never occur) and hence often included. It has been suggested that the inclusion of many different possibilities was a result of a desire to include all the possible combinations for completeness. This feature is by no means restricted to eclipses, as many *EAE* tablets include all kinds of astronomical impossibilities. However, in this case, not all days in the month are featured with equal significance. Why the days 19, 20, and 21 were included, and not other days in the month, is an outstanding question.[21] While some recensions of *EAE* use only the reasonable days, others (even in later times) include the impossible, which may indicate that there were practitioners who had some understanding of the occurrences of lunar and solar eclipses and others who did not.

Other features of eclipses are mentioned in *EAE*,[22] which are obviously not treated on an empirical basis. For example, the colors of eclipses: various colors can be observed during certain eclipses, but many of the colors mentioned in these texts are at best imaginative. Other features include time of impact and duration, magnitude,[23] direction of impact, and other phenomena that can be associated with eclipses such as meteorological features (wind, clouds, rain, thunder) and natural events (earthquakes, mudslides) for lunar eclipses. Eclipses were cataloged in the text according to a "schematic" basis, that is, they were creatively organized into human-generated divisions (watches, cardinal directions, up-down, paths,[24] etc.). that were already in existence. Aspects that could accompany an eclipse in the protasis-statement could be drawn from the following schematic divisions:[25]

Time	Months (12), days (30), watches (3)
Color	White, black, red, yellow (and later variegated or dark)
Directions	North, south, east, west
Location	Paths of Anu, Enlil, Ea

As well as displaying a certain degree of responsiveness to empirical data, there were more dominant forms of organization at work in the expression of eclipses in omens. This can be seen, for example, in the identification of the direction of the impact of an eclipse. This direction could be given as any one of the four cardinal directions. However, these allocations sit rather uncomfortably with the fact that only one of them is possible. With respect to a terrestrial observer, the moon moves in an easterly direction, and therefore an eclipse can only be observed to begin on the eastern side of its disk. However, the desire to include all four directions is maintained for a logical completeness, and perhaps more importantly because it provides the vital connection to the material included in the apodosis. Four cardinal directions on the disk of the moon reflect, for ominous purposes, the four terrestrial regions on which the effects of the eclipse will befall. The importance of this connection and the ominous significance of a complete set prevailed over the evidence suggested by observation.

One can also locate the emergence of the recognition that eclipses were recurring and thus predictable. This came with a search to identify those events that may be relevant in indicating the future occurrence of an eclipse. These came in two forms: those details that were useful for forecasting an approaching eclipse, and an interest in the spacing between successive eclipses. In the former case, elements were suggested as relevant to reveal whether or not an eclipse was imminent in the near future. For example, a criterion given for the prediction of lunar eclipses in *EAE* is based on the last visibility of the moon:[26]

> *ina* UD.28.KAM *rēš tāmartišu tammarma ina Simāni* UD.14.KAM
> *antalâ taqabbi*
> *bubbulu antalâ ukallamka*
>
> [Observe his last visibility on the 28th and on the 14th of Simānu;
> you will predict an eclipse.
> The day of last visibility will show you the eclipse.]
> (III-Recension A Composite text (9), translated by
> Rochberg-Halton [1988] p. 190)

There is also evidence that these astronomers noticed the possible time interval between successive eclipses. There exists a text[27] which reveals that the Mesopotamians at this time had a crude understanding that (both) lunar

and solar eclipses occur in periods six months apart or a multiple thereof. The text is very fragmentary; however, its structure seems clear. For example, line 7 (to which lines 1, 4, and 5 [fragmentary] seem parallel) reads:

> DIŠ *ina* ITI.KIN UD.14.KAM AN.MI GAR-*in ina* ITI.ŠE U[D . . .]
> [If in month VI the 14th day there is an eclipse, in month XII, d[ay . . .].]
> (82-5-22,501, as translated by Rochberg-Halton [1988] p. 283 Appendix 2.3)

The text also contains statements concerning a six-month period that elapses with no eclipse occurring. The word used to indicate this period is *adannu*, a term also associated with the expected day of an eclipse.[28]

Predictions were employed in the case of solar eclipses as well. For example:

> MAN *ina* ITI.BÁR UD.1.KAM *ina* IGI.LÁ-*šú* GIM *di-pa-ri* SA_5
> *u* IM.DIRI BABBAR *ina* IGI-*šú* . . . *ina* ITI BI UD.28.KAM UD.29.KAM
> UD.30.KAM AN.MI MAN GAR-*ma*
> [If the sun is red like a torch when it becomes visible on the first of Nisannu, and a white cloud moves about in front of it, etc. . . . in that month on the 28th, 29th, or 30th an eclipse of the sun will take place.]
> (Tablet 23 (24) Nisannu 1, as translated by Van Soldt [1995] p. 5)

In all probability, the criteria for the prediction of solar eclipses were not based on empirical data, but rather imitated (albeit embellished) the scheme used for lunar eclipses ("the day of last visibility will show you the eclipse"). Despite the infrequency of solar eclipses, at this point the compilers had no reason to believe that the solar scheme should be any different from the lunar, a claim that seems to hold true in later periods.

Thus, in the comprehensive nexus of protasis-apodosis relations, significant details concerning eclipses were noticed and articulated in omen series. The recognition of their periodic nature, some simple features, as well as remarks which reveal an attempt to predict their successive occurrences were included. Besides being tightly interwoven as part of greater aspirations to compile and relate all natural phenomena with their ominous significance, such treatment of eclipses laid an essential foundation for inquirers to come, launching them toward an ever more ramified but careful consideration of these phenomena and thus toward a fuller and sharper accountancy of the appearances.

3.3 Assyrian Eclipse Reckoning: The Letters and Reports

The Letters and Reports[29] are closely connected with *EAE* and other celestial omen collections; however, they represent a higher level of sophistication in terms of engagement with the astronomical phenomena. Indeed, the more

personal, intimate nature of the communications, combined with the fact that the composers were not constrained by a protasis-apodosis format, means that the evaluation of the level of astronomical activity is much more straightforward. Both observations and predictions of eclipses are contained in the Letters and Reports, as well as material on eclipses excerpted directly out of *EAE*.

By the time of the Letters and Reports, the fact that lunar eclipses occur on the 14th or 15th days and solar eclipses on the 29th or 30th is recognized as indisputable.[30] Dates outside this margin are almost always quotes from omen sources, usually *EAE*, and are expressed in the following form:

1 *ina* ITI.SIG$_4$ TA UD.1.KÁM EN UD.30.KÁM AN.MI GAR
AN.MI LUGAL URI.KI
A.KAL ŠÚ GÁL-*ma* BURU$_{14}$ KUR ^{d}IM RA-*iṣ* ERIM GAL ŠUB-*ut* . . .
1 *ina* ITI.SIG$_4$ *ina la mi-na-ti-šú* AN.MI GAR LUGAL ŠÚ ÚŠ-*ma* ^{d}IM RA-*iṣ* . . .
[If there is an eclipse in month III from the 1st to the 30th day: eclipse of the king of Akkad; there will be a peak flood, and Adad will devastate the harvest of the land; a great army will fall . . . If there is an eclipse in month III at an unappointed time: the king of the universe will die, and Adad will devastate; a flood will come . . .[31]]
(Issar-šumu-ereš, VIII:4, as translated by Hunger [1992] p.7)

In a similar way, the problem of the arrangement of eclipse possibilities during the year, a less trivial challenge, was grappled with. It was perhaps partially understood that lunar eclipses are spaced regularly in six-month intervals with the occasional longer interval that was not a multiple of six months. The recognition of the six-month interval can be seen in the following letter:[32]

AN.MI ^{d}30 . . . *šá ina* ITI.SIG$_4$ GAR-*an*
. . . *ši-na ù a-du-ú ina* ITI.GAN *a-ga-a* AN.MI *i-ša-šak-kan*
[An eclipse of the moon . . . in month III will take place . . . and now in this month IX, an eclipse will take place.]
(Unassigned, VIII:502, as translated by Hunger [1992] p. 278)

Another example[33] again conveys the recognition of the six-month interval:

DIŠ *sin ina lā minā̀ti-šu atalā iškun-un-ma šá VI arhe lā samū-ú u ūmu XII-kan ūmu XIII-kan atalū iššakan-ma*
[If the moon is eclipsed at an unappointed time, (it means either) the six months have not yet passed, or alternatively an eclipse occurs on the 12th or 13th day.]
(*ACh.* Sin 3, 26)

It has been argued that by this time simple eclipse periods had been developed.[34] However, this claim is dubious due to a lack of direct evidence.[35] While it is plausible that some of the more confident scholars had an idea of certain periodic arrangements, the majority of the citations in the Letters and Reports reveal that the scribes had little appreciation for such techniques.[36]

At the very least, though, it was clear that they understood that eclipses recur at certain intervals, but the precise nature of these intervals was still a constant source of uncertainty. For example, the Assyrian astrologer-scribe Nabû-ahhe-eriba comments:

> [AN.MI *ina* Š]À UD.DUG$_4$.GA.MEŠ *a-da-an-na-a-te*
> [*us-se*]-*et-iq*
> [*ur-ki*] 5[?] ITI.MEŠ
> ITI.APIN *ma-ṣar-tu-šú*
> *ú-ma-a* ITI.KAN
> *ni-na-ṣar*
> [[Eclipses] cannot occur [dur]ing certain periods. [After] 5[?][37] months, there was a watch in Marchesvan (VIII) and now, in the month Kislev (IX) we will (again) keep watch.]
> (Nabu-ahhe-eriba X:71, as translated by Parpola [1993] p. 53)

This account reveals that the astrologers would wait five months after an observable eclipse to see if there was another one, and, if not, six months more. Another Letter also shows such an interval:

> [*il-ta-kan in*]*a* ITI.APIN ^{d}30 AN.MI
> [*ul-te-ti-i*]*q ina* ITI.BARAG *en-n[a]*
> [In Arahsamnu (VIII) the moon [let] the eclipse [pass] by. Now then, in Nisannu (I), . . .]
> (Bel-naṣir X:159, as translated by Parpola [1993] p. 120)

The critical element of these accounts is the identification of the five-month interval. It has been demonstrated by several detailed studies[38] that seek to reproduce the conditions in which eclipses were visible at the latitude of Babylon, that beginning from the early eighth century BCE an eclipse was *never* visible five months after a previous occurrence for almost the next half millennium.[39] In general, the phenomenon is very rare. Eleven-month intervals occur seldom, but larger intervals of six or eleven or more are the norm.

Thus, the astrologer-scribes knew that the five-month interval was the smallest interval between lunar eclipses, despite the fact they could never verify it empirically, and furthermore their observations would not have suggested it to them in the first place. This insight is remarkable. That they could propose

this connection may perhaps be explained in the following way: even though observers would only see eclipses separated by six or eleven (or larger appropriate multiples thereof) months, given the pressures of their profession, it seems likely that it was prudent to predict more eclipses, and thus shorten an eleven-month interval into a six- and a five-, than to run the risk of missing an eclipse occurrence. This is a noteworthy insight somewhat facilitated by wider divination-inspired concerns.

In the absence of more abstract geographical considerations, early thinkers had little means to address the disparity in frequency between lunar and solar eclipses, a feature which it seems they noticed. Some Letters and Reports may indicate more difficulty with regard to the prediction and observation of a solar eclipse, but no mention is made of why this should be the case. The following letter suggests that the scribe is less than certain about when the solar eclipse should occur, giving two consecutive months in which he will look for an eclipse:

ina UGU *ma-ṣar-ti*
šá ^{d}UTU *šá* LUGAL *be-lí*
iš-pur-an-ni
ITI EN.NUN *šá* ^{d}UTU
šu-u 2-šú EN.NUN-*šú*
UD.28.KÁM *ša* ITI.APIN
UD.28.KÁM *ša* ITI.KAN
ni-na-aṣ-ṣar
ki-i ha-an-ni-i-e
EN.NUN *šá* ^{d}UTU 2 ITI.MEŠ
[Concerning the watch of the sun about which the king, my lord, wrote to me, it is (indeed) the month for a watch of the sun. We will keep the watch twice, on the 28th of Marchesvan (VIII) and the 28th of Kislev (IX). Thus we will keep the watch of the sun for two months.]
(Balasî, X:45, as translated by Parpola [1993] p. 34)

Also:

ina UGU *ma-ṣar-ti šá* ^{d}UTU
šá LUGAL *be-lí*
iš-pur-an-ni
LUGAL *be-lí la ú-*[*da*]
ki-i da-a'-nu-tú-[*ni*]
UD-*mu ša ši-i-*[*a-ri*]
[*š*]*u-ú*
ú-de-e-šú
ma-ṣar-tú ga-[*am-rat*]

ú-še-taq-ma
la i-šak-kan
[Concerning the watch of the sun about which the king my lord, wrote to me does the king, my lord, not know that it is being closely observed? The day of tomor[row] is the only (day left); once the watch is over (this eclipse), too, will have passed by, it will not occur.]
(Balasî, X:46, as translated by Parpola [1993] p. 35)

Similarly, the following Letter may reveal some confusion as to whether or not a prediction should be made:

a-na UGU AN.MI
^{d}UTU *šá* LUGAL *iš-pu-ra-an-*[*ni*]
um-ma i-šak-ka-nu
ul i-šak-ka-nu
a-mat pa-ri-is-tú
šup-ra AN.MI ^{d}UTU
ki-i šá ^{d}30
a-na ŠU.2-*ia*
ul a-ṣi
it-ti-i
la eb-bi-ma
u am-qu-tam-ma
ul a-lam-mad-su
[Concerning the solar eclipse about which the king wrote to me: "Will it or will it not take place? Send definite word" An eclipse of the sun, like one of the moon never escapes me; should it not be clear to me and should I have failed (to observe it), I would not find out about it.]
(Unassigned, X:170, as translated by Parpola [1993] p. 130)

In their attempts to address the relations between solar and lunar eclipses, all that the scholars had recourse to were previous observations and the deductions that could be generated from these. From such accountancy of the appearances, some patterns were recognized. There does seem to be a connection made between a lunar eclipse and a solar eclipse occurring before or after it by the interval of a month. Solar eclipses were to be watched for around the conjunction preceding and following a lunar eclipse:

[*ina* ITI.KIN UD.]28.KÁM ^{d}UTU AN.MI
[*il-ta-kan in*]*a* ITI.APIN ^{d}30 AN.MI
[[On] the 28th of [Tešrītu] the sun made an eclipse; in Arahsamnu the moon [let] the eclipse [pass] by.]
(Bel-naṣir, X:159, as translated by Parpola [1993] p. 120)

and also:

> AN.MI ^{d}30 *u* ^{d}UTU *šá ina* ITI.SIG$_4$ GAR-*an*
> [An eclipse of the moon and the sun in month III will take place.]
> (Unassigned, VIII:502, as translated by Hunger [1992] p. 278)

Although most of the statements recording the observations of eclipses are very brief, giving only the day number and perhaps the month, there are several that give more information. A report (the author remains unknown) gives the following details of an observed eclipse:

> *ina* ITI.SIG$_4$ [UD].14.KÁM
> 30 AN.MI [*ina*] EN.NUN.UD.ZAL
> *i-sa-kan*
> *ina* IM.U$_{18}$.LU *i-sa-kan*
> *ina* IM.U$_{18}$.LU *ú-zak-ki*
> *ina* ZAG-*šú a-dir*
> *ina* KI MUL.GÍR.TAB *a-dir*
> MUL.*ku-mar-ru*
> *ša* MUL.UD.KA.DU$_8$.A
> *ziq-pu*
> 2 ŠU.SI AN.MI
> [x x] *i-sa-kan*
> [A lunar eclipse took place on the 14th of Sivan (III), [during] the morning watch. It started in the south (of the moon) and cleared up in the south. Its right side was eclipsed. It was eclipsed in the area of the constellation Scorpius. The *shoulder* of the constellation Panther was culminating. An eclipse of two fingers (magnitude) took place.]
> (Unassigned, X:149, as translated by Parpola [1993] p. 113)

and also a detailed observation[40] included in a lengthy letter from the Babylonian astrologer-scribe, Munnabitu:

> ITI.SIG$_4$ . . . UD.14.KÁM
> . . . *taš-ri-tum a-šar ú-še-er-ru-ú ul ni-i-du*
> *mi-ni-tú* AN.MI-*šu a-na* UGU IM.U$_{18}$.LU *u* IM.MAR.TU *il-ta-ha-aṭ*
> . . . *ul-tu* IM.KUR.RA *u* IM.SI.SÁ
> KI *šá im-mir*
> . . . *gab-bu-šú šá i-ri-mu*
> . . . *ina* AN.MI [^{d}30]
> MUL.SAG.ME.GAR GUB-*iz*
> [(the eclipse occurred in) Sivan (III) . . . the 14th day . . . the beginning, where (the eclipse began) we do not know. (The moon) pulled the amount of its

eclipse to the south and west . . . it became clear from the east and north . . . it covered all of the moon . . . In the eclipse [of the moon], Jupiter stood there.]
(Munnabitu, VIII:316, as translated by Hunger [1992] pp. 178–179)

At the beginning of the same letter, Munnabitu stipulates the elements important for his purposes:

šá AN.MI HUL-*šú a-na* EN ITI EN UD-*mu* EN *ma-aṣ-ṣar-tum*
EN *taš-ri-tum*
a-šar ú-šar-ru-ú ù a-šar ^{d}30 AN.MI-*šú i-šah-ha-ṭu-ma i-na-as-su-ku*
[The evil of the eclipse affects the one identified by the month, the one identified by the day, the one identified by the watch, the one identified by the beginning, where (the eclipse) begins and where the moon pulls off its eclipse and drops it.]
(Ibid., lines 3–4)

Thus, the elements deemed important to take note of are the day number, the month, the direction of impact and of clearing, the magnitude, the lunar position, as well as other notable celestial events (presence of planets, culminating constellation). Also added to this list is duration (although it is not given in the above two observations, it is recorded in another report (VIII:104, in which the lunar eclipse is observed to have lasted 2;30 BERU[41]).

Here, the influence of *EAE* is visible. The elements deemed important by these astrologer-scribes are those that are in the protasis material. This emphasis persists and is passed on to generations of astronomers and their treatment of eclipses. It is precisely these features that are deemed important by the Greek astronomers and others and much attention is given to constructing models that can predict these elements. Notably, the "direction" of impact, a measurement that has little use astronomically, but was of great importance in Mesopotamian omens, continued to be computed beyond the Mesopotamian tradition in which it originated. It was called *prósneusis* by Ptolemy, and the *valana* by the Indians, and in these later times was given serious theoretical consideration and graphical representation.

Lunar eclipses are frequently predicted.[42] Aside from the day number, the month, and occasionally the watch, the prediction of any other elements of an eclipse, such as magnitude, duration, and so on, is not recorded in any extant sources. For example:

ša LUGAL *b*[*e-lí*]
iš-pur-a-[*ni*]
mu-šu an-n[*i-ú*]
ina EN.NUN.UD.ZA[L.LA]
ma-ṣar-tu-šú

AN.MI EN.NUN.UD.ZAL.LI
i-šá-kan
[[Concerning the watch for a lunar eclipse] about which the king, [my lo]rd, wrote to me, its watch will be (kept) tonight, in the morning watch. The eclipse will occur during the morning watch.[43]]
(Nabû-ahhe-eriba, X:79, as translated by Parpola [1993] p. 58)

For the purposes of the astrologer-scribes, the watch in which the eclipse occurred was precise enough, as the division of the night into three watches provided the smallest unit of time with which the omens were classified in *EAE*. There was no motivation to record the time any more accurately in this context.

There are several eclipse reports in which the contents are puzzling. One letter could indicate some fairly accurate predictions, despite the fact that it states otherwise. This unassigned letter (the author remains unknown) contains a quote from the Assyrian astrologer-scribe Akkullanu who asserts that he witnessed a solar eclipse of two fingers in magnitude:

^{m}*ak-kul-la-nu šu-u*
is-sap-ra ma-a ^{d}UTU
ina na-pa-hi-šu ma-a
i-ba-áš-ši a-ki
2 ŠU.SI AN.MI *i-sa-kan*
[A certain Akkullanu has written: "The sun made an eclipse of two fingers at the sunrise . . .".[44]]
(Unassigned, X:148, as translated by Parpola [1993] p. 113)

What is surprising about this report is the detection of a solar eclipse of such a slight magnitude. In general, solar eclipses present the observer with more challenges than lunar eclipses. Solar eclipses are extremely brief—totality (if it occurs at all) never lasts more than seven and a half minutes and solar eclipses cannot be detected (due to the brightness of the disk of the sun) until at least nine-tenths of the disk is covered. Therefore, even if the time of occurrence is well established (the determination of which has its own challenges), articulation of the features of solar eclipses is difficult. Thus whether this statement was the result of some prediction procedure or a lucky coincidence, it is impossible to tell (if indeed it did correspond to an actual eclipse!).

But perhaps more revealing in an attempt to reconstruct the methods that were used to predict eclipses are those predictions that forecast the *nonoccurrence* of a lunar eclipse.[45] It was understood that a lunar eclipse could occur around the time of opposition, and that if the eclipse did not occur, then instead the moon would be seen in the sky as well as the sun:[46]

[*x* AN.MI] *it-ti-iq*
[^{d}30 KI] ^{d}UTU *in-nam-mar*
[The eclipse will pass by and the moon will be seen together with the sun.]
(Munnabitu, VIII:321, as translated by Hunger [1992] p. 181)

Implicit in the above statement is the comprehension of the relative positions of the sun and the moon at the time of opposition, and perhaps, by extension, something about their configuration during a lunar eclipse. The Report then seems to give Munnabitu's justifications for this prediction; lamentably, however, the text is fragmentary:

AN.MI *it-ti-iq la* GAR-*an*
ki-i LUGAL *i-qab-bu-ú um-ma*
mi-nu-ú i-da-tu ta-mur
[*x x x* DINGIR].MEŠ *a-ha-meš la* IGI.MEŠ
[*x x x*]*a-na mu-ši x*
[1. The eclipse will pass by, (it) will not make it. Should the king say: "What signs did you see?"
4. [. . .the god]s did not see each other.
5. [. . .] for the night.
(Ibid.)

Another clue as to the way in which they could have determined the nonoccurrence of an eclipse can be seen in the following Report:

^{d}30 *ina* ITI.KIN UD.15.KÁM
[TA] ^{d}*šamaš in-na-mar*
[AN.M]I *ú-še-taq*
[. . .]
[ITI.KI]N UD.13.KÁM
[The moon will be seen [together with] the sun in Elul (VI) on the 15th day, it will let [the eclip]se pass by. [. . .] [Mo. V]I[?] 13th day.]
(Nabû-ahhe-eriba VIII:42, as translated by Hunger [1992] p. 26)

Nabû-ahhe-eriba wrote this on the 13th of the month for a prediction that concerned an eclipse two days later. This suggests that there was some method that could confirm whether or not an eclipse would occur a few days before the eclipse. Another report is more detailed:

[*ina* ITI.G]UD UD-*mu ú-šal-lam*
[UD.14].KÁM ^{d}30 *u* ^{d}UTU
it-ti a-ha-meš IGI.MEŠ
[UD].13.KÁM *mu-ši šá* UD.14.KAM
[UD]-*mu* EN.NUN *u* AN.MI NU GAR
pu-t[*ú a-*]*di* 7-*šú na-šá-ka*

AN.MI *ul iš-šak-kan*
a-mat pa-ri-is-tum
a-na LUGAL *al-tap-ra*
[[In month I]I, (the moon) will complete the day; [on the 14th] day the moon and sun will be seen together. The 13th [day], the night of the 14th day, is the [da]y of the watch (to be held), and there will be no eclipse. I guarantee it seven times, an eclipse will not take place. I am writing a definitive word to the king.]
(Ṭab-ṣilli-Marduk, VIII:447, as translated by Hunger [1992] p. 251)

which also hints at the practice of observing the sun and the moon on the days leading up to an eclipse. This idea was first pointed out by Steele,[47] who argues that once it was established that the month contained an eclipse possibility, the astrologer-scribes would have to determine whether the eclipse would be seen or whether it would occur below the horizon and thus not be visible. They could do this, he argues, by observing the time interval between the time of sunset on the western horizon and the moon's crossing of the opposite horizon—this interval is directly related to whether the moon will reach opposition sooner or later. If the time was small, the eclipse would follow closely in time, if great, the eclipse might occur the following day, and thus not be visible.

They might have been more precise, and formulated the following rule: if, on the 15th day of the month, at the time of sunset, the moon is more than approximately 6;30° from the horizon, then opposition will not occur during that night, but rather the next day. One could extend this rule so that it would give the required information further back in time, so that for Nabû-ahhe-eriba's statement on the 13th day about an eclipse passing by, he could see that the moon was more than $2 \times 13° + 6;30° = 32;30°$ above the eastern horizon. A similar rule-of-thumb method was suggested by Pannekoek such that if the moon rose before sunset already seeming completely full, a lunar eclipse might occur that night.[48]

There is another interesting account of an opposition that was stated to have occurred, although it was cloudy. Again, it is hinted that the above method was used, which involved observing the moon several days prior to opposition:

LUGAL *be-la-a la i-qab-bi um-ma*
IM.DIRI *ak-ka-a-'i-i ta-mur*
mu-ši a-ga-a a-ṣu-ú-šú šá a-mur
ina ŠÀ-*bi* UD-*mu i-ṣu it-ta-ṣa-a*
qaq-qar-šú šá ina ŠÀ-*bi in-nam-ma-ru*
ik-ta-šad i-da-at šá a-ma-ru ši-i
šá še-e-ri ki-i UD-*mu pe-tu-ú*
LUGAL *im-mar* 1-*en* KASKAL.GÍD UD-*mu*

it-ti ^{d}UTU *iz-za-az*
[The king my lord must not say as follows: "(There were) clouds; how did you see (anything)?" This night, when I saw (the moon's) coming out, it came out when little of the day (was left), it reached the region where it will be seen (in opposition with the sun). This is a sign that it is to be observed. In the morning, if the day is cloudless (literally "open"), the king will see: for one "double-hour" of daytime (the moon) will stand there with the sun.]
(Nabû-iqiša VIII:293, as translated by Hunger [1992] p. 162)

In addition, if Parpola is correct in placing the following letter a day or two before a lunar eclipse, it too hints at the practice of observing the moon several days before opposition in order to determine whether an eclipse would occur or not:

ù ina UGU *ta-mar-ti an-ni-*[*ti*]
ša ^{d}30 *šu-ú* TA ŠÀ-*bi-*[*ia*]
ad-du-bu-ub mi-il-[*ki*]
lu-u šu-ú šum-ma DÙG.GA
me-me-ni nu-še-ši-ib MI [*ša* UD.15.KÁM]
il-lak li-ip-tu-šú [*ina* ŠÀ-*bi*]
e-pi-iš ú-ba-a[*l-laṭ-ka*]
[I am also worried about the impending observation of the moon; let this be [my] advice. If it is suitable, let us put somebody on the throne. (When) the night [of the 15th day] comes, he will be afflicted [by it]; but he will save your life.]
(Marduk-šakin-šumi X:240, as translated by Parpola [1993] pp. 190–192)

This letter refers to the practice of the previously mentioned "Substitute King Ritual" in which an individual was set on the throne in place of the king, so that the king would escape the potential bad effects, which would fall instead upon the substitute. As seen in this letter, the gravity of the threat to the monarch provided compelling incentive for scholars to sharpen their prediction skills.

In a similar fashion, there seems to be a connection between the observation of the moon and the occurrence of a solar eclipse. This connection may be hinted at by the Babylonian astrologer-scribe Rašil the older, in the following report:[49]

1 30 *ina la mi-na-ti-šú bi-ib-lum ú-bil*
AN.MI GAR-*an*
UD.24.KÁM 30 *i-tab-bal-ma*
[If the moon is carried off at an inappropriate time: there will be an eclipse—
The moon disappeared on the 24th day.]
(Rašil the older, X:382. as translated by Parpola [1993] p. 219)

and in this report from the Babylonian astrologer-scribe Ašaredu:

[1 UD.N]Á.ÀM *ina* NU ŠID.MEŠ-*šú* GÁL-*ši*
[*šal*]-*pú-tim gu-ti-i* GAR-*an*
[*ša*] UD.27.KÁM 30 TÙM-*ma*
1 UD.NÁ.ÀM *ina* ITI 3-*šú* [*x x*]
AN.MI GAR-*ma* DINGIR.MEŠ *ki* [*x x*]
[If the day of the disappearance[50] of the moon is at an inappropriate time: the ruin of the Gutians will take place. That means the moon disappears on the 27th day. If the day of disappearance of the moon in the third month [. . .], there will be an eclipse
(Ašaredu, VIII:346, as translated by Hunger [1992] p. 197)

Also linking the observation of the moon at the end of a month with a solar eclipse is the following letter from Esarhaddon's agent in Babylonia, Mar-Issar:

a-na MAN EN-*iá liš-ru-ku* UD.27.KÁM
^{d}30 *iz-za-az* UD.28.KÁM
UD.29.KÁM UD.30.KÁM *ma-ṣar-tu*
ša AN.MI ^{d}UTU *ni-it-ta-ṣar*
ú.se.ti-iq AN.MI *la iš-kun*
[On the 27th day the moon stood there; on the 28th, 29th, and 30th we kept watch for the eclipse of the sun, (but) he let it pass by and was not eclipsed.]
(Mar-Issar, X:363, as translated by Parpola [1993] p. 300)

Correspondence from Assyrian scholars reveals in many respects a new engagement with old material. Through these Letters and Reports, one witnesses scholars clearly responding to the exigencies of their profession, namely, providing their community with dependable astronomical forecasts and the corresponding portentous information. This demand encouraged scholars anew to engage with the phenomena in ever more ambitious ways. Given the alleged impact of these events, the ability to predict eclipses became critical—but attempts at improving this capability were never at the expense of relaxing the connection with an event's astrological significance. Details of eclipses were secured. Their appearance at specific times in the month was established, along with the noticing of the six-month recurring interval and tentative suggestion of the occasional five-month interval. This particular insight was a product of the circumstances; scholars were encouraged to somewhat overcompensate in their reckoning of reccurrence, given the disastrous consequences that missing an event would entail.

That solar eclipses seemed to present their own challenges seems to have been partially recognized, and ingenious techniques that could be employed

in the days leading up to eclipses to determine their occurrence or nonoccurrence were proposed. Methods remained very much rule-of-thumb and reveal a dependency on trends made in previous observations. The connections scholars were making were based on alignments and configurations whose geometric details they were nowhere close to understanding. They could verify that their proposals seemed to work, but they were in no position to articulate *why* they worked. Their reliance on empirical coincidences indicates their orientation and engagement with the phenomena. The pressing urgency to detail the patterns in the facts directed them to refine and deepen their accountancy of the appearances. The persistent and ultimate incentive of the prediction and interpretation of divine signs meant that they had little encouragement to investigate these events any more deeply. Nowhere was there inspiration to consider a reality behind the facts. Given the human-centered orientation, questions such as how the heavenly bodies interact, and how they determine what celestial phenomena will be viewed could not fully emerge. More abstract questions in which human concerns were muted never surfaced in this culture.

3.4 Babylonian Eclipse Reckoning

3.4.1 Early Techniques

The Assyrian astrologer-scribes were concerned with eclipses in the context of omens and were largely committed to texts such as *EAE*. In contrast, in other parts of Mesopotamia, thinkers were engaging with slightly more adventurous methods of eclipse prediction. While the Assyrians had concentrated on the ability to predict successive eclipse potentialities employing rule-of-thumb type techniques which focused largely on the immediate future, later generations of Babylonian scholars were more ambitious. Their main preoccupation was with a larger comprehensive reckoning of the patterns and periods of eclipse possibilities (EPs), that is, months in which eclipses were *not* to be excluded. Babylonian practitioners furthered their accountancy of the appearances with some impressive results. Their ability to predict eclipses was increasingly commanding and successful. However, their achievements did not amount to the articulation of theory. Theory implies depth, and although their efforts are praiseworthy and endearing, they remained grounded in patterns, numerical period relations, and arithmetical manipulations. They did not result in the formulation of larger unifying principles with respect to astronomical phenomena; prediction took priority over explanation.

A corpus of texts known as Late Babylonian texts which are housed in the British Museum[51] reveals sophisticated and innovative activity with respect to

eclipse reckoning. The texts include eclipse lists, individual eclipse reports, goal-year texts, and astronomical Diaries. From these, there is evidence of activity concerning lunar eclipses from around 750 to 50 BCE, and solar eclipses from 350 to 50 BCE.[52]

Steele argues that early on Babylonian astronomers had in their possession a fairly extensive record of observed eclipses, beginning in the mid-eighth century. Simply having a record of these eclipse occurrences is not enough to develop schemes, but there must have been some further investigation of the layout of information, for example, arranging them in a chronological order, so that patterns of their occurrences would emerge visually. Eclipses that were not observed because of factors such as cloudy weather, or the like, could be imagined to have occurred, because of an existing agreement with the pattern. Simple schemes for the arrangement of eclipse possibilities were developed.

From records such as the Diaries and their related texts, the ways in which practitioners dealt with their material can be quickly appreciated. For example, texts that outline larger eclipse cycles show a preoccupation with the distribution and layout of information in order to generalize knowledge gained empirically. It can only be speculated, but seems highly plausible, that other features of which we do not have evidence were also in existence. These could include an accurate month count and a substantial, but perhaps not necessarily complete, archive of lunar eclipses, perhaps along with other information such as a listing of varying magnitudes, directions of impact and other aspects which were deemed important.

Indirect evidence for the archive of lunar eclipses includes the following: the whole notion that sequences of eclipses are periodic in some sense and various aspects such as series of magnitudes, times of occurrence in the day, duration, will repeat in various cycles; the recognition of the regularity of the five-month interval (despite the fact that no eclipse was actually visible five months after a previous observation) occurring after seven or eight eclipse possibilities that were spaced by six months. Ptolemy also confirms the existence of an archive for deriving theoretical information about eclipses. The highly controversial reference by Pliny to Hipparchus being in possession of a 600-year list of eclipse observations may also give weight to this claim.[53] Smaller versions of such eclipse lists are in existence.[54] The earliest reference is contained in the fragment BM 41985 (LBAT 1413) which lists the observations of lunar eclipses in −747/6 and −746/5. The most substantial record is BM 32238 (LBAT 1414)[55] which is a compilation of lunar eclipse observations from −730/29 to at least −316/5.[56] This tablet is more than just an observation text. Given that it spans at least four centuries, it is a compilation of observations from presumably many sources into one large whole.

It has been argued that these lists, probably extracted from the Diaries, were compiled as a means of testing their forecasting efforts. Indeed, such a process which extracts information from a larger archive and distributes it into a layout that can then be directly compared with a predictive scheme shows the facility and sophistication with which the Babylonians refined their accountancy of the appearances. That they turned their attention to ever larger sets of observations, and not just successive occurrences, shows their ever increasing motivation to determine the patterns in the facts. Superior archives would allow them to further their capture of these patterns.

As seen in the previous section, eclipse forecasting was founded on the premise that eclipses occurred separated by some multiple of six months ($6k$) or, on occasion, one month less than such a multiple ($6k - 1$). Once this realization had been firmly established, the main focus became the determination of a distribution that would correctly place these five-month intervals, known as *5 itu* (literally "5[th] month"). A combination of empirical data and mathematical extrapolation must have eventually indicated to them that this interval follows either seven or eight six-month EPs. That is to say, on the seventh or eighth EP, all of which were six months apart, the next EP would occur five months later:[57]

$$1 \xrightarrow{6\ mos} 2 \xrightarrow{6\ mos} 3 \xrightarrow{6\ mos} 4 \xrightarrow{6\ mos} 5 \xrightarrow{6\ mos} 6 \xrightarrow{6\ mos} \mathbf{7} \xrightarrow{\mathbf{5\ mos}} 8 \xrightarrow{6\ mos} 9 \xrightarrow{6\ mos} 10 \ldots$$

or

$$1 \xrightarrow{6\ mos} 2 \xrightarrow{6\ mos} 3 \xrightarrow{6\ mos} 4 \xrightarrow{6\ mos} 5 \xrightarrow{6\ mos} 6 \xrightarrow{6\ mos} 7 \xrightarrow{6\ mos} \mathbf{8} \xrightarrow{\mathbf{5\ mos}} 9 \xrightarrow{6\ mos} 10 \ldots$$

As previously discussed, no eclipse visible in Babylon occurred five months after a previous eclipse in this time period.[58] Similarly, eclipses very seldom occur at an interval of eleven months (−678; −537; −396; −385; −338; −291). Thus the interval of five months is not as immediately obvious as it would seem, but has to be deduced from the assumption that eclipse possibilities occur every six months but sometimes certain multiples of six less a month (i.e., $6k - 1, k \geq 1$).

Once this five-month interval was recognized (as confirmed by the constant repetition of the phrase 5 *itu*), the next step was to determine the pattern of their arrangement. Did this five-month separation occur regularly and if so how often? A comparison of the visible eclipses with all the eclipse possibilities reveals that five-month intervals occur either after seven EPs separated by six months or after eight EPs separated by six months. In the best-case scenario, assuming all the visible eclipses in Babylon beginning in −746 were in fact observed and compiled in a single list, this trend would have been observable as early as ca. −720. As a comparison, a simple mathematical reckoning reveals that the distribution averages out to be a little over 7.5. This confirms that the

five-month interval distribution is sometimes after seven six-month EPs and sometimes after eight, with a distribution of slightly more eight EP intervals.

Bearing in mind these details, various arrangements of six-month and five-month intervals[59] can be used to predict eclipse possibilities, with varying degrees of reliability over time. Larger distributions (sometimes referred to as eclipse cycles), which combine several strings of seven and eight six-month intervals with a five-month interval in between, will generally be more successful, particularly those that have slightly more strings of eight six-month intervals than seven. In order to explore these possible arrangements in more detail, hypothetical arrangements are presented in the table below.

The length (in months) of each arrangement is given in the second column. The total number of eclipse possibilities and their arrangement into strings of seven and eight are given in the third and fourth columns. The "Breakdown" column highlights the fact that these larger cycles are simply combinations of smaller cycles. The next column gives the number of eclipse possibilities in each cycle separated by five months, and the last column offers, as a means of comparison, a technical calculation of the eclipse period—dividing the number of months—(column 2) by the number of eclipse possibilities—(column 3).

No.	No. of months	EPs	Distribution	Breakdown	No. of 5 itu	Period = m/n
1	41	7	7	1	1	$P = 5;51,25,42, \ldots$
2	47	8	8	2	1	$P = 5;52,30$
3	88	15	8-7	3 (= 1 + 2)	2	$P = 5;52$
4	129	22	8-7-7	3 + 1	3	$P = 5;51,49,5, \ldots$
5	135	23	8-7-8	3 + 2	3	$P = 5;52,10,26, \ldots$
6	217	37	8-7-8-7-7	3 + 4	5	$P = 5;51,53,30, \ldots$
7	223	38	8-7-8-7-8	3 + 5	5	$P = 5;52,6,18, \ldots$
8	358	61	8-7-8-7-8-7-8	5 + 7	7	$P = 5;52,7,52, \ldots$

For example, row 5 sets out a 135-month eclipse cycle that contains 23 eclipse possibilities—20 of these eclipse possibilities are separated by six-month intervals and three by five-month intervals. The arrangement of these eclipses through this large cycle is eight eclipses separated by six-month intervals, then one eclipse separated by a five-month interval, then seven eclipses separated by six-month intervals, then one eclipse separated by a five-month interval, and then eight eclipses separated by a six-month interval. The cycle then repeats.

The above list of periods is, of course, hypothetical. However, there is evidence in the sources to suggest that eclipse cycles of such a nature were utilized. Several are directly attested in cuneiform sources and were named, well-known

cycles. Several can be "matched" to ancient sources, but it is moot as to whether the authors of these sources were aware of the more general pattern. Others yet are so basic that in all likelihood they were used as rule-of-thumb methods. For the remainder we have no evidence that they were used, even though they are natural extensions of known cycles.

The most notable and commonly used of the above cycles was the so-called Saros cycle, measuring 223 months and containing 38 EPs.[60] The compilation of the Saros cycle must have been a result of experimentation with smaller cycles and the recognition of the inaccuracies caused by these smaller cycles. Such deficiencies accumulated over a short period of time could have provided the impetus for a longer more accurate period from whence the Saros was developed.

Evidence for the 135-month cycle is seen in the text ACT 93 = BM 34705 + 34960.[61] This text is simply a listing of numbers. Modern reconstructions reveal that the relations between the numbers are similar to a modern-day "saw function," with a maximum of 39, 0 and a minimum of 21, 0 and the (monthly) difference between successive entries is $d = 3,4$. It has been convincingly argued that the lines refer to successive syzygies and several underlying assumptions can be extracted from the text. These assumptions reveal the structure of the scheme—23 eclipse possibilities over 135 months, with a distribution 8-8-7.[62] There is nothing preserved in the text to indicate whether the text deals with lunar or solar eclipses.[63]

This text is simply the laying out of a linear pattern. It has no parameters such as longitudes that would allow one to assign a historical date to it. This in itself is interesting. For here is a text that is unconnected to what was directly observed. It reinforces the Babylonians' interest in exploring and extracting mathematical schemes that underlie observational phenomena. This is probably the result of the Babylonians investigating mathematically what they already knew empirically.

Further (tentative) indirect evidence for the 23:135 month cycle may be suggested in an eclipse magnitude scheme on the reverse of the tablet of BM 36651 + 36719 + 37032 + 37053 (the so-called Text L).[64] It has been argued that implicit in the eclipse magnitude scheme exhibited in this text is an eclipse cycle comprising 46 EPs over 270 months (two periods of 135).[65]

A 41-month period and a 47-month period, and perhaps their combination, the so-called 88-month period, have been hypothesized by various historians[66] to be implicit in some eclipse predictions. If a text predicts an eclipse for a date 41 or 47 months after a known visible eclipse (or some combination of multiples of 41 and 47), this can be taken as evidence that astronomers may have recognized the 41-month and 47-month eclipse cycles. Evidence from

predictions based on observations of visible (lunar) eclipses taken 41 months, 47 months, or a combination such as 82, 88, 94, 123, or 129 months prior to the prediction are looked for as a possible justification for the present prediction. As an example, the predicted eclipse 13 III 8 Cyrus = 15 June −530 which did not occur[67] has been the source of some debate regarding its origin.[68] Hunger and Pingree suggest that the justification for this prediction may have been from the fact that there was a visible lunar eclipse in XI 4 Cyrus = 21 Feb −533, which implies the 41-month cycle, or from the visible lunar eclipse II 1 Cyrus = 4 May −537 which implies the 88-month cycle.

Another plausible instance of the 47-month rule at work, this time in the case of a solar eclipse, is the eclipse of VII 28 of 40 Artaxerxes I = 23 October −424 which was not visible in Babylon. Van der Waerden[69] observes that 47 × 3 (= 141) months previously on 31 May −435 = II 28 of 29 Artaxerxes I there was a solar eclipse in Babylon which was total (mag = 1.00[70]). Van der Waerden then pairs a lunar eclipse prediction for III 15 Nebuchadnezar = 4 July −567 with the observation of a lunar eclipse on VI 12 of 33 Nebuchadnezar = 14 September −571, 47 months earlier. However, these correlations must be treated with a degree of caution because larger cycles, such as the Saros, are made out of these smaller eclipse periods. Thus it is impossible to confirm absolutely that these relations are not part of some larger scheme, when only fragments remain.

Further experimentation with the arrangements of eclipse intervals within larger cycles are attested. For example, BM 37149 (the so-called Text D),[71] the "Solar Saros" text which gives the months of solar EPs over the interval comprising five Saros cycles contains a Saros cycle in which the groups are distributed 8-6-8-8-8, instead of the expected 7-8-7-8-8.[72] BM 36599 + 36941, BM 36737 (+) BM 47912 and BM 36850 (the so-called Text S), first published as one of the "two lunar texts" by Aaboe and Sachs[73] and later redone and identified as a solar eclipse scheme by Britton[74] has 38 eclipses in the arrangement 7-8-8-7-8 (a rearrangement of 7-8-7-8-8).

Although the Saros was the most accurate cycle that is described, it too becomes inaccurate over a period of many cycles. In fact, the Saros cycle is slightly too short. Cycles that were too short (i.e., the Saros) could be combined with cycles that were too long (i.e., the 135-month cycle) to develop a larger, more accurate period relation, but there is no evidence to show that the Babylonians considered this. It is not attested in the sources that they thought to combine these periods to create even longer cycles.

By the time Babylonian lunar theory reached its peak in the Seleucid period, the astronomers were able, with success and a certain degree of accuracy, to forecast the occurrence of lunar eclipses along with some of their features.

Attempts were similarly made to predict series of solar eclipses. Interestingly enough, it seems that they treated solar eclipse possibilities in an analogous manner to the schemes they had developed for predicting lunar eclipse possibilities. This is in spite of the clear observational discrepancies between the two. Indeed, from the vantage point of the Babylonians, for every four or five lunar eclipses, there was roughly only one solar eclipse that could be observed because of the location-specific visibility constraints (and this would be the best-case scenario; the proportion was probably even less, given weather conditions, magnitude factors, etc.).

Thus it is surprising, given this discrepancy, that solar eclipse schemes that were similar to lunar eclipses were adopted. Textual evidence suggests that sequences of solar eclipse possibilities, which mimicked the periods of lunar eclipse periods, such as the Saros, were in existence. Text S gives evidence of this.[75]

Along with a complete set of eclipse records, the cyclical nature of other features was no doubt recognized, including that of magnitude. Generally speaking no two successive lunar eclipses are the same, but there is a natural, observable pattern of magnitudes, which loosely repeat. A group of eclipses (say five or six), in general, commences with one or two partial eclipses (often of increasing magnitude) followed by one to three total eclipses, and concluding with one or two partial eclipses (often of decreasing magnitude).[76] This trend was no doubt noticed and perhaps was worked into a rule-of-thumb method, such that if a partial eclipse occurred with no predecessor six or twelve months previously, one could assume that a new group had begun. This would not take long to confirm.

Texts which confirm an interest in cycles of magnitudes include the so-called Text S which contains 38 Solar eclipse possibilities from Xerxes 11 VIII to Artaxerxes I 8 IV (−474 to −456). This text has similarities with later-appearing schemes (such as "System A" which will be discussed below), as well as two otherwise unattested functions relating to eclipse magnitudes.

3.4.2 Nonmathematical Astronomical Texts: The Astronomical Diaries

An important source of the nature of astronomical observations as well as of the predictions that the Babylonian astronomers were carrying out is the text genre known as the astronomical Diaries. The Diary tradition spans an impressive time period, with the earliest extant text dated in −651, and the latest in −60.[77] The Diaries, as their modern name implies, usually contain a day-by-day account of various astronomical phenomena. A single tablet will most commonly cover a six-month period. Among the astronomical phenomena therein

recorded were measurements of the lunar six (the various intervals between moonrise and set and sunrise and set), the passage of the moon among the so-called Normal Stars, material related to Sirius, planetary phases, the length of the month, solstices and equinoxes, occasionally comets and meteors, as well as eclipses. For the most part, the material recorded is from observations, but some phenomena are obviously predictions (whether explicitly so identified or not). Solstices and equinoxes were always computed—there are no visual indicators to mark either events. Thus, for completeness, when the weather prevented an observation being made, the records of other phenomena also were based on computation rather than observation.

As Steele notes,[78] most eclipse material from the nonmathematical astronomical texts (NMAT) can be grouped into five main sub-divisions:

- Straightforward lists of consecutive eclipse observations and predictions
- Lists of consecutive eclipse observations and predictions arranged in 18-year cycles
- Texts containing consecutive eclipse observations and predictions together with other material
- Reports of individual eclipse observations
- "Theoretical"[79] texts

It has been argued that the origin of the predictions that are included in the Diaries is the Almanacs and the Normal Star Almanacs, a different group of astronomical texts. Texts of this genre set out phenomena that have been computed for the following year—that is, they do not contain observations, only predictions. The astronomical material recorded in the Diaries appears to have been excerpted and compiled into further separate texts. In the case of eclipse phenomena, these are the Goal-Year texts[80] and the individual "Eclipse texts."[81]

In general, the terminology often indicates whether or not the recorded eclipse was an observation or a prediction.[82] An eclipse of the sun or the moon that has been observed is referred to as

šamáš AN.KU$_{10}$ or *sin* AN.KU$_{10}$[83]

In contrast, when a prediction is referred to, the terms are in the reverse order:

AN.KU$_{10}$ *šamáš* or AN.KU$_{10}$ *sin*

Other indications in terminology that the eclipse has been predicted rather than observed is the phrase *ki* PAP NU IGI, which translates as "when I watched

I did not see it." This suggests that an eclipse which had been predicted was not in fact seen. Otherwise, the phrase *ša* DIB (literally "omitted") adjacent to a record of an eclipse meant that the eclipse occurred, but could not be seen because it occurred while the luminary was below the horizon.

As Steele has noted,[84] occasionally the phrase *ša* DIB was replaced by BAR DIB,[85] and he shows, using LBAT 1426 (+) 1427 as an example, that there is a deliberate difference in the nuances of the two terms. He convincingly concludes that BAR DIB is used for an eclipse prediction made when an earlier visible eclipse had not recently occurred, and, in contrast, *ša* DIB, one or two entire Saros cycles (that is, 18 or 36 years apart) after a visible eclipse.[86] This supports the idea that the Babylonians were able to jump not only from one successive eclipse possibility to the next, but also between eclipse possibilities separated by a complete Saros cycle. This may argue for the existence of the arrangement of material in tabular format which contained one complete Saros cycle of eclipse possibilities down the rows, and multiple Saros cycles along the columns. From row to row successive eclipse possibilities were tabulated, and from column to column, the eclipse possibilities were separated by a Saros cycle.

As Steele observes,[87] an eclipse prediction is often accompanied by a time interval relative to sunset or sunrise, a time that gives the moment of the *beginning* of the eclipse, rather than the instant of syzygy or mid-eclipse. This shows that these predictions were tied to observation. More mathematically oriented approaches that were to develop later gave the moment of mid-eclipse, an instant that has no distinct visual marker.

The earliest prediction of an eclipse preserved in an astronomical Diary has the date of −651.[88] It has been argued that by at least the sixth century BCE Babylonian astronomers were using the Saros period of 223 months to predict their eclipses. Calling this period simply 18 MU.MEŠ (literally "18 years"), they observed that eclipses 223 months apart were very similar.

Once the arrangement had been established, the next challenge was to determine where exactly in such a cycle the eclipse fell, or more precisely, the date at which such a cycle began. An answer to this question can be seen in several tablets. Walker[89] has suggested that a certain set of tablets (LBAT *1420, LBAT *1414 LBAT 1415 + 1417, and LBAT *1419) form part of a series containing records of eclipses from 747 BCE to 315 BCE. Steele suggests the first eclipse in the 8-7-8-7-8 arrangement was the eclipse occurring on 6 February 747 BCE.

Steele[90] also observes that when the eclipse possibilities from the Saros are compared to other NMAT sources, there is a direct similarity between all the eclipse possibilities until about 250 BCE. After this date the scheme began to fail and alternatives were introduced, largely involving an adjustment to the beginning of the 8-7-8-7-8 arrangement.

The above arrangement was not the only scheme used to distribute the Saros EPs; other alternatives were in existence.[91] Steele suggests that, as a new scheme was introduced around 250 BCE, it is likely that several existed before one was actually adopted.

The actual time at which the eclipse was going to occur was also predicted. Increasingly, it seems astronomers were no longer satisfied with simply stating the watch (a period of about four hours) in which the eclipse would occur. The Saros cycle accumulated a difference of about eight hours, so that a rough estimate from an eclipse in one Saros to the next could be predicted by adding eight hours to the beginning of the former eclipse prediction. Steele observes that the actual accuracy of the predicted eclipse times does *not* significantly improve over the whole Babylonian period.[92] This could imply that the methods used to determine this time were the same and there was no innovation throughout the Babylonian period, despite the inception and development of mathematical astronomy that we see during the Seleucid era with the Astronomical Cuneiform Texts (ACT) material.

In general, though, Babylonian scholars were able to predict eclipses with increasing confidence. In particular, they developed broad predictive schemes the foundation for which they had extracted from archives of existing observations and projected these far into the future, taking note of when they worked as well as when they did not. Periodicity was recognized not just from one successive possibility to the next, but in rather more ambitious schemes—periods that spanned almost two decades were investigated. The ability to make long-range forecasts with confidence was largely a result of the Babylonian commitment to the compilation of large archives of observed phenomena. Comprehensive archives of observational phenomena were not enough, however. Babylonian scholars excerpted relevant material from these archives and extracted more general patterns from the sets of individual recorded instances of phenomena. But despite the sophistication in compiling and managing their observational archives, their efforts would remain limited to an ever more sophisticated accountancy of the appearances. Aberrations in the pattern forming were noticed, but attempts to deal with these resulted at the most in further mathematical corrections, and did not direct these scholars to consider a deeper picture which could explain these events, beyond their search for increasingly confident expectations of their occurrences.

3.4.3 Mathematical Astronomical Texts: Astronomical Cuneiform Texts

In Mesopotamia, the progress of the moon was important to many aspects of daily life, none more so than the grouping of days into months and months

into years. The beginning of the month in the civil calendar was based on the visibility of the first crescent, which is a nontrivial challenge to predict. In fact the main drive behind much of Babylonian lunar investigations was to be able to predict this event accurately and confidently, and the techniques and methods developed to do this were the driving force for the progress and development of ancient mathematical astronomy as a whole.

From early on, practitioners had developed and employed a number of different techniques for determining the visibility of the lunar crescent. But a distinctly different approach to this problem arose just before the beginning of the Seleucid era (311 BCE). A new genre of texts, referred to today as Astronomical Cuneiform Texts[93] saw predictive techniques in the ancient Near East emerge with an entirely new foundation. The basis of this approach was centered on the simple premise that phenomena of a periodic nature, albeit complex, can be modeled in terms of the result of a combination of several less complex periodic schemes.[94] Such schemes in this context are what are known today as periodic step functions and linear zigzag functions (fig. 3.1) and their cumulative effect accounts for the phenomena in question. This mathematical astronomy contained in ACT texts can be divided into two distinct categories, system A and system B, with the major difference between the two systems lying in their characterization of the yearly motion of the sun. The former system uses a step function as a basis for solar velocity, the latter a linear zigzag function. The first extant ACT text was composed in 319 BCE and the latest almost four hundred years later in 42 CE.

This genre has been split into three groups, known as "ephemerides," "procedure texts," and "auxiliary functions." Ephemerides[95] are entirely numerical and give tabulations of successive phenomena (such as conjunctions or oppositions). The procedure texts set out the rules and demonstrate the computations behind these numerical patterns, and auxiliary texts present the various mathematical relations that are needed for the ephemerides (such as solar velocity, various corrections, etc.).

The primary motivation for mathematical astronomy of the ACT type was to determine the precise moment of conjunction. However, this approach could be and was useful for other astronomical phenomena as well. From a consideration of conjunctions and oppositions, the conditions for the occurrences of lunar and solar eclipses can be produced, by factoring in additional information such as the latitude of the moon. Although the determination of the exact time at which a syzygy occurred was the primary aim, eclipse theory was an important secondary feature.

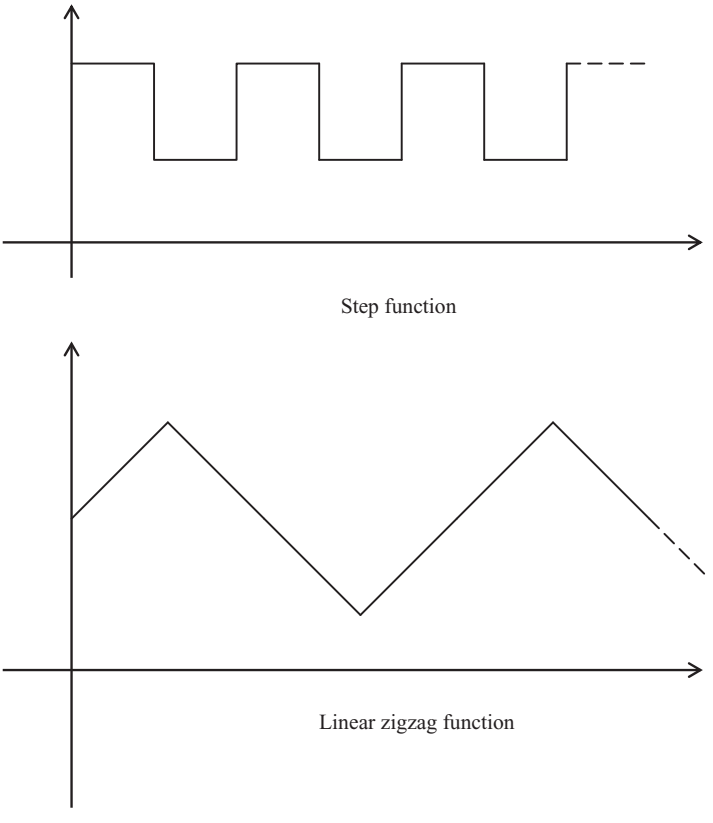


Figure 3.1 The step function and linear zigzag function, which reflect geometrically the ways in which rates of change were computed for system A and system B, respectively.

The lunar ephemerides are immediately recognizable, because of their distinct layout of information. Each text deals either with conjunctions (from which information about solar eclipses can be derived), oppositions (from which information about lunar eclipses can be derived), or a combination of the two. The progressions of numbers are arranged in columns, in which (for the most part) each successive column is mathematically dependent upon the previous column(s).[96] Each column has a specific purpose, tabulating dates, longitudes, latitudes, velocities, and their various corrections. Generally, later columns are dependent on previous columns for their tabulations. For example, the column that tabulates the longitude of the moon is necessary for the later column of lunar latitude. The columns are not labeled by their authors, so modern scholars gave each column a capital letter designation,

roughly alphabetical (but not consistently) for the order in which they generally appeared.[97]

Out of the many columns, those which are (directly) necessary for eclipse computations are as follows:

Column T: the dates of the syzygies in months and years of the Seleucid Era
Column B: the approximate longitudes of the two luminaries at syzygy
Column E: the latitude of the moon at syzygy
Column ψ: the so-called magnitude function
Column M: the precise moment of syzygy

The basic information required to predict an eclipse is when syzygy occurs (which is given by columns T and M) as well as a check for the alignment of the two bodies (which can be ascertained from columns B and E). With respect to the time, column T provides the date and column M gives the instant of syzygy, and therefore could provide the details of when the middle of the eclipse was going to occur. There is evidence from the extant tablets that column M was used for such a purpose. A system B tablet (ACT 136 = U 132) which lists eclipse possibilities, presumably excerpted from larger ephemerides, has simply included (in the fragment remaining) column T and column B, and on the reverse, column ψ and column M. The fragment (albeit incomplete) tabulates only the columns T, B, M, and ψ, in the following arrangement:

Obverse	I	II	Reverse	I	II
	T	B		M	ψ

This reveals that the authors of the ACT texts were indeed concerned with eclipses themselves.

Beyond the basic information, other columns that provide additional information, beyond the basic, pertinent to eclipse reckoning. Column F (velocity of the moon) and following columns provide elements that can be used as a basis from which to compute the conditions of the actual visibility of an eclipse—conditions such as the position of the moon with respect to the horizon. To know whether or not the eclipse will be visible one needs information about the exact time of the syzygy (whether it falls during the day or at night).

Despite the fact that the primary aim of the ACT texts was to predict the visibility and conditions of the first crescent, there is further evidence to show that the compilers were also interested in eclipses. Column ψ has no

other function but to indicate an eclipse possibility. Furthermore, a sub-set of these ephemerides are devoted exclusively to eclipse computations, texts in which compilers have excerpted the relevant syzygies which contain eclipse possibilities from the larger general lunar ephemerides. In fact, for example, ACT 135 includes a colophon in which the author states that this text is "on eclipses of the moon" (*šá AN.KU$_{10}$ sin*). As the ephemerides themselves are arranged in a series of successive conjunctions or oppositions, one only needs to consider the latitude of the moon from column E and extract those lines in which there is an eclipse possibility to compile a list of successive eclipses. This was made even simpler by the inclusion of visual indicators, in particular the logogram HAB which was placed next to each syzygy (in column ψ) that contained the possibility of an eclipse. System A and System B could therefore be used to predict the occurrence, the time of mid-eclipse, and, indirectly, the magnitude.

Such tablets dealing primarily with eclipses as part of the ACT corpus are sparse. In system A, there are so far nine texts, of varying degrees of preservation, and in system B only three.

System A	Lunar	60, 51a
	Solar	50, 51, 61a, 52
	Both	53
	Either	54, 61
System B	Lunar	135, 136
	Solar	130

An Outline of the Columns and Their Functions

Though the primary aim of ACT calculations was not to predict eclipses, as mentioned, five columns can be extracted for that purpose. From these five, two are directly pertinent to eclipse computations; these are column E and column ψ. Mathematical analysis can give a good impression of what the numbers in each stand for and how they interact with one another. However, what is less understood is exactly how these columns were conceived. While much work has been done on determining the numerical patterns that are presented in the ACT material, many questions remain as to the origin of these numbers.

One feature that is notable in the presentation of the numbers in the various columns of the ACT ephemerides is their distribution of different units from one column to the next. For example, three columns that pertain to eclipse theory are measured in three different units. Column B, the column that deals with the longitude of the moon at syzygy, is measured in degrees (UŠ). Column E, the latitude function for the moon, is measured in barley

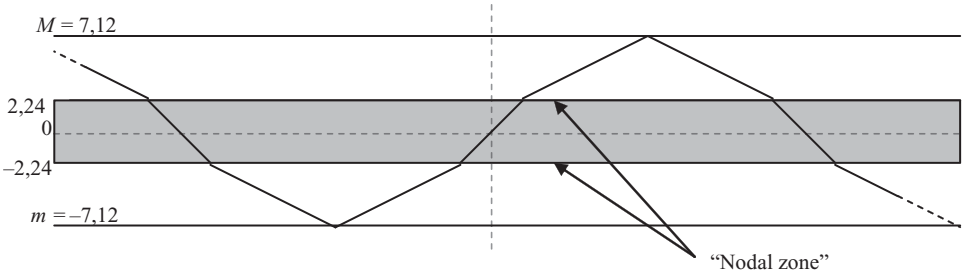


Figure 3.2 Graphical rendering of column E.

corns (ŠE), and column ψ, the so-called eclipse-magnitude function, is measured in fingers or digits (ŠU.SI). Such variation within one system seems a little puzzling, until the physical layout of the ephemerides themselves is taken into account. The adoption of distinct units can be easily explained by their usefulness in distinguishing one column from the next. One can, at a glance, tell what column one is looking at because the number and limits are particular to the column they represent. Conversions are given by the following:[98]

	Fingers	Barleycorns	Degrees
1 finger	1	6	0;05
1 barleycorn	0;10	1	0;0,50
1 degree	12	72	1

Column E

Column E is a latitude function, giving the latitude of the moon at syzygy.[99] Although this column is a single series of numbers, an understanding of the relationship between successive tabulations requires extrapolation of several separate schemes. These underlying individual patterns combine together in a complex way to cumulatively form the series of entries in column E. Briefly, column E is not a straightforward zigzag function[100] (fig. 3.2). Successive entries have variable differences—differences that are derived from what can be characterized as similar to a modern step function, known as ΔE. The jumping points of this step function[101] ΔE are identical to those of column B. In this way, column E in effect takes into account the variable velocity of the sun, but contains the assumption that the lunar velocity is constant.

The jumping points of ΔE occur at the following points:

ΔE_1	↑ Virgo 13	ΔE_2	↑ Pisces 13
	↓ Pisces 27		↓ Virgo 27

where ΔE_1 refers to conjunctions and ΔE_2 to oppositions, with the following values for each of the steps (where D is the maximum and d the minimum):

$$D = 2,6,15,42,$$

$$d = 1,58,45,42.$$

E is computed by adding successive values of ΔE. When the maximum (M) or minimum (m) is reached, the increments are reflected at these extrema. The extrema of E are $\pm$7,12 ŠE which is equivalent to 6°. This is an acceptable value for the extremes of lunar latitude.

There is an additional layer of complexity to column E however. The above process holds for only two-thirds of the values of E. A region (often referred to as the "nodal zone," for reasons that will become clear shortly), can be identified a namely,

$$\text{-2,24 ŠE} < E < \text{2,24 ŠE},$$

in which the rate of change is *double* that outside this region. The limit $\pm$ 2,24 ŠE seems to have been selected by dividing the whole band of E (14,24 ŠE) into three, to get three smaller equal-sized strips of 4,48 ŠE, so that the upper band runs from 7,12 ŠE to 2,24 ŠE, the middle (the nodal zone) from 2,24 ŠE to -2,24 ŠE, and the lower from -2,24 ŠE to -7,12 ŠE.[102]

Column E is also notable because it is one of the few columns that contains "technical terms" alongside the series of numbers. Along with each value of E, there is one of the following four pairs of logograms:[103]

LAL	LAL	"Positive"	"Increasing"
LAL	BUR_3	"Positive"	"Decreasing"
BUR_3	BUR_3	"Negative"	"Decreasing"
BUR_3	LAL	"Negative"	"Increasing"

in which each of the two relevant logograms, LAL and BUR_3, has been translated carefully to reflect the context in which they seem to be intended—distinct in each instance. There is inherent in the above layout an idea of the "orientation" of the moon's position with respect to an intersection point of the moon's path with the sun's path. The above system gives four distinct positions for the moon in relation to the sun's path: being near the ascending node while moving away from it (positive and increasing), having reached the maximum distance from the sun's path and now moving toward the descending node (positive and decreasing), having crossed the descending node and moving away from it (negative and decreasing), and having reached the maximum distance from the sun's path in the opposite direction and now moving back up toward the ascending node (negative and increasing). This layout

suggests the conception of a point at which the latitude function changes sign—and it is this layout that is the closest the Babylonians come to defining the moon's node.

It has been argued that such terminology accompanying the numbers is rather cumbersome, and the motivation behind their inclusion has been questioned. No other column in the ephemerides has two explanatory logograms accompanying it; most have none. There may be a simple explanation, from the point of view of using this column to search for eclipse possibilities. In terms of these eclipse possibilities, of critical interest are those positions of the moon that are close to the node. One needs only to scan down the list in a general ephemerides text and identify where a LAL changes to a BUR_3 or a BUR_3 to a LAL in the first position. This indicates in effect that the moon has changed its orientation from south to north of the sun's path, or north to south, or in other words, that it has crossed the node. Thus, by a simple scan down the column, one can find the row (or two rows) pertinent to an eclipse possibility.

In addition, these orientations can be used to determine what part of the disk is going to be eclipsed first. A BUR_3 LAL designation will indicate that the northern part of the disk will be eclipsed first, a LAL BUR_3 designation, the southern part of the disk. The direction of impact was important to those calculating the circumstances of an eclipse, as has been seen from its emphasis in the omen series in *EAE*. The use of the logograms in this context is telling. It shows that the concepts of relative orientations were utterly bound in arithmetical patterning. It was the rising and falling of numbers and their relative relations numerically that indicated alignment, and not a geometrical approach.

A procedure text gives further (rare) insight into how they imagined the path of the moon:[104]

> *12 DAGAL ma-lak* ^{d}*Sin*
> [12 (degrees) is the width of the path of the moon.]

This suggests that they viewed the extrema of lunar latitude at $\pm 6°$. Column E measures its extreme at $\pm 6°$, and the tablet BM 41004 (the so-called Text E), confirms this. Although the recognition of the change of sign in the middle of this band would support the idea of the ecliptic conceived as a line through the middle of this band, there is no evidence to sustain the notion that the Babylonians viewed the latitude of the moon in terms of the moon moving on an orbit that was related to the ecliptic, or that the ecliptic was in the middle of this band. It seems rather that they were interested in defining the path of the moon, as one can view it at night against a backdrop of fixed stars.[105] This is less easy with the sun, of course. In very early texts such as MUL.APIN the path of the sun is imagined as the same as the path of the moon.[106] It is not

clear that they consistently viewed the sun as traveling on the single band, as in ACT there exist expressions such as

> 7 šu.si ^{d}*Šamaš* NIM *u* SIG. . .[107]
> [7 fingers of the sun positive or negative latitude. . .]

Eclipses occur only near the nodes—the intersection points between the paths of the sun and the moon. Whether explicit or not, a reckoning of the nodes is fundamental for gauging the success of long-term prediction patterns.[108] The lunar nodes themselves move with respect to the ecliptic and embedded in column E is a very accurate value for this motion (sometimes referred to as nodal elongation). This parameter is nowhere explicitly attested, but it is undeniably the basis of the latitude scheme presented in column E and its difference function, the step function ΔE. The nodal elongation can be extracted and is:[109]

$$\eta = 1;33,55,30^{\circ} \text{ per month.}$$

It is not yet known how this parameter was derived. It has been suggested by Britton[110] that this value may be derived from an eclipse period, larger than the Saros, which combines Saros and the 135-month periods. He argues that they may have noticed that the Saros, with its period $P = 5;52,6,18,\ldots$ (see section 3.4.1) was a little too short, and in order to improve this they combined it with a period that was a little too long in order to produce a more accurate eclipse cycle, which would in turn suggest a more precise parameter for nodal retrogression.

In reconstructing this period, there needs to be found an appropriate combination (n) of the two periods (223 months and 135 months) with an even distribution of the two, such that

$$\text{months} = 223n + 135(n-1),$$

$$\text{EPs} = 38n + 23(n-1).$$

Using the following value for the mean monthly progress of the moon:

$$\text{System A: } \Delta\lambda = 360/12;22,8 = 29;6,19,0,55,37,24,\ldots\,[111]$$

Britton observes that $n = 5$, that is five Saros cycles interspersed with four 135-month periods, will give

Month:EPs	n	P		$\eta = 360/P$	k
1655:282	5	1655/141	11;44,15,19,8, . . .	30;40,14,30,5, . . .	1;33,55,29,9, . . .

so that $k = 1;33,55,29,9\ldots \approx 1;33,55,30$ was derived in this way. Nodal elongation is not explicitly described as a separate phenomenon, but seems to have

been understood in the context of the determination of whether or not eclipses would occur.

Column ψ

The next column of ACT, column ψ, is a particularly difficult column to unravel.[112] This column seems to relate nodal elongation and eclipse limits, and is frequently called the eclipse-magnitude function.

In contrast with column E, column ψ is measured in fingers. This column is handled quite differently by systems A and B, with its treatment in system A being better understood than that in system B.

Column ψ is closely connected with column E, as can be immediately seen (fig. 3.3). In fact it is simply a linear *translation* of column E. In system A, ψ is manifested in two ways that are mathematically identical, but are simply formatted differently; one computes the value just for those values of E that fall inside the nodal zone (ψ), the other, for all values (ψ'). ψ, which is defined only for the smallest absolute value of E, is calculated by

$$\psi = 17;24 \begin{cases} +\frac{E}{6} & \text{around the ascending node} \\ -\frac{E}{6} & \text{around the descending node.} \end{cases}$$

In system A, ψ has one tabular entry at the value of E inside the nodal zone, which is its smallest absolute value, and is empty everywhere else. E has at

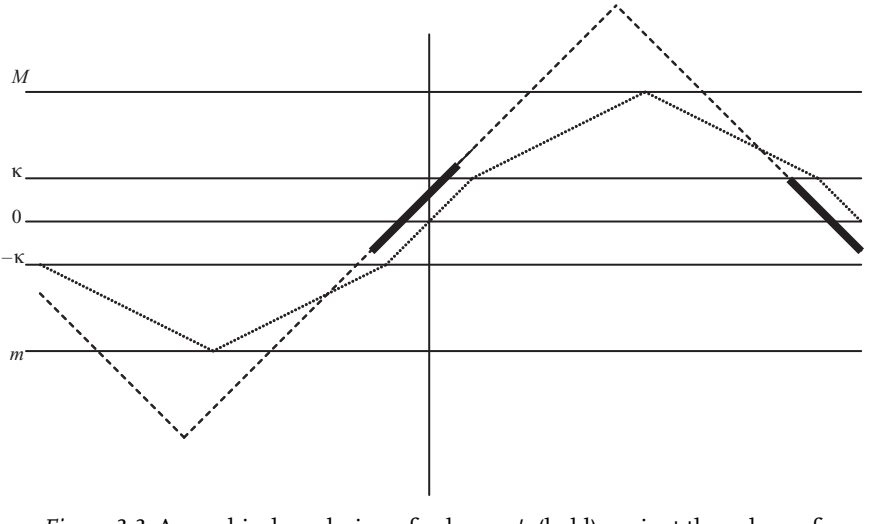


Figure 3.3 A graphical rendering of column ψ (bold) against the values of column E (dotted) inside the "nodal zone." Column ψ has been extended (dashed) to produce ψ'.

least one value inside the nodal zone and sometimes two, but never three. The divisor 6 simply converts units of ŠE, in which E is measured, to units of fingers in which ψ is measured.

The other function, referred to as ψ', is a *continuous* zigzag function and is calculated for *all* values of E, both in and outside the node, and not just ones to which an eclipse-possibility was ascribed. If ψ is an eclipse-magnitude function of some sort, then these extra values are superfluous. For values of E inside the nodal zone, the above formula is used. However, to continue ψ outside the nodal zone, some adjustment must be made to E, so that the same gradient of ψ is preserved. Because the gradient of E is half as steep outside the nodal zone, E is multiplied by a factor of 2 so that the gradient of ψ' is continued. ψ' itself is a simple zigzag function, out of phase with lunar latitude and with extrema $\mathrm{M} = -\mathrm{m} = 2{,}0$ fingers ($= 12{,}0$ ŠE $= 10°$). As is immediately evident, ψ and ψ' are essentially the same function; it is simply a question of how often they have been tabulated with respect to E (see fig. 3.3).

From the point of view of someone who wished to select the smallest absolute value of E in order to calculate ψ, the designations of LAL and BUR_3 next to the values of E would have been very helpful. All one has to do is glance down the column of E and when the second LAL or BUR_3 changes to a BUR_3 or LAL,[113] this is the place at which the value of ψ should be calculated.

As is clear from the diagram, ψ is simply a function of nodal elongation, translated by 17;24 fingers. Because of this, it is puzzling why it has been included as a separate column. Inside the nodal zone, ψ reflects the lunar latitude measured in fingers and out of phase. Perhaps the reason ψ has undergone a phase shift is because the Babylonians wanted the zero point of E to be the inferior eclipse limit.

ψ has always been considered a "magnitude function." However, some arithmetical manipulation must be performed before the values of ψ make any sense as an eclipse magnitude. As a consequence of this, Neugebauer proposed[114] another function (a modern invention which is never attested anywhere in these texts) called ψ^*, such that

$$\psi^* = \psi \qquad \text{if } \psi \leq 17;24^f$$
$$\psi^* = 2c - \psi \quad \text{if } \psi \geq 17;24^f$$

(where $c = 17;24^f$), so that an eclipse was possible (fig. 3.4) if

$$-6;36^f \leq \psi^* \leq 17;24^f$$

and accordingly the maximum eclipse limit was

$$E(0) = \pm 17;24 \text{ fingers} = \pm 1;27°.$$

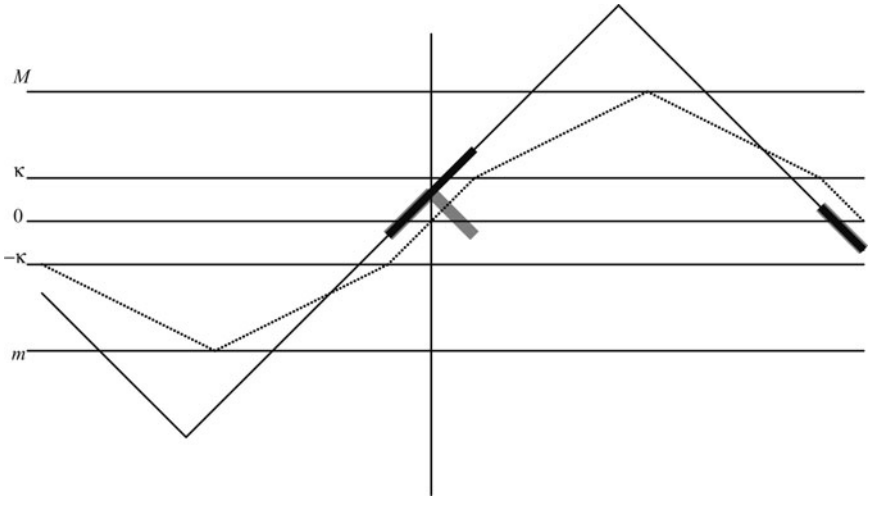


Figure 3.4 A graphical rendering of $\psi*$.

However $\psi*$ is still not useful as an eclipse limit as it is not symmetrical about the axis of latitude (around the nodes) as it should be. Furthermore, the value of $17;24^f$, its origin and significance, remain a mystery.

To investigate further, consider $17;24^f$ in its wider context as an eclipse limit. Its effectiveness as an eclipse limit can be considered via its relation to the sizes of the luminaries:

$$r_u + r_m = 1;27^\circ (= 17;24^f).$$

Supposing a ratio of 2;30 for moon to shadow, as seen in early Greek and Indian texts, the following amounts can be derived for the radii:

$$r_m + 5/2 r_m = 1;27$$

$$7/2 r_m = 1;27$$

$$\Rightarrow r_m = 0;24,51,25,48$$

$$\approx 0;25^\circ$$

which is not a terribly accurate figure (since $r_m \approx 0;15$).

However this value is better than the one attested value in which the measure of the moon's diameter is given as:[115]

> *šá SIN 12 šu-si la-an-šú*
> [The size of the moon is 12 fingers.]

12^f is 1° and elsewhere in the sources the size of the moon is related to the number of times it fits into the lunar orbit, which is given as both five and six. This reflects the recognition that the lunar path deviated from the solar by sometimes as much as 5° or 6° on either side. The division of the lunar disk into 12 may provide an origin for such a division used in the Greek tradition and perhaps also the early Indian tradition (see, for example, the *Pauliśasiddhānta* (see 5.5.1) in which one is instructed to draw 13 lines across the diameter of the moon, which produces 12 segments).

Unfortunately, this value is too large—in fact it is almost double its real size. It is difficult to comprehend how the astronomers would make such an error, when the 12° for the width of the path of the moon is a good estimate.

As is immediately clear, ψ as it stands is not helpful for giving information on eclipse magnitudes without manipulation; thus it is preferable to consider ψ a function of nodal elongation, translated by a specific amount to make it useful for determining eclipse magnitudes. In light of this it seems difficult to avoid the conclusion that ψ is redundant, simply because it is a linear translation of E. In fact, the information given by ψ could easily be obtained from E. ψ is unchanged for conjunctions and thus not very useful at all for solar eclipses.

System B examples of column ψ are more difficult. Although they use the number $c = 18$, rather than the more mysterious 17;24, the discontinuities in their zigzag function of ψ are more puzzling (fig. 3.5). However, the choice of 18, combined with the assumption seen in BM 41004 (Text E), that $r_m = 6$, gives a simple 2:1 ratio between the lunar diameter and the "shadow" so that its derivation is easier to explain than the more precise 17;24.

Column ψ has several logograms associated with it. One of these, the logogram HAB, is used in a variety of contexts outside of the ACT material. However, as with all words used within scientific contexts, it is not the existing broader lexicography that defines the precise nuance of the meaning; rather

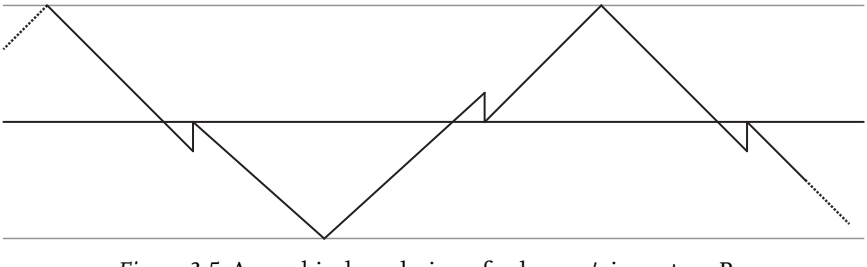


Figure 3.5 A graphical rendering of column ψ in system B.

its meaning should be determined by the function it plays within the text.[116] The sign is represented by the arrangement of four cuneiform wedges in the shape of a square.[117]

The function of HAB seems fairly clear; it is placed next to a value of ψ for which an eclipse is possible. In system A tablets, sometimes, for small values of ψ, a BE[118] sign is used instead. This seems to indicate that although the value of ψ is calculated from within the nodal zone the eclipse will not happen. Interestingly enough, one can observe that the BE sign is almost always next to a 5 itu row.

ACT 135, a system B eclipse text, includes a column ψ which has two logograms next to each value. The right-hand sign is either LAL or SIG, which refer to the ascending node or the descending node respectively. The left-hand sign is one of the signs HAB, BAR, or BE. The latter two in particular are reminiscent of the deliberate technical usage in Diary reports for eclipse predictions (see section 3.4.2). A pattern immediately emerges: a group of six or more HAB designations followed by two (sometimes three) designations of BE and BAR. HAB, as in system A, indicates an eclipse possibility. From the usage in this text, it seems that BE or BAR is used when the moon is in the nodal zone, but an eclipse is to be excluded. BE is used when the value of ψ is positive and BAR is used when the value of ψ is negative.[119] As ψ is a function that is based on column E, there is a small portion of it that has negative values. As outlined above, one can make the tentative comparison:[120]

	Diary	ACT
BE	One Saros after last visible eclipse	Positive values of ψ
BAR	Many Saroi after last visible eclipse	Negative values of ψ

Values of ψ within the nodal zone that are positive yet exclude an eclipse would suggest the general rule that the last visible eclipse was recent, as opposed to values of ψ within the nodal zone that are negative, which would suggest that the last visible eclipse was further back. So, although in the case of the Diaries the terms are used in the context of successive Saros cycles and in ACT in the context of consecutive eclipse possibilities, it seems the underlying concept being referred to by these terms may be the same.

HAB is also used in the procedure texts, where its interpretation as an indication of values in column ψ (regardless of whether an eclipse could occur) seems certain. Here it is seen both as HAB and with its phonetic complement HAB-rat. It is used on ACT 135 under the ephemerides:

HAB-rat-meš 18 ana 18 ana IGI-ka

which Neugebauer translates as[121] "in order for you to find the eclipse magnitudes (from) 18 to 18 (years)." Following this title, a scheme is given with zodiacal signs and a numerical sequence, the exact function of which remains a puzzle.

To establish eclipse limits based on the sizes of the disks was not an appropriate approach here, given that there was on interest in the literature in the relative sizes of the bodies. Nonetheless, despite this absence, the Babylonians seemingly had a good estimation for the eclipse limit. Perhaps the way they approached the problem was through period relations, using relations of time intervals with respect to EPs.

3.4.4 Comparison of Mathematical and Nonmathematical Astronomy Methods

Interestingly enough, the ACT methods were not adopted by "mainstream" astronomers (such as the Diary keepers, who presumably continued to use the same methods discussed in the previous section). Despite the development of the calculationally superior ACT methods, which were available and copied in the Seleucid era, these parameters and techniques were not adopted by the astronomers who were predicting eclipses.

The question as to why these methods were continued then must be answered. Perhaps they provided validation to other methods? Perhaps it hints at a broader academic scene motivated by elitism? More precise numbers and greater complexity suggested a higher degree of accuracy and demanded a greater level of respect and awe. However, as is observed by Steele[122] and as will be seen below, in fact these complex computations were not any more accurate—and in some cases were less so.

Other possible reasons could include the following: there was no need to use these complex ACT calculations if the object was simply to predict the day or night in which the astronomer was to watch, for the older cruder Saros methods were fully adequate. Furthermore, although the ACT methods could have given more precise times, the means to measure these times could not achieve the same degree of precision. The Babylonian water clocks were hardly reliable time keepers. Also, the ACT methods gave the values for mid-eclipse, but the Diaries always gave the time for the beginning of the eclipse.

Finally, the compilers of the Diaries were extremely conservative. Over the period of the hundreds of years in which they were kept, the Diaries underwent only several minor changes in format. Perhaps it was this alone that accounted for the fact that the compilers of the Diaries were not interested in taking

any other innovations into account, and thus did not adopt the ACT methods. Tradition alone may suffice to explain the reluctance to change.

But regardless of the status they achieved among their contemporaries, the ambitions of the ACT scholars represented something new and unprecedented. Their fundamental insight that complex astronomical phenomena are the result of the cumulation of other, more basic, phenomena, is significant. These scholars were engaging with the phenomena as never before. The realization that eclipses are the product of the numerical alignment of more basic arithmetical patterns was profound. These scholars pushed mathematical-astronomical techniques to the utmost: the positing of eclipse limits, without any recourse to cosmography; the description of relative orientations of heavenly bodies, without any clear geometrical conception of path.

A Comparison of NMAT and MAT Eclipse Predictions

The following set of tables compares ACT system A and system B predictions with corresponding predictions and observations in NMAT sources. The purpose is to ascertain whether in fact the ACT predictions were any more accurate than the results achieved by the techniques used in the NMAT sources. For practical reasons, only eclipses after −300 have been considered. Times for mid-eclipse have been taken from column M, which gives the instant of syzygy, and the possibility of an eclipse indicated from column ψ or column E.

What is important to note is that the eclipse predictions in the NMAT sources give the time of the beginning of the eclipse, whereas the time given in the ACT material from column M is for the instant of syzygy, mid-eclipse. Thus, as Steele has observed, the ACT predictions would not have been immediately useful to the compilers of the Diaries, who were interested in recording the beginning of the eclipse. Other important features in the Diaries for eclipses that were observed were the intervals between significant phases in the eclipse, which could not be directly compared to anything in the ACT sources without significant manipulation. The records for the NMAT material have been selected from the lists compiled by Steele.[123]

With respect to the predictions, both lunar and solar, Steele shows[124] that there is no evidence for an improvement of accuracy of the predictions as time progressed, which suggests that the same methods for making the predictions were used in the early as well as in the late stages.

Out of all the surviving Diary eclipse predictions and ACT texts, there is only one eclipse that has a record in the Diaries with a corresponding one in *both* systems A and B. It is a solar eclipse and the details are as follows:

Date (J)	Date (S.E.)	NMAT text	System A	System B
−102 Jul 8	209 III	30° before sunset	13;1° before sunset [125]	14° before sunset[126]

Timed Solar Eclipse Predictions

This table selects all solar eclipse predictions recorded in the Diaries after −300[127] that have a corresponding prediction in an ACT source which includes a tabulated value for column M.

Date (J)	Date (S.E.)	NMAT text	System A	System B
−204 Jun 25	107 III	87° before sunset		67° before sunset[128]
−193 May 26	118 II	77° after sunset		41° after sunset[129]
−185 Nov 20	126 VIII	65° before sunrise		8° before sunrise[130]
−183 May 6	128 I	8° after sunrise		79° before sunset[131]
−168 Jan 22	142 XI	93° after sunset	71;13° before sunset[132]	
−164 Oct 29	147 VIII	64° after sunset	309;59° before sunset[133]	
−132 Aug 7	179 V	9° after sunset		Not predicted[134]
−131 Feb 1	179 X	21° after sunset		60° before sunset[135]
−122 Feb 7	188 XI	23° after sunrise	210;23° before sunset[136]	
−122 Jul 19	189 IV	71° before sunset	49;54° before sunset[137]	

Timed Lunar Eclipse Predictions

This table selects all lunar eclipse predictions recorded in the Diaries after −300,[138] which have a corresponding prediction in an ACT source that includes a tabulated value for column M.

Date (J)	Date (S.E.)	NMAT text	System A	System B
−194 Jun 20	117 III	15° before sunset		22° before sunset[139]
−194 Nov 16	117 VIII	45° after sunset		77° after sunset[140]
−193 May 11	118 II	94° after sunrise		91° after sunrise[141]
−191 Apr 19	120 I	2° after sunset		20°[142]
−185 Jun 11	126 III	48° before sunset		78° before sunset[143]
−131 Jan 17	179 X	60° after sunset		81° after sunset[144]

Solar Eclipse Timings

This table selects all solar eclipse observations which included the time of the beginning of the eclipse that have a corresponding prediction in an ACT source which includes a tabulated value for column M.

Date (J)	Date (S.E.)	NMAT text	System A	System B
−189 Mar 14	121 XII	30° after sunrise		24° after sunrise[145]
−165 May 17	145 XII_2	13° C1-Max.	Not predicted[146]	

Lunar Eclipse Timings

This table selects all lunar eclipse observations which included the time of the beginning of the eclipse that have a corresponding prediction in an ACT source which includes a tabulated value for column M.

Date (J)	Date (S.E.)	NMAT text	System A	System B
−193 Nov 5	118 VIII	12° before sunrise		15° before sunrise[147]
−189 Feb 28	121 XII	30° before sunrise		27° before sunrise[148]
−188 Feb 17	122 XI	34° before sunrise		40° before sunrise[149]
−184 Nov 24	127 VIII	44° after sunset		40° after sunset[150]

Where appropriate the predictions from the Mesopotamian sources have been compared against modern computations[151] taking the geographical co-ordinates of Babylon to be 44.42° E and 32.55° N. All times used are local times (LT). 1° has been approximated to 4 minutes. From this small number of examples, the following comparisons will show that the Diaries are more accurate than the ACT calculations. In the case of lunar eclipses, syzygy has been approximated to the middle of the eclipse. Obviously the accuracy of an observation of an eclipse in a NMAT source is going to be better than a prediction from an ACT computation; however, for interest, the comparison has nonetheless been made. The mathematical details are presented in Appendix B, and show that the MAT methods were no more accurate than the NMAT sources.

Such comparisons are to be treated with caution; the lack of sample size, the exigencies of the assumptions made, and so on, mean that they cannot be take as conclusive. However, that ACT results were not significant improvements over those generated by NMAT sources can be supported. From the five solar eclipse comparisons, the NMAT predictions are slightly more accurate than the ACT ones. From the eight lunar eclipses half of them were better predicted by NMAT sources and the other half by ACT texts (all eight predictions here were those made under system B) and in general the NMAT gave

better all-around accuracy. Therefore, despite being more deliberate, more intricate, and dedicated to taking into account more factors, ACT texts could not produce consistently better predictions that could compete with existing traditional methods.

While ACT calculations accounted for many of the aspects necessary to predict eclipses, there were several noticeable features that were completely absent. This too is a product of an overwhelming orientation toward an arithmetical accountancy of the appearances, at the expense of geometrical reckoning. These include a lack of consideration of the relative diameters of the sun and moon, and the shadow (if they even recognized such a feature), and the lack of any direct treatment of their relative distances. Also missing in their work is an active reckoning of geographical considerations. There is no (active) accounting for terrestrial coordinates in their calculations. Related to this is the fact that these scholars had no concept of parallax. This is critical to the prediction of solar eclipses. Without factoring in parallax, while practitioners could *exclude* the possibility of a solar eclipse, they could only tentatively furnish details on the actual occurrence of a solar eclipse.

The methods devised and utilized for the reckoning of eclipses in the ancient Near East, taken as a whole, are worthy of great respect for several reasons. The succession of the various proposals, the periodic reengagement with phenomena, and the sense of development that can be detected in these efforts are emblematic of a broader unfolding of the intellectual culture. Since technology of writing was newly acquired, the earliest forms of eclipse reckoning were heavily invested with human significance. Writing opened up to the earliest scholars new potentialities of recording and evaluation of data, but their orientation remained tightly connected to the mytho-poetic thought-forms of antecedent preliterate ruminating. Indeed, the astronomical tradition in the ancient Near East never quite shook free of the persistent epistemic authority of the arts of divination, astrology, and horoscopy. Importantly though, divination provided an essential impetus and medium for the ever critical consideration of eclipse phenomena, and scholars became increasingly ambitious to account for the appearances, compiling huge archives and extracting and rearranging empirical data to further refine their command over the celestial patterns.

The ability to record data also meant that these peoples were in a position to take account of subtler nuances within the phenomena than would ever reveal themselves otherwise and notice periodicities that they could tentatively forecast into the future; at first, the immediate instances, but soon after, well into the future. Late in the picture, a more mathematical approach

was furthered. Babylonian scholars were among the first to recognize that the heavenly alignments which had so much significance for them in everyday life—visibility of first crescent, eclipses, and so on—could be captured by mathematical combinations of simpler arithmetical patterns. The culmination of these patterns at significant instants in determinable numerical limits could uncover the occurrence of the phenomenon in question. This step was profound and scholars increasingly pushed mathematical astronomy to its limits. Remarkably, in spite of the significant developments in the reckoning of eclipses (and indeed all celestial phenomena) and the ensuing increasingly successful attempts to predict their occurrences, the cosmographical picture remained largely unchanged through this long time span, which seems incongruent given the sophistication of their mathematical attempts to model celestial phenomena. The overwhelming task of establishing the first accounts meant that it was beyond the purview of inquirers to consider a physical reality beyond the facts. Furthermore, the orientation of the divination tradition, being human centered, did little to open up such avenues of inquiry. That the focus of astronomy was arithmetical and predictive, and not geometrical and explanatory, is telling of its continued close association with the arts of divination. Such practices demanded answers as to when and where celestial events occur. In many respects, it closed practitioners off from questions such as how or why it should be so.

Despite the sophistication of these mathematical models, ancient Near Eastern practitioners were firmly bound to an accountancy of the appearances. The realization of the theoretical orientation of mind in ancient Mesopotamia was thus limited in extent. The exclusive focus of engaging with the phenomena at the level of sky-arithmetic, and intricate numerical schemes and quantitative patterns meant that theorizing could not emerge in this culture. However, their parameters and period relations proved utterly essential for successive cultures. Vast observational records, astronomical conventions, and features introduced to demarcate the sky, as well as mathematical techniques and standards were adopted as foundational by later practitioners. The legacy that was passed on was to prove to be vital and inspirational.

CHAPTER FOUR

Beyond the Appearances
Greek Eclipse Reckoning

4.1 Overview

Theon of Smyrna says that Babylonian astronomers using arithmetical methods, succeeded in confirming the observed facts and in predicting future phenomena, but that, nevertheless, their methods were imperfect, for they were not based on a sufficient understanding of nature, and one must also examine these matters physically.

—As paraphrased by Evans (1998) p. 214, from Theon of Smyrna, *Mathematical Knowledge Useful for Reading Plato* (III, 30).

περὶ δὲ τὸ τοσοῦτον ἀκριβεύεσθαι κενοδόξου μᾶλλον ἢ φιλαλήθους ἂν εἴη.
Scrupulous accuracy about such a small amount is a sign of vain conceit rather than love of truth.

—Ptolemy, *Almagest* VI, 7, as translated by Toomer.

In the fifth century BCE, methods to predict eclipses as developed by Mesopotamians were far more advanced than those of their Greek contemporaries. There is no evidence to suggest that the early Greek astronomers were experimenting with numerical patterns in any way comparable to the Mesopotamians at this time. However, as was even recognized in their own times, as in Theon of Smyrna's account centuries later, ancient Greek thinkers were engaged with the celestial phenomena, and in ways that were new and unprecedented. Astronomical alignments beckoned to them in ways that drew them beyond the surface appearances to considerations beyond what they observed directly. With such aspirations, these early thinkers thus initiated for the first time theoretical dimensions for the investigations concerning eclipses.

Nascent philosophical reflections had set the groundwork for thinkers to be mindful of the distinction between facts as they might manifest themselves apparently and what they might be really. Explanation emerged as an ambition, as increasing attention was paid to conditions not within the appearances but able to be inferred from them. Greek ambitions reveal that once theory begins to direct people's consideration of celestial appearances then the functions that astronomical observations can serve are soon deepened. Those inquirers who first dealt with empirical considerations by theoretical modes of questioning and reflection, initially experienced countervailing pressures to their efforts, but in time encountered richly compounding inducements to further that new mode of investigation.

Among the earliest astronomical questions posed by Greek inquirers were those that concerned the calendar, and the connection between the structuring of the year and the rising, setting, and general patterns of movement of the stars and planets. Such a focus was common to many early cultures. However, what set the Greek inquirers apart from all other peoples at the time was the emergence of questions about the physical structure of the earth and the heavens, and their relation to these calendrical questions. When calendar problems were combined with concurrent cosmological speculations, such as the proposal that the heavens are rotating concentric spheres, first articulated by Eudoxus of Cnidus in the fourth century BCE, an entirely new mode of investigation in astronomical research was unleashed. Investigations into when and where the stars and planets rise, set, and move across the sky, were addressed primarily by a qualitative study in celestial spherics—a search beyond the appearances.

Accordingly, early sources reveal attempts by the Greeks to comprehend, on a cosmographical level, the physical causes that brought about eclipses, something that their Mesopotamian contemporaries (as far as the extant evidence suggests) were not doing. In fact, as emphasized in the previous section, the sophisticated arithmetical patterns of the Mesopotamians are an example of the limits to which astronomy can progress without the existence of any suppositions about the shape and dimensions of the earth and the celestial realm. It was the cosmographical proposals of the early Greek astronomers considering that the earth and the luminaries are spherical in shape, their relative positions, their illuminations and so on, that gave direction to early astronomical researches. With such propositions in hand, the Greeks began to experiment with geometrical models to describe the movements and patterns of the heavenly phenomena—those very features that the Mesopotamians had advanced by means of numerical accountancy.

These geometrical conceptions were able to emulate the general qualitative patterns of the celestial motions, but a further step was needed to make

these models truly replicate the astronomical phenomena. This was to apply numbers and magnitudes to these models so that sizes, distances, and timings were consistent and reflected what was being observed. Perhaps surprisingly, Greek thinkers were slow to seek such quantitative potential for the models. Inspiration came from afar. With no significant sources in their own tradition, the Greeks initially drew from the arithmetical period relations developed by the Mesopotamians and imbued their models with quantitative predictive power. In addition, the large collection of observations that had been produced by ancient Near Eastern peoples, reaching back many centuries, provided the Greek practitioners with much raw data from which to extract further parameters for their models. Greek astronomy was ripe for such input and the arithmetic period relations now could be accounted for by geometrical kinematic models. Such knowledge from Mesopotamia was greatly to impact and further stimulate development. What resulted from this transmission was a synthesis between numerical patterns and parameters and a geometrical concentric cosmography, a fusion which was to prepare the way for the peak of theoretical astronomy in the ancient world with Ptolemy's *Almagest*.

Indeed, the geometrical orientation provided practitioners with structure and depth that arithmetical reckoning alone could not offer, and for this reason such inquiries can be deemed theoretical. However, a geometric model also locks its users in to the geometric and physical constraints of whatever particular configuration has been settled upon, and as well as potentialities, has limiting effects. The physical principles of concentricity and uniformity, for example, in Ptolemaic models required complex, and often unsatisfactory, additional mechanisms to account for nonuniform motions. Some phenomena simply could not be accounted for adequately within the chosen configuration. Later cultures, placing less of an emphasis on strict conformity to some basic geometrical principles in their astronomical models, could often account for the phenomena better and more simply.

Often noted are references in Greek literary sources to descriptions which have been interpreted to refer to eclipses. For example, some have interpreted Homer's description: "The shining light of the sun fell in the ocean, dragging black night over fertile land"[1] as referring to a solar eclipse;[2] many other examples have been suggested. If indeed these are references to eclipses, they are of little interest in this context, as these accounts are simply a description of the effects of an eclipse, not attempts to penetrate into the causes of these celestial events. There were certain pieces of vital information that needed to be secured before an adequate account of eclipses could be given. Along with the discovery of the sphericity of the earth came aspirations to explain eclipses with reference to unifying celestial geometrical models. A fundamental component of

the increasingly theoretical consideration of eclipses was now a cosmographical element, that is, the celestial physical conditions that brought about lunar and solar eclipses.

Thus the quest of the early Greek practitioners for an explanation of eclipses combined information about the sphericity of the heavenly bodies with the discovery that the moon was illuminated by the sun. This, paired with the assertions about their respective positions, provided the right pieces of information for a physico-spatial account of the causes of eclipses. By the time of Aristotle, it seems that there was a wide acceptance of the sphericity of the earth, and Aristotle's demonstration of its spherical shape by pointing out that the shadow on the moon during a lunar eclipse was a convex arc seems to have been a compelling verification.[3] Before this, though, there were many accounts that attempted to explain the cause of eclipses. Flat earth cosmologies and celestial bodies with fiery vents gave rise to many creative propositions. The disparity in frequency between lunar and solar eclipse observations led many to introduce the existence of additional (invisible) celestial bodies which eclipsed the moon in addition to the earth's shadow into an already satisfactory account.

However, the recognition of the significance of this simple empirical fact was critical and eventually led to a further essential discovery by the Greeks with respect to eclipse reckoning. This was the displacement effect that observing on the surface of the spherical earth as distinct from the center of the earth has on the observation of the moon and the sun against a fixed backdrop of stars. The possibility that the circumstances of the observer might impact the viewing of an eclipse was no doubt astonishing. Such an effect, given the name *parallaxis* or parallax by the Greeks, meant that solar eclipse phenomena could at last be given serious theoretical consideration. Furthermore, this effect could not be properly accounted for before the accurate determination of lunar latitude—i.e., around the time of the second-century BCE astronomer Hipparchus. The resulting kinematic geometrical proposals were refined and quantified, and culminated in the systems presented in Ptolemy's *Almagest* in the second century CE. This work represented the limits to which mathematical-astronomical approaches within a geocentric orientation could reach. Thereafter, only a complete overhaul of the model could significantly further the scope, accuracy, and insightfulness of astronomers' reckonings of heavenly motion.

The regularity of the heavens was a source of inspiration to Greek thinkers in many respects. The periodicity that could be discerned in celestial phenomena was esteemed not just because it was predictable, but because it turned the mind toward rationally apprehensible order and beauty. In this way, astronomy epitomized that which humans should aspire to. "With regard to virtuous conduct in practical actions and character," Ptolemy reflects,[4] "this science, above

all things, could make men see clearly; from the constancy, order, symmetry, and calm which are associated with the divine, it makes its followers lovers of this divine beauty, accustoming them and reforming their natures, as it were, to a similar spiritual state." Plato throughout the *Timaeus* expresses this sentiment also.

4.2 Early Greek Geometrical Celestial Conceptions

Greek thinkers were responsible for vast innovations in astronomy by bringing geometry to bear upon that science. The earliest of them, the so-called pre-Socratics, determinedly sought the *arche*, or essence, of all things. They looked for that which was fundamental and beyond what they observed. Accordingly, they approached the heavenly phenomena with the same sort of demand—to uncover that which was more essential as it was suggested by appearances. It is difficult to confidently relay the details of their points of view since so few facts concerning the pre-Socratics are known, and often little or even none of their original writings still exist. Despite this, the sketchy particulars, anecdotes, and doxographic accounts, combined with broader, anthropological considerations, indicate the substantial intellectual changes taking place across those ancient times. Their views bear witness to the ground-breaking character of the initial, halting, yet inchoately theoretical preoccupations. This pattern of inquiry then compounded itself, ultimately giving rise to theoretical purposes that go ever deeper and become ever richer and more exacting.

4.2.1 The Pre-Socratics

Thales of Miletus (625[?]–547[?] BCE), whose reputation with respect to eclipses rests on one notorious solar eclipse prediction,[5] has been similarly credited with other such cosmological discoveries that are hardly sustainable. In particular, first-century BCE ancient Greek thinker Aetius claims that Thales was the first to state that the sun is eclipsed when the moon comes in a direct line with it, and furthermore that he, as well as Anaxagoras, Plato, Aristotle, and the Stoics, maintained that the moon is eclipsed when it travels into the shadow made by the earth when the earth is in between the two bodies.[6] Such a claim is doubtful for many reasons. Among them is the seeming incongruence with Thales' assertion that the earth floats on water and the fact that none of his Ionian successors make any mention of such claims.

Also to be rejected is Aetius' assertion that Thales believed the earth to be spherical and centrally placed. The ancient biographer of Greek inquirers, Diogenes Laertius, states that according to some, Thales was the first to declare that the apparent size of the sun was $\frac{1}{720}$ part of the circle described by it.[7]

This relationship was later credited to Aristarchus by Archimedes, although a different value appears in his extant work (see section 4.3).

Anaximander (ca. 611/10–ca. 547/6 BCE) asserted that the earth was placed free-standing in the center of the universe.[8] The sixth-century CE neo-Platonist commentator Simplicius reveals:

> Ἀναξιμάνδρου πρώτου τὸν περὶ μεγεθῶν καὶ ἀποστημάτων λόγον εὑρηκότος . . . τὰ δὲ μεγέθη καὶ τὰ ἀποστήματα ἡλίου καὶ σελήνης μέχρι νῦν ἔγνωσται ἀπὸ τῶν ἐκλείψεων τὴν ἀφορμὴν τῆς καταλήψεως λαβόντα, καὶ εἰκὸς ἦν ταῦτα καὶ τὸν Ἀναξίμανδρον εὑρηκέναι
> [Anaximander was the first to find the ratio of the sizes and distances . . . Now the sizes and distances of the sun and the moon as determined up until now were ascertained starting from (observations) of eclipses, and the discovery of these things might reasonably be supposed to go back as far as Anaximander.]
> (Simplicius *de Caelo* 471.1 in *Vors.* i, p. 86.36–40)

However, Simplicius' implication that Anaximander used empirical data from eclipse observations in order to derive the sizes and distances sits rather uncomfortably with Anaximander's theory of the causes of eclipses, namely that:

> . . .γίγνεσθαι δὲ τὴν ἔκλειψιν τοῦ στομίου τῆς τοῦ πυρὸς ἐκπνοῆς ἀποκλειομένου
> [that the eclipse (of the sun) occurs with the opening being shut off by an exhalation of fire.]
> (Diels [1929] p. 354 [Aet. ii.24 2])

and

> Ἀναξίμανδρος τοῦ στομίου τοῦ περὶ τὸν τροχὸν ἐπιφραττομένου
> [Anaximander (says that there is an eclipse of the moon) when the opening in the rim of the wheel is stopped up.]
> (Diels [1929] p. 359 [Aet. ii.28 1])

Great cosmic wheels were imagined to be responsible for the orbits of celestial phenomena to some pre-Socratics. Furthermore, for Anaximander, the moon was illuminated from fires within, and did not receive its light from the sun. Anaximander apparently had estimations for the celestial sizes, variously reported as follows:

> Ἀναξίμανδρος κύκλον (τοῦ ἡλίου) εἶναι ὀκτωκαιεικοσαπλασίονα τῆς γῆς
> [Anaximander (says that) the orb (of the sun) is 28 times the size of the earth]
> (Diels [1929] p. 348 [Aet. ii.20 1])

…εἶναι δὲ τὸν κύκλον τοῦ ἡλίου ἑπτακαιεικοσιπλασίονα
[that the orb of the sun is 27 times as large (as the earth).]
(Diels [1929] p. 560 [Hippol. *Refut.* i 6 5])

Ἀναξίμανδρος κύκλον (τῆς σελήνης) εἶναι ἐννεακαιδεκαπλάσιον τῆς γῆς
[Anaximander (says that) the orb (of the moon) is 19 times as large as the earth]
(Diels [1929] p. 355 [Aet. ii.25 1])

Heraclitus (fl. ca. 500 BCE) had a fairly primitive understanding of the universe attributed to him by the doxographers; his conceptions of the heavenly bodies were very much inspired from earthly everyday objects. The moon and the sun were shaped like basins or bowls (*skaphai*) with their concave sides facing the earth, in which fires burn. Eclipses of the sun and the moon were said by him to occur when the bowls rotate so that their convex sides are facing the earth, and the concave side, and thus their internal fires, are facing away.[9] Heraclitus also believed that the diameter of the sun was one foot wide. This view was adopted by Epicurus and criticized by Aristotle and Cleomedes.

It is often stated that Pythagoras (b. ca. 560 BCE d. ca. 480 BCE) held that lunar eclipses are caused by the interposition of either the earth or the counter-earth[10] between the sun and the moon.[11] Solar eclipses are only caused by the moon "getting under the sun."[12]

Parmenides (b. ca. 515 BCE d. after 450 BCE) is said to have had a strong affinity with the Pythagoreans, to the extent that some discoveries are credited both to Parmenides and to Pythagoras. However, there are several elements in his cosmology which differ from that of the Pythagoreans. While the Pythagoreans held that beyond the celestial sphere there was a limitless void, Parmenides is said to have argued against the existence of such boundless empty space.

It is generally accepted that the centrally placed spherical earth and celestial sphere were discovered by Parmenides. It is also reported that he stated that:

(τὴν σελήνην) ἴσην τῷ ἡλίῳ καὶ γὰρ ἀπ΄ αὐτοῦ φωτίζεται
[(The moon) is equal to the sun; for indeed it is illuminated by it.]
(Diels [1929] p. 357 [Aet. ii.26 2])

This was probably not in fact first observed by Parmenides, but rather by Anaxagoras.

Anaxagoras (b. ca. 500 BCE d. 428 BCE[?]) is commonly stated to have been the first to correctly enunciate the causes of eclipses, because of his

conclusion that the moon receives its light from the sun. Plato confirms this, stating that:

> ὃ ἐκεῖνος νεωστὶ ἔλεγεν, ὅτι ἡ σελήνη ἀπὸ τοῦ ἡλίου ἔχει τὸ φῶς
> [what he (Anaxagoras) recently stated, that the moon receives its light from the sun.]
> (*Cratylus* 409A)

Third-century CE commentator Hippolytus elaborates:

> τὸ δὲ φῶς τὴν σελήνην μὴ ἴδιον ἔχειν, ἀλλὰ ἀπὸ τοῦ ἡλίου . . . ἐκλείπειν δὲ τὴν σελήνην γῆς ἀντιφραττούσης, ἐνίοτε δὲ καὶ τῶν ὑποκάτω τῆς σελήνης, τὸν δὲ ἥλιον ταῖς νουμηνίαις σελήνης ἀντιφραττούσης . . . οὗτος ἀφώρισε πρῶτος τὰ περὶ τὰς ἐκλείψεις καὶ φωτισμούς
> [The moon has a light which is not its own, but comes from the sun . . . The moon is eclipsed through the interposition of the earth, and, at times, of the bodies below the moon, the sun is eclipsed at the new moons through the interposition of the moon . . . This man (Anaxagoras) was the first to bring to light the facts about eclipses and illuminations.]
> (Diels [1929] p. 562 [Hippol. i.8 8–10])

Similar sentiments are repeated by Aetius and Plutarch.

Anaxagoras may have had a correct explanation of lunar and solar eclipses, but he seemed still confused about the discrepancy in frequency between lunar and solar eclipse observations. As a result, he coupled his explanation with the unnecessary addition that sometimes other dark bodies that are invisible to observers on the earth cause eclipses. Perhaps this sentiment was influenced by his predecessor Anaximenes or the Pythagoreans with their counter-earth. On this subject, Aristotle comments:

> ἐνίοις δὲ δοκεῖ καὶ πλείω σώματα τοιαῦτα ἐνδέχεσθαι φέρεσθαι περὶ τὸ μέσον, ἡμῖν δὲ ἄδηλα διὰ τὴν ἐπιπρόσθησιν τῆς γῆς. διὸ καὶ τὰς σελήνης ἐκλείψεις πλείους ἢ τὰς τοῦ ἡλίου γίγνεσθαί φασιν: τῶν γὰρ φερομένων ἕκαστον ἀντιφράττειν αὐτήν, ἀλλ´ οὐ μόνον τὴν γῆν.
> [It seems possible to some that more bodies of such a kind may move about the center but are invisible to us because of the interposition of the earth. Thus, they say, there are more eclipses of the moon than of the sun, for each of the bodies obscures it (the moon), and not only the earth.]
> (*de Caelo* ii.13, 293 b 21–25)

Aristotle notes also that the Pythagoreans did not posit that their position of observation, on the surface of the earth and not at its center, made any difference:

ἐπεὶ γὰρ οὐκ ἔστιν ἡ γῆ κέντρον, ἀλλ´ ἀπέχει τὸ ἡμισφαίριον αὐτῆς ὅλον, οὐθὲν κωλύειν οἴονται τὰ φαινόμενα συμβαίνειν ὁμοίως μὴ κατοικοῦσιν ἡμῖν ἐπὶ τοῦ κέντρου, ὥσπερ κἂν εἰ ἐπὶ τοῦ μέσου ἦν ἡ γῆ. οὐθὲν γὰρ οὐδὲ νῦν ποιεῖν ἐπίδηλον τὴν ἡμίσειαν ἀπέχοντας ἡμᾶς διάμετρον.

[For since the (surface of the) earth is not the center, but is at a distance (equal to) the whole of its hemisphere, they think nothing prevents the phenomena from occurring in the same way to us who live away from that center, as if the (surface of the) earth was at the center. For, it is clear that we being removed (from the center) by half the diameter (of the earth) makes no difference in this.]

(*de Caelo,* ii.13, 293 b 26–30, as translated by Heath [1932] pp. 30–31)

Indeed, the Pythagoreans could not support the concept of parallax because of such beliefs. Any further development in terms of Anaxagoras' eclipse theory was hampered by his belief that the earth and the heavenly bodies were flat, not spherical.

From the outset, Greek thinkers responded to eclipse phenomena in ways that were distinctly different from those of their ancient Near Eastern predecessors. Most importantly, the nature of their ruminations laid the groundwork for their successors. Initial empirical investigations generated some creative suggestions to account for the phenomena, in many cases halting, but groundbreaking in intent. Central to eclipses was the establishment of the source of illumination of the heavenly bodies, therein accounting for the seeming loss of this illumination at the time of an eclipse. Proposals such as exhalations of fire, opening and closing of vents, and additional heavenly bodies, were pioneered. Suggestions could be markedly rationalistic and resourceful in their approach but far from where later researches would go—of note is Heraclitus' conception that the moon and the sun were concave bowls which housed fires, obviously inspired by analogy from more earthly domestic affairs. Key to the refinement of their considerations was a growing agreement that the earth was centrally placed, and this, combined with an examination of the transfer of light between celestial bodies, naturally brought these early thinkers to consider the relative configurations of heavenly bodies. Other vital details were posited, such as the recognition of the fact that observers were located on the surface of earth and not at the center. However, this recognition did not lead them to infer the displacing effect of parallax which would affect what they saw in the sky. This example reveals how innovative yet halting their proposals were. But, regardless of content, the explanatory intent and ambitions of these early thinkers were of great consequence.

4.3 Sizes, Distances, and Measurement: Aristarchus and Archimedes

Aristarchus (ca. 310–230 BCE) is a figure hailed by modern thinkers for his promotion of a heliocentric system for the visible universe—a suggestion too radical for his time, it appears, as it found little popularity among his successors and largely languished for over a millennium and a half. Coming a generation after Euclid, and recognized as a highly skilled mathematician, Aristarchus represents one of the first Greek thinkers to seriously advocate the careful application of mathematics to address those details about heavenly phenomena that the observations of his time could not speak to. This sentiment proved foundational for later practitioners and is thoroughly embraced by Ptolemy, who reflected:[13] "Only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry."

That Aristarchus asked questions such as "at what angle relative to the sun is the moon half full," and "what are the various astronomical distances and dimensions" and that he invoked mathematical conjecture and deduction to address these questions represents the first real success of Greek theoretical endeavors. His conceptions represent a culmination of those physical conceptions of the universe that had been initiated by his predecessors in the sixth century, but now these conceptions had acquired a mathematical orientation as well as increasingly determined and organized measurement making of the heavenly phenomena. Indeed, his predecessors had simply mused about the sizes of the sun and the moon and their relative positions; Aristarchus demonstrated that they could be measured.

In terms of eclipse reckoning, Aristarchus' work was pivotal. His one preserved treatise *On the Sizes and Distances of the Sun and the Moon*[14] is the earliest evidence for the treatment of the sizes and distances strictly geometrically. Poor observational data meant that many of his ratios were inaccurate, and yet his efforts point to the new direction Greek astronomy was taking.

Aristarchus opens his work with several propositions and then derives limits and ratios with respect to the geometrical models accordingly. Among the propositions are the assumptions that the moon is illuminated by the sun (prop. 1) and that the orbit of the moon is smaller than the orbit of the sun. Furthermore, Aristarchus states that the earth has the relation of a point with respect to the sphere of the moon (prop. 2) with the consequence that diurnal parallax can effectively play no role in his astronomy. The radius of the shadow is twice the apparent radius of the moon (prop. 5):

$$r_u = 2r_m.$$

Even though, despite his book's title, Aristarchus never gives values for the distances, he deals with the problem of determining the various sizes of the earth and the luminaries by employing geometrical techniques. He differentiates between the apparent and the actual sizes of the luminaries. In the context of eclipses, he employs a shadow cone (prop. 13) and uses lunar eclipse occurrences to give bounds for the actual sizes of the luminaries (prop. 9). Good approximations for the relative sizes and distances of the sun and the moon are of great importance in the context of eclipses, especially in order to answer the question of the degree to which the solar disk can be obscured by the lunar disk. He discounts the possibility of an annular eclipse by the material set out in proposition 8, namely, that when there is a total eclipse of the sun, the sun and the moon are perfectly aligned and contained within the one same shadow cone which has its vertex at the observer's eye.

Archimedes reveals that:

> Ἀριστάρχου μὲν εὑρηκότος τοῦ κύκλου τῶν ζῳδίων τὸν ἅλιον φαινόμενον ὡς τὸ εἰκοστὸν καὶ ἑπτακοσιοστόν
> [Aristarchus discovered that the sun's apparent size is about one 720th part of the zodiac circle.]
> (*Arenarius* 10 p. 222 lns. 6–8.)

One 720th part of the zodiacal circle is half a degree, which is a good approximation. However, in Aristarchus' work, proposition 6 reads:

> That the moon subtends 1/15 part of a sign of the zodiac
> (*Peri Meg.* pp. 352–53, as translated by Heath [1932] p. 101)

One-fifteenth of a zodiacal sign gives $d_m = 2°$. However, this result is about four times too large. It is difficult to imagine that Aristarchus obtained it empirically; with the techniques available at this time it would have undoubtedly been within his capabilities to obtain a better estimate. Furthermore, the relation of the shadow size to the lunar diameter as 2:1 would result in the maximum duration for a lunar eclipse being about 12 hours, a duration which is significantly too long. Nevertheless, the important point is that the sizes and distances of the luminaries were now being considered from a geometrical perspective, with the first attempt at numbers being introduced (rather unsuccessfully at this point) into these geometrical models. At this stage there is no consistency with actual empirical evidence—this leads one to conclude that either Aristarchus was using unusually poor observational data or he was incorporating data that were not based on observations at all, but rather values selected for their computational facility or some other similar reason.

The late Greek commentator Pappus of Alexandria (ca. 290–ca. 350) makes comments[15] on Aristarchus' propositions and compares and contrasts them with the statements made by Aristarchus' successors Hipparchus and Ptolemy. Pappus notes that propositions 1, 3, and 4 are in agreement with Hipparchus and Ptolemy. However, he criticizes Aristarchus' value of 2° for the diameter of the moon. He gives Ptolemy's figures from the *Almagest* and Hipparchus' estimation of the lunar diameter as 1/650 of its orbit, and sustains that these different amounts give different ratios for the sizes and distances.

In the same spirit, Archimedes (286–212 BCE) continued to validate the use of geometry to model the cosmography and motions and applied data gained from observations to these models. His work, the *Arenarius* (*Sand-Reckoner*), traditionally known for its systematization of notation for large numbers, includes a section that is important for its content on the sizes and distances of the sun and the moon.

Archimedes states that the diameter of the sun is about 30 times the diameter of the moon and no greater. Regarding the diameter of the sun, he states a ratio he attributes to Aristarchus and then comments:

> αὐτὸς δὲ ἐπισκεψάμενος τόνδε τὸν τρόπον ἐπειράθην ὀργανικῶς λαβεῖν τὰν γωνίαν, εἰς ἂν ὁ ἅλιος ἐναριμόζει τὰν κορυφὰν ἔχουσαν ποτὶ τᾷ ὄψει
> [I myself tried, by a method which I will now describe, to find "by instruments" the angle subtended by the sun and having its vertex at the eye.]
> (*Arenarius* 10 p. 222 ll. 8–11)

From his observations, which he describes in detail, he concludes that the apparent solar diameter (d_s) lay within the following limits (*Arenarius* 16):

$$\frac{1}{200} < d_s < \frac{1}{164}.$$

Archimedes then describes a procedure that compares angles having their vertex at the center of the earth with those having their vertex at the surface of the earth. Using this difference he begins a detailed examination of the changes in angle when one shifts the points of observation from the center of the earth to its surface, an effect of what was soon to be known as parallax. Thus, by the time of Archimedes, this effect was beginning to be taken seriously and explored.

Aristarchus and Archimedes, and others active at the time, are a testament to a new response to astronomical phenomena and eclipse reckoning, and their efforts reveal the initial successes in addressing the phenomena using theoretical suppositions. Their achievements were a product of inquiry increasingly informed and driven by theoretical ideas. Where observational input remained tenuous, theoretical conjecture supplied insight. Inherent

difficulties with measurement still challenged astronomers, but these were no longer a distraction from greater insights about the heavenly phenomena. Aristarchus and Archimedes set the tone for the next phase of Greek astronomy, that is, the predominantly mathematical approach pursued by Apollonius, Hipparchus, and Ptolemy, albeit under the assumptions of traditional geocentrism.

4.4 Early Greek Mathematical Astronomy

Various considerations with respect to the solar model compelled Greek astronomers to revise their simple geometrical designs in which the sun traveled uniformly on a circle whose center was the earth. As a response, increasingly sophisticated kinematic geometrical models were developed, still with the requisite regular, uniform, and circular motion in mind. These models included the eccentric model, which shifted the center of circular rotation away from the earth, and the epicyclic model, which made the sun travel uniformly on a smaller additional circle, which itself was carried around on a larger circle, called a deferent circle, concentric with respect to the earth.[16]

It has been traditionally assumed that these models were developed by Hipparchus (190–120 BCE), although Ptolemy[17] credited Apollonius with demonstrating the equivalence of the epicyclic and eccentric models in the third century BCE. Thus these models were well known by his time. However, Apollonius, it seems, was interested above all else in reproducing the general patterns of the celestial motions, without any compulsion to imbue these designs with numerical significance. It is supposed that Hipparchus was the first to demand this. He was determined that these models could reproduce astronomical phenomena in the details, and to do this, magnitudes needed to be added to these models.

The inspiration to do so seems to have been strongly influenced by Hipparchus' contact with Babylonian sources. Indeed, he must have been greatly impressed with the successes of the Babylonians in predicting astronomical phenomena, but (if he had access to a full impression of their efforts) could have been confused about the lack of attention to cosmography, which had been the foundation of Greek ambitions. The allure of predictive power motivated Hipparchus to expect the same from the Greek model and thus for the first time in Greek astronomy came a desire for a grounded physical explanatory model which could also function as a precise tool for prediction and account for the details of the appearances. Accordingly, Hipparchus has been credited as "the man who transformed Greek astronomy from a purely theoretical into a practical predictive science."[18] To this end, Hipparchus played a vital

role in introducing Babylonian parameters and period relations into Greek astronomy. His resulting efforts were significant, but a proper integrated system that addressed the details of the sun and the moon, as well as the five planets, was not achieved until the work of his prodigious successor, Ptolemy.

4.4.1 *Hipparchus*

Despite the dearth of extant material written by Hipparchus[19]—a commentary written by him on Aratus, references to him by Ptolemy, and a handful of anecdotal material—there is no doubt as to the considerable importance of this astronomer and the profound impact he had on the development of Greek astronomy. The most valuable source of knowledge about his theories available is Ptolemy, who refers to him frequently in the *Almagest*. Clearly, Ptolemy had a thorough knowledge of all the details of Hipparchus' theories; he mentions him with both admiration and criticism. Ptolemy's intent, however, was not to preserve his material untouched, but rather to use it, to build and improve upon it, and perhaps even to unify ideas presented by Hipparchus in many different treatises into his own all-inclusive single astronomical work.

One of the most important mathematical devices used in early astronomy was the computation of the angles and sides of plane triangles. Greek solutions involved trigonometric methods involving the chord. Hipparchus authored a work on chords.[20] His circle had a circumference that was divided into 360°, and each degree into 60′, so that the radius, which was also expressed in minutes, was

$$\begin{aligned} C &= 2\pi \cdot R \\ \Rightarrow R &= \frac{360.60'}{2\pi} \\ &\approx 3438'. \end{aligned}$$

This value was transmitted to India, where it first appears in the *Paitāmahasiddhānta* in the fifth century (see section 5.6.1). It can be shown[21] from Hipparchus' calculations, which are preserved by Ptolemy (*Almagest* IV, 11), that Hipparchus used a chord table, similar to those used in Indian works, with values of the angle computed in increments of 3¾°.[22] But Ptolemy improved this and replaced Hipparchus' table with his own improved chord table, now with $R = 60^\circ = 1{,}0$, which was computed at intervals of half a degree.

Hipparchus wrote a treatise called *On Sizes and Distances* in two books. This material was important for the prediction, limits, and circumstances of eclipses, and for the computation of parallax, a topic which he addressed in another treatise. From comments made by Ptolemy (*Almagest* IV, 9) and later by fourth-century CE mathematician and commentator Pappus, it can be concluded that the following relations were accepted by Hipparchus:

i. The apparent diameter (d_m) of the moon at mean distance from the earth is 1/650 of its circle.[23]
ii. The diameter of the shadow (d_u) of the earth is 2;30 times the diameter of the moon at mean distance.

The first relationship gives the following value for the diameter of the moon:

$$d_m = \frac{360}{650} = 0;33,13,50,\ldots^\circ$$

The proportion

$$d_u = 2;30 d_m$$

combined with the relationship given in (i), gives

$$d_u = 2;30 \times 0;33,14$$

$$= 1;23,\ldots$$

In a similar way to this, the "Canobic Inscription" of Ptolemy[24] apparently expresses the relationship of the respective diameters:

$$d_m = \frac{90}{162} = \frac{360}{648},$$

$$d_u = \frac{90}{65},$$

so that

$$d_m = \frac{5}{9} = 0;33,20 \text{ and } d_u = \frac{18}{13} = 1;23.^{25}$$

Hipparchus' maximum latitude of the moon was 5°,[26] found "by calculation" according to Ptolemy and backed up by observations of the moon "whether taken with respect to the stars or taken with instruments." The first method, observing the moon with respect to fixed stars to determine the greatest possible latitudinal deviation, was probably how it was determined in Mesopotamia[27] (see section 3.4.3).

Ptolemy remarks (*Almagest* IV, 2) that Hipparchus "proved" various period relations by "Babylonian and his own observations." These include 4267 months = 4612 revolutions in longitude (Ptolemy indirectly showed that upon division by 17, this produces the relation 251 synodic months = 269 anomalistic months), as well as 5458 months = 5923 returns in latitude. That Hipparchus could have achieved this by these means is surely an exaggeration. In fact, as was first observed early in the twentieth century,[28] all the parameters he gives can be directly derived from those given in Babylonian astronomical cuneiform ephemerides. Hipparchus was clearly dependent on

Mesopotamian astronomical material and had some sort of direct knowledge of the contents of the sources. It is certain that he was very familiar with system B.

It seems likely that Hipparchus was the first to use Babylonian material for such purposes. But it is not certain how he gained knowledge of such parameters or how these period relations were transmitted to him. It has been established that he (and indeed Ptolemy) was in possession of a complete (or almost complete) list of lunar eclipses beginning with the reign of Nabonassar (747 BCE). He also had Babylonian observations of planets.[29] As Ptolemy credits Hipparchus with these parameters, it would seem that Hipparchus had access to some text(s) that Ptolemy was unaware of—otherwise Ptolemy would have cited those more original sources instead of Hipparchus.

It is difficult to suppose that Hipparchus would have obtained the parameters straight from the astronomical cuneiform texts themselves. It seems more plausible that they were extracted and translated by a very competent Babylonian commentator who was familiar with these techniques. The existence and identity of such an individual is as yet unknown. There exist some sources that reveal traces of the nature of the transmission of Mesopotamian astronomical and astrological ideas into Greece. There are countless references to the so-called "Chaldeans"[30] but most are vague and reveal little. Mentions of specific individuals are found in the works of Vettius Valens, who refers to Sudines and Kidenas.[31] Strabo and Pliny mention the Chaldean astronomers Kidenas, Naburianus, and Sudines.[32] A Greek excerpt[33] also mentions period relations which were said to have been discovered by Kidenas.

Astronomy modeled by spherical geometry poses problems such as the positions of celestial bodies which should be measured by arcs on the celestial sphere. Early approaches in Greek astronomy simply reduced or approximated these spherical triangles by plane triangles. In most cases this was done without a great loss of accuracy. However, problems concerning time-keeping, rising times, and parallax required a more precise approach; one that involved the ability to handle spherical triangles mathematically. It seems that problems in spherical trigonometry were known at least by the time of Menelaus of Alexandria (ca. 100 CE) whose name is commonly attached to such results. However there is evidence to suggest that these ancient trigonometric methods antedated him and were known at least as far back as Hipparchus.[34] Concepts such as spherical triangles, the so-called Theorem of Menelaus, which sets out and proves the relationship between arcs (and as a direct consequence, angles) of great circles on a spherical surface, and the application of this geometry to astronomical phenomena were frequently used by astronomers who followed him.

Furthermore, it seems likely that among the techniques that Hipparchus used was the analemma.[35] The analemma is a technique that collapses the arcs and points of the three-dimensional celestial sphere onto corresponding straight lines and points in the two-dimensional plane by means of orthogonal projection.[36] This may be what he means when he speaks of having solved a problem *dià tōn grammōn* (literally "by means of straight lines").[37] This technique was used extensively in India, so much so that in Arabic texts it was referred to as the "Indian circle."[38]

Hipparchus was well aware of the problems of parallax. He wrote, according to Ptolemy (*Almagest* V, 19), two books on parallax (if not more). Methods based on the theorems set out by Menelaus were employed in the *Almagest*. It is not known who initially isolated and enunciated the effects of parallax on planetary positions in Greek astronomy, but by the time of Hipparchus, the theoretical make-up of the problem had been developed to a moderate degree of sophistication, so it seems likely that it predated him by a significant amount of time. Ptolemy gives us insight into how Hipparchus approached the problem of parallax, which seems to be related to his consideration of the relative distances:[39]

> Now Hipparchus used the sun as the main basis of his examination of this problem. For since it follows from certain other characteristics of the sun and moon (which we shall discuss subsequently) that, given the distance to one of the luminaries, the distance to the other is also given, Hipparchus tries to demonstrate the moon's distance by guessing at the sun's. First he supposes that the sun has the least perceptible parallax, in order to find its distance, and then he uses the solar eclipse which he adduces; at one time he assumes that the sun has no perceptible parallax, at another that it has a parallax big enough [to be observed]. As a result the ratio of the moon's distance came out different for him for each of the hypotheses he put forward; for it is altogether uncertain in the case of the sun, not only how great its parallax is, but even whether it has any parallax at all.
> (*Almagest* V, 11, as translated by Toomer [1984])

As can be seen here and elsewhere (*Almagest* V, 19), Ptolemy is critical of Hipparchus' treatment of solar parallax, which motivates his own description (fig. 4.1). Let ABG be a segment of the ecliptic, AD a segment of the lunar orbit, A the node, D the center of the moon, DB the perpendicular dropped from the center of the moon to the ecliptic, and E the pole of the horizon or the zenith. DH is then the parallax of the moon, and the latitudinal component of this parallax is $D\Theta$ and the longitudinal ΘH.

Ptolemy then explains how to derive these arcs. Supposing that the total parallax is proportional to the angular distance of the moon from the zenith,

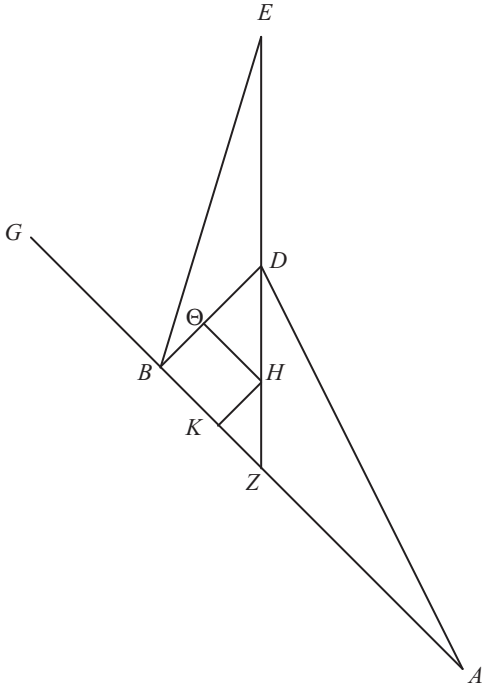


Figure 4.1 Ptolemy's account of parallax.

he states that the total lunar parallax, arc *DH*, is proportional to arc *ED*. Then he notes that the two components of longitudinal and latitudinal parallax (Θ*H* and *D*Θ, respectively) can be computed if the angle *GZE* is given, that is, the angle made by the lunar zenith distance circle (the great circle running through the zenith and the moon) and the ecliptic. Ptolemy then observes that the previously described methods calculate not *GZE*, but *GBE*, that is, the angle made by the solar zenith distance circle and the ecliptic.

At the time of the eclipse, the distance between the two luminaries is negligible, that is, *B* and *D* are very close to one another, and thus the difference between these two angles will be very small. However, Ptolemy observes that Hipparchus was aware of this discrepancy and made attempts to correct it ("in a very careless and irrational way" in the estimation of Ptolemy [*Almagest* V, 19]). He states that Hipparchus' attempts were on several fronts. The first was that Hipparchus corrected this problem for a single value of *AD*, rather than for a selection of values, or all possible values.

Second, Ptolemy states that in Book I of his work on parallax, Hipparchus showed that, in order to demonstrate that if *ED* is given *DH* can be found, *ED* itself should be found by finding arc *EZ* and angle *EZG*. This is done,

Ptolemy adds, in Book II, where Hipparchus calculates ZD and takes ED as the difference between EZ and ZD. Ptolemy criticizes this because the moon's corresponding point on the ecliptic (i.e., its position in longitude) is taken at B, that is, along a line perpendicular to the ecliptic, and not at Z, despite the fact that the whole point of making the correction was to notice that arc EZ and angle EZG are distinct from arc EB and angle EBG. Ptolemy then goes on to give what he believes to be a better procedure.

Pappus, in his commentary on the *Almagest*, gives a long discussion of this section and claims to present a description of Hipparchus' approach to parallax. It has been argued[40] that Pappus' description is of such poor quality that it is likely that he did not have any more information than Ptolemy's description of Hipparchus' procedure, and on this basis was trying to reconstruct some procedure which Ptolemy might have criticized. The example Pappus says Hipparchus gives is for the clime of Rhodes, with the moon in the beginning of Pisces, with latitude +2;30. It is tempting to identify this with the "one" example Ptolemy says that Hipparchus gives. Elements of this problem are similar to those given by Ptolemy's description of an observation made by Hipparchus[41] in −126 May 2,[42] which may be the same example.

The aim of the procedure (fig. 4.2) is to find the zenith distance of the moon ($E\Delta$). This is found by taking the difference:

$$E\Delta = EZ - \Delta Z.$$

E is the zenith for Rhodes, A is the ascending node, and Z is the beginning of Pisces. EZ is found from a table; Pappus gives it as 65;48°.[43] AZ, the distance between the node and the beginning of Pisces, is given as 30° and the angle $ZA\Delta$, the inclination of the lunar orbit (i), is given as 5°.

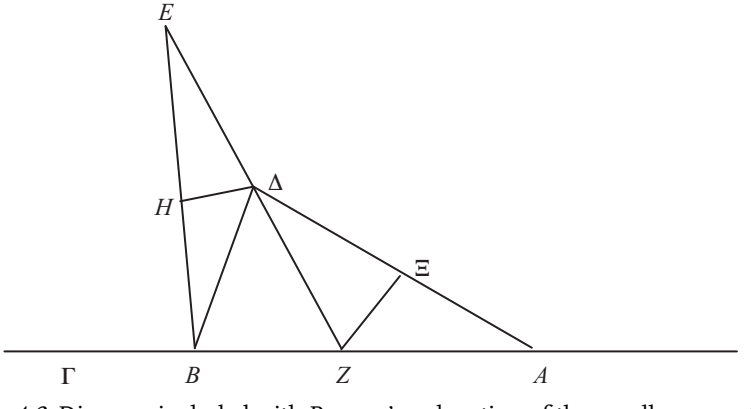


Figure 4.2 Diagram included with Pappus' explanation of the parallax correction.

The procedure Pappus sets out finds the necessary values by means of ratios which are based upon faulty assumptions. First ZΞ is computed by means of the following relations:

$$\frac{AZ}{180} = \frac{Z\Xi}{10} \Rightarrow \frac{AZ}{30} = \frac{Z\Xi}{1;40^\circ}$$

where $10 = 2 \cdot i = 2 \times 5$ and $180 = 90 \times 2$. This relation is mathematically invalid. The left-hand side of the relation seems to be based on the formula for arcs and angles on spherical triangles, expressed in modern notation as

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}.$$

However, it is not correct to take factors of this relationship in this way, that is, a factor of an arc is not equal to the same factor of the angle:

$$k \cdot \sin(A) \neq \sin(kA)$$

(one can do it for a chord, but not an arc). If this is indeed Hipparchus' procedure, it reveals that he may have been familiar with certain spherical trigonometric approaches, but used them imperfectly. Thus, this example gives a glimpse into the early stages of spherical trigonometrical inquiry. These techniques were in the early stages of development and were not applied completely consistently as they were by the time of Ptolemy.

Pappus then turns his attention to $Z\Delta\Xi$ which he calls *trípleuron* (literally "three-sided").[44] ΔZ is found by the same false assumption to be 1;44°. Thus, the zenith distance of the moon, Pappus continues, is the difference

$$\begin{aligned} E\Delta &= EZ - \Delta Z \\ &= 65;48 - 1;44 \\ &= 64;4^\circ, \end{aligned}$$

which is the result supposedly given by Hipparchus.[45]

What is notable is that the procedure rests upon two inexact proportions which seem to combine assumptions for plane triangles while retaining an underlying spherical approach. It may be useful to point out that the following single proportion, a consequence of considering the problem strictly by spherical trigonometry, would have produced the desired result in one step:

$$\frac{\Delta Z}{\sin i} = \frac{AZ}{\sin \nu},$$

where i is the inclination of the lunar orbit and $\nu = \angle A\Delta Z$. Thus, supposing that Pappus' description reflects Hipparchus' approach, it may be possible to conclude that Hipparchus was working at a time when practitioners were experimenting with computations on the sphere but before the rules and

assumptions for spherical trigonometrical formulas had been properly set out and rigorized.[46]

Because Hipparchus was aware of the issues of parallax and had good values for the sizes and distances, he was in a position to predict with success the occurrences of both lunar and solar eclipses. A commentator[47] on his work reveals that Hipparchus worked on establishing the intervals between both lunar and solar eclipses and concluded that lunar eclipses can occur at five-month intervals and solar eclipses at seven-month as well as one-month intervals (but not at the same locality). Ptolemy provides a detailed proof of these intervals (*Almagest* VI, 6), computations which are likely to be based on Hipparchus' work. Perhaps the interest in these intervals was sparked by seeing the examples of the eclipse tables derived from Babylonian sources.[48]

4.4.2 Cleomedes

Cleomedes (ca. 200 CE[49]), a Stoic astronomer, was not notable for producing original work, but rather used material excerpted from more technical astronomical treatises for the contents of his lectures, and is thus a useful source for information on earlier thinkers. His chief authority is the second-century BCE Greek Stoic Posidonius, although his work *Caelestia* contains a collage of collected references which he either endorses or refutes according to Stoic scientific principles. He clearly knows Hipparchus' works, but seems unaware of Ptolemy. Therefore, despite the fact that he is much later in time, his subject matter seems to reflect that of a period contemporary with Hipparchus. He introduces his discussion of eclipse theory with the following statement, which obviously refers to some of the folklore of the day:

> τούτοις ἑξῆς τὸν περὶ τῆς ἐκλείψεως αὐτῆς ποιησόμεθα λόγον, ὅπως μὴ παραπλησίως τοῖς γραϊδιος καὶ ἡμεῖς δοξάσωμεν, ὅταν ἐκλείπῃ ἡ σελήνη, ὅτι φαρμακίδες εἰσὶν αἱ καθαιροῦσαι αὐτήν.
>
> [We shall next conduct our discussion concerning the eclipse of the moon, our object being to avoid sharing with old hags the belief that at eclipses witches drag the moon down.]
>
> (II.5.150–153, as translated by Bowen and Todd [2004] p. 153)

In the *Caelestia*, eclipse theory is dealt with in II.6. Lunar eclipses are dealt with first. The shadow has a conical shape (II.6.9–10, 90–108), although earlier in the work, Cleomedes uses an argument for the relative sizes of the luminaries and the earth which implies a cylindrical shadow (II.1.286–294). It is in this place that he asserts (following Aristarchus—prop. 5) that the ratio of the diameters of the moon (r_m) and the earth's shadow (r_u) is

$$r_u = 2r_m.$$

Cleomedes states that the moon is eclipsed by falling into the Earth's shadow

> ἀλλ΄ οὐχ ἕτερον τρόπον. . .ἀπὸ τῶν φαινομένων αὐτῶν πάρεστιν ὁρᾶν
> [and only in that way, <as> can be seen from the phenomena themselves.]
> (II.6.35–36, as translated by Bowen and Todd [2004])

This is an interesting admission from a Stoic author, who should be more inclined to deny sensory observation as a criterion for validity. In this sentiment, he spends much time criticizing Epicurus and his estimation that the sun is the size it appears to be (II.1) (further into his work he states that this diameter is a foot wide [II.1.76]):

> αὐτῇ τῇ διὰ τῆς ὄψεως φαντασίᾳ κατακολουθήσαντες καὶ ταύτην τοῦ μεγέθους αὐτοῦ κριτήριον ποιησάμενοι.
> [they followed only the sense presentation caused by sight: that is, they made this presentation a criterion of its size.]
> (II.1.2, as translated by Bowen and Todd [2004])

Cleomedes' work reveals a comprehension of the different rates of duration of eclipses as being dependent on where the moon is with respect to the shadow cone. The apparent size of the sun as well as the moon is set by him at 12 digits (II.3.18–19).

Most notable, though, is Cleomedes' detailed attention (II.6.109–196) to the claim that there was an occurrence of a lunar eclipse when both the luminaries were above the horizon, a so-called paradoxical eclipse (see section 1.3.2). He initially seems to give some credit to the idea as he states:

> ὥστε δεήσει ἡμᾶς, ἂν ταῦτα οὕτως ἔξῃ, ἑτέραν εἶναι τὴν αἰτίαν τῆς περὶ τὴν σελήνην ἐκλείψεως ἀποφαίνεσθαι.
> [If this [theory] is correct, we shall have to lay claim to a different cause for the moon's eclipse.]
> (II.6.120–121, as translated by Bowen and Todd [2004])

This problem was obviously taken seriously by some astronomers before Cleomedes, as he mentions the attempts of earlier scientists (οἱ παλαιότεροι τῶν μαθηματικῶν) to explain this phenomenon. Their explanation, Cleomedes remarks, was that because the earth was a sphere, both divinities could be observed on the horizon when they were directly opposite one another. Cleomedes rejects this explanation, stating that the line of sight is obscured by the size of the earth.

Cleomedes also admits that the appearance of the sun above the horizon may be due to some effect resulting from an atmospheric condition, such as a dense cloud illuminated by the sun's rays which appears to be the sun itself

(II.6.171), or a "counter-sun" (*anthelios* [ἀνθήλιος]) (II.6.173), or that the moist air causes refraction to such an extent that the sun still appears above the horizon even after it sets (II.6.174–177).

Solar eclipses are not given their own section and Cleomedes does not directly talk about or describe the effects of parallax. However, he does include an account of an observation of a solar eclipse that was said to have been total at the Hellespont; in Alexandria the sun was observed to be eclipsed beyond ⅓ of its diameter, an amount that Cleomedes says is equivalent to more than two digits.[50]

4.5 The Peak of Greek Mathematical Astronomy

Hipparchus' ambitions to give predictive potential to existing astronomical models came many centuries after the very first attempts to reveal the physical causes of astronomical phenomena among Greek thinkers, and it was to be several more centuries before his aspiration was fully reached. This goal, representing the climax of Greek mathematical astronomy, was achieved by Ptolemy (ca. 100–175 CE), who worked in Alexandria, in his seminal treatise, later called the *Almagest*. At its heart was a complete set of geometrical kinematic models which were given quantitative significance; their sources were drawn from ancient as well as contemporary data, updated by the utilization and refinement of the leading mathematical techniques of the time. The establishment of these sizes and magnitudes was very much guided by theoretical insight; for example, among the many observational records Ptolemy had at his disposal, selections were made as suggested by key geometrical alignments of the models which would minimize or maximize a particular element. The completeness of the exposition and the presentation of the essential results in tabular format, with instructions for their use as well as explanations of their structure and derivation, allowed astronomers to compute astronomical details of choice for any time and any circumstance. This work singularly dominated and directed astronomical efforts for more than a millennium to come, with versions being made in Greek, Latin, Arabic, and even Sanskrit (among other languages). The reception this work received ensured in many senses the concentration and continuity of the early astronomical tradition that was passed on to the modern world.

4.5.1 Ptolemy's Almagest

Eclipse observations were intrinsically useful to Ptolemy, as to Hipparchus, for establishing the parameters he adopted for his geometrical models. Groups of

carefully selected eclipse observations and the astronomical alignments they attested to were vital.[51] But eclipses also received significant attention in their own right. In terms of exposition of eclipse theory, Ptolemy's approach is clear and coherent. First, he calculates the mean oppositions and conjunctions, and from these he determines the true syzygies. Then, applying the eclipse limits he calculated, which take into account the sizes and distances of the celestial bodies combined with information on the inclination of the lunar orbit, he determines the possibility of an eclipse. He next gives mathematical validation of the intervals between eclipses, verifying the six-month interval for solar and lunar eclipses, the five- and seven-month intervals for lunar eclipses, the five- and seven-month intervals for solar eclipses, and the one-month interval for solar eclipses (the last two of which were first put forward by Hipparchus[52]). Next, he gives a way to calculate the particular features of the eclipse such as half-duration and magnitude. Finally, he gives the method for calculating the angle of "inclination" or *prosneusis* which is closely related to the direction of impact. This method he says is of use in astrological procedures.

Ptolemy's exposition of eclipse theory is found in the middle of the *Almagest*, in the last half of Book V and all of Book VI. His theory of parallax is treated in the final chapters (11–19) of Book V, along with information concerning the sizes and distances of the sun and the moon. With these elements established, his general eclipse theory follows in Book VI. It is quite understandable that he treats parallax first as it fits into his general progression of determining the mean position of the moon, then the true, and then with the theory of parallax applied, the so-called "apparent" position. Ptolemy's treatment of parallax in the *Almagest* contrasts with his exposition in another of his works, the *Handy Tables*. There he computes the combined parallax of the sun and the moon and not the individual values; this combined amount is only useful when the sun and the moon are very close to each other, i.e., during an eclipse.

The other assumptions critical to eclipse theory (given in previous sections in the *Almagest*) are the following:

i. Length of the synodic month = $29;31,50,8,20^{d}$
ii. Information about the inclination of ecliptic ($\epsilon = 23;51^{\circ}$) and lunar orbit ($i = 5^{\circ}$).
iii. Mean motions: in particular the nodal elongation = $12;11,26,41,20,17,59^{\circ/d}$ (IV 4)

Ptolemy's length of the synodic month is taken from Hipparchus, who took it from system B as previously mentioned.[53] Ptolemy is also familiar with the Mesopotamian Saros cycle:

> The even more ancient astronomers use the somewhat crude estimate that such a period could be found in 6585⅓ days . . . they called this interval the "periodic."
> (*Almagest* IV, 2)

6585⅓ days = 223 synodic months = 239 anomalistic months = 243 draconitic months = 241 returns in longitude +10;40°.[54] Thus the mean synodic month 29;31,50,18,50,2,41, . . . can be derived. Ptolemy remarks that Hipparchus noted that this relation was not accurate and states that he proposed the following relation:

$$126007^d 1^h = 4267 \text{ months},$$
$$4573 \text{ anomalistic months} = 4612 \text{ rev. of moon in long. } -7;30°.$$

This gives a mean synodic month of 29;31,50,8,9,20,12,22, . . . Ptolemy says it gives "approximately" 29;31,50,8,20 days (*Almagest* IV, 2). As discussed previously, Ptolemy then gives another interval from Hipparchus:

$$5458 \text{ synodic months} = 5923 \text{ draconitic months or}$$
$$1,30,58 = 1,38,43.$$

This interval, he claims, has eclipses that are identical with respect to size and duration, and accumulates no difference in anomaly. This is the period which underlies the "eclipse magnitude" function ψ in system B (see section 3.4.3).[55]

Ptolemy's account of parallax is the first detailed and complete description that is available, despite the fact that certain aspects for its calculation predated him. V, 12 is given over to a detailed description of the construction of an instrument for measuring zenith distances. The readings from this instrument could be used for calculating parallax from any observed zenith distance.[56]

In V, 17, Ptolemy presents his geometrical computations of parallax of the sun and the moon from the amount of their geocentric distances. He introduces the section by carefully explaining the need for such a correction. At the outset he states that "the earth does not have a ratio of a point to the distance of the moon's sphere" which is probably a response to claims such as those that Aristarchus makes (see above, prop. 2). This entails, as he observes, that the "apparent position does not coincide with its true position" (V, 11). Ptolemy quite clearly states that the apparent position is the position as seen from the standpoint of the observer, that is, with respect to some point on the earth's surface, which is distinct from the true position, given with respect to the center of the earth. He establishes that the ratio of the distance of the body to the earth's radius gives the amount of parallax. This implies that celestial bodies whose distances have the ratio to the earth's radius of a line to a point have no parallax. For practical purposes he then adds, one must first determine the

parallax in a particular situation and then from that deduce the ratio, which, he argues, is otherwise undeterminable.

The concept and computation of parallax are straightforward[57] (fig. 4.3). For an observer on the surface of the earth (at point *A*), the difference between the zenith distance of a celestial body for their location (the apparent zenith distance, ζ') and the zenith distance with reference to the center of the earth *O* (the true zenith distance, ζ) is the measure of parallax (π):

$$\pi = \zeta' - \zeta.$$

Curiously, this is a case in which the theoretical understanding of a phenomenon greatly surpasses the technology available to measure it. In contrast with the simplicity of the theoretical illustration of parallax by Ptolemy, the actual physical measurement of the phenomenon presented much difficulty. The distinctions in position are, for the most part, so slight, that the instruments employed to make observations were sufficient to detect only the lunar parallax with any satisfactory degree of accuracy. In fact, Ptolemy straight away concludes that the eccentricity of the solar orbit has no effect on the measure of parallax, and thus the amount of its distance can be kept constant (1210 earth radii) for the purposes of calculating parallax. The sun's parallax ranges from 0;0,7° at an argument of $\zeta = 2°$, up to a maximum ($\zeta = 90°$) of 0;2,51.

With respect to the moon however, the situation is more complicated. As a consequence of the setup of the lunar model presented by Ptolemy, lunar parallax is dependent on three variables: zenith distance, mean elongation (the distance between the mean positions of the sun and epicycle), and true anomaly (the position of the moon on the epicycle). Second, unlike the sun, the moon's varying geocentric distance must be taken into consideration, as

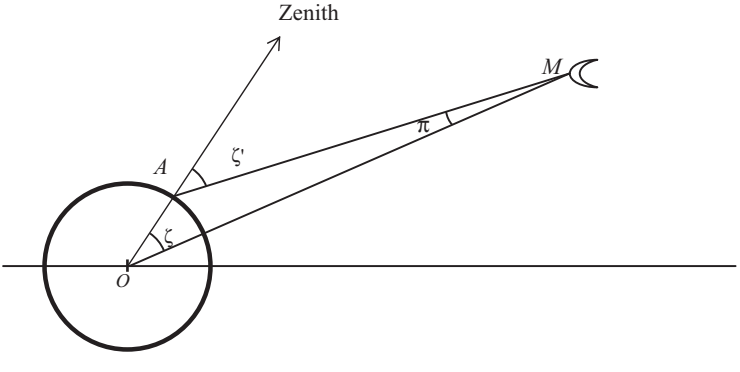


Figure 4.3 Lunar parallax and its relation to zenith distance.

it has a large impact on the measure of lunar parallax. Ptolemy has skillfully presented the values for parallax in a table with multiple columns, so that each of these factors can be accounted for.[58] His maximum lunar parallax is 0;53,34.

Ptolemy has a further element of computation of parallax: this is his concept of epiparallax (ἐπιπαράλλαξις). The idea behind epiparallax is that, once the longitude of the moon is corrected with respect to parallax, the time of conjunction too will have to be corrected, leading to a new zenith distance, which will in turn give an updated value of longitudinal parallax. A similar idea was used by Indian astronomers (see section 5.5.1). They recognized that the longitudinal parallax correction affects the time of syzygy and that as a result there is need to iterate to find the correct time of syzygy.

Ptolemy's approach to the determination of the occurrence of an eclipse is to calculate all the possible conjunctions and oppositions from his epoch (the first year of Nabonassar), and to present the results in a table along with the information about lunar anomaly and latitude at those times. Then he gives limits for which eclipses are possible, and by comparing these limits against the data given in the table, one can almost directly ascertain the syzygies at which eclipses can occur. The tables are grouped into fifteen 25-year periods, (Nab. 1–Nab. 1101),[59] twenty-four single-year entries, and twelve single-month entries. This 25-year interval is advantageous for the treatment of mean syzygies because the date between successive entries almost always remains the same.[60]

From the mean syzygies, one must make corrections to determine the true syzygies. Ptolemy explains the procedure to do so in the second half of VI, 4. Having calculated the mean conjunction and the appropriate anomalies, the corresponding true longitudes of the sun and the moon are easy to compute. However, after determining these true positions, Ptolemy notes that the sun and the moon will no longer be in at syzygy more often than not. Now the time until true conjunction will differ from the mean time by the amount

$$\pm\Delta t = \frac{\eta}{(\nu_m - \nu_s)} \text{ hours},$$

where Δt is measured in hours and correspondingly ν_m and ν_s are the velocities of the moon and sun, respectively, per hour and η the elongation between the sun and the moon. Ptolemy assumes that because the motion of the sun is so small, its variation is trivial, and thus he notes "we take the difference between the bodies in longitude, expressed in degrees, and increase it by a twelfth part of itself, to account approximately for the additional motion of the sun [between mean and true syzygy]." This is easily shown by approximating the ratio of the sun's motion to the moon's by the constant 1/13, and rearranging the above

expression, so that it becomes

$$\nu_m \left(1 - \frac{\nu_s}{\nu_m}\right).$$

After the constant is substituted, the denominator becomes

$$\nu_m \left(1 - \frac{1}{13}\right) = \frac{12.\nu_m}{13}$$

and thus the time until true conjunction becomes a function of the lunar velocity only, calculated by the formula

$$\pm\Delta t = \frac{13\eta}{12\nu_m} \text{ hours.}$$

The problem thus becomes to find ν_m for the given anomaly α, and Ptolemy carefully reveals how to compute this. Hence the time of the true syzygy can be found.

Ptolemy is critical of the previous attempts to obtain adequate values for the apparent diameters (V, 14), naming those who measure "the luminaries by measuring [the flow of] water or by the time [the bodies] take to rise at the equinox." Cleomedes (see above) refers to the same techniques, stating that

> διὰ γὰρ τῶν ὑδρολογίων καταμετρούμενος εὑρίσκεται μέρος ἑπτακοσιοστὸν καὶ πεντηκοστὸν τοῦ οἰκείου κύκλου...λέγεται δ΄ ἡ τοιαύτη ἔφοδος ὑπὸ πρώτων τῶν Αἰγυπτίων ἐπινοηθῆναι.
> [When the sun's size is measured out by means of water clocks, it is determined as 1/750th part of its own circle . . . such a procedure was reportedly first conceived of by Egyptians.]
> (*Caelestia* II.1, as translated by Bowen and Todd [2004], pp. 185–191)

A similar estimate for the solar diameter is reported by Archimedes who attributes it to Aristarchus; this is the solar diameter as measured by 1/720 of its orbit.

In order to establish his diameters, Ptolemy uses two pairs of eclipse observations, two near the apogee and two near the perigee. Using a diopter, the design of which he attributes to Hipparchus, he concludes that the diameter of the sun remains constant and also that the moon is the same size as the sun when it is at the apogee of its epicycle. The second statement he qualifies: it is "in contradiction to the hypotheses of my predecessors [who assumed that it subtends the same angle as the sun at full moon] when it is at mean distance." (V, 14). Unfortunately, these "predecessors" are unnamed.[61] Furthermore, as Neugebauer points out, Ptolemy thus denies the possibility of an annular eclipse.[62] According to Ptolemy, the moon at its farthest distance looks equal to the sun and so at all other distances will look larger than the sun (and, which is important for annular eclipses, never smaller).

Ptolemy's approach to establishing the diameters does not use arbitrary observations of the sun and moon, but rather employs several observations of the luminaries at certain selected circumstances. He purposefully and insightfully chooses observations at points at which a certain element is minimized or zero so that it has no effect on his calculations. For example, in this case, he chooses eclipses that occur either at the apogee or the perigee so that the anomaly (α) is zero. He then combines these observations with their preassumed theoretical circumstances and computes details so that accurate figures can be derived.

From data extracted from two eclipses that occurred near the apogee,[63] Ptolemy shows (V, 14) that the moon's diameter subtends an arc measuring 0;31,20°. He uses two further eclipses to get the diameter at perigee.[64]

The idea behind Ptolemy's calculations is to compute the distance (b) from the center of the moon to the center of the shadow at mid-eclipse, a distance perpendicular to the lunar orbit (as distinct from the latitude, which is perpendicular to the ecliptic). He then utilizes the relationship between magnitude (μ) and diameter (fig. 4.4):

$$\frac{1}{2}(d_u + d_m) - b = \mu.$$

With the values from each pair of eclipses, Ptolemy uses the fact that the difference in magnitudes is equal to the difference of mid-eclipse distances:[65]

$$b_2 - b_1 = \mu_1 - \mu_2.$$

Accordingly, Ptolemy gives the distances between the two centers in the first eclipse pair as $b_1 = 0;48,30°$ and $b_2 = 0;40,40°$ respectively. The difference between the two eclipse magnitudes is

$$(6/12 - 3/12)d_m = d_m/4$$

so that

$$\begin{aligned} \frac{1}{4}d_m &= b_1 - b_2 \\ &= 0;48,30 - 0;40,40 \\ &= 0;7;50, \\ d_m &= 0;7,50 \times 4 \\ &= 0;31,20° \end{aligned}$$

and

$$\begin{aligned} r_u = \frac{1}{2}d_u &= 0;48,30 - \frac{0;31,20}{2} + \frac{0;31,20}{4} \\ &= 0;40,40 \end{aligned}$$

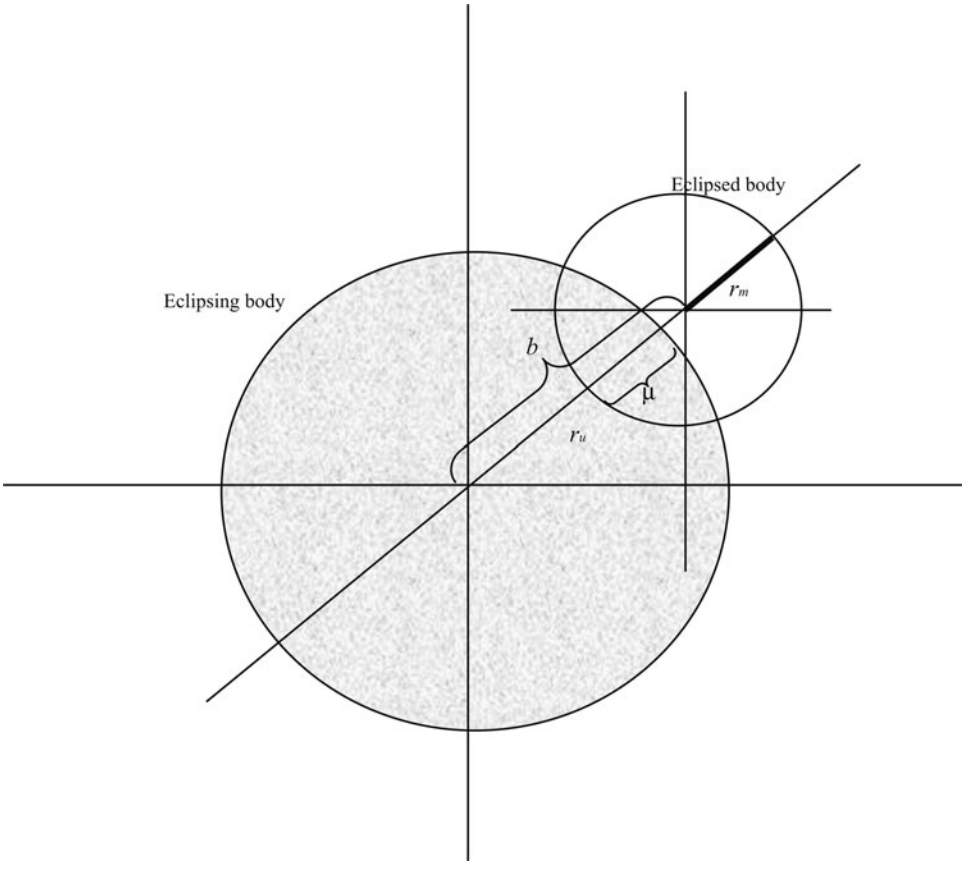


Figure 4.4 Apparent diameters and magnitudes.

(so that, incidentally, the second eclipse is just total, that is, $b_2 = r_u$, the shadow's radius).

Similarly, for the apparent diameters at perigee:

$$\left(\frac{7}{12} - \frac{3}{12}\right) d_m = \frac{1}{3} d_m,$$

$$\begin{aligned} d_m &= 3(0;54,50^\circ - 0;43,3^\circ) \\ &= 3 \times 0;11,47^\circ \\ &= 0;35,21,0^\circ \\ &\approx 0;35,20^\circ, \end{aligned}$$

and

$$\begin{aligned} r_u = \frac{1}{2}d_u &= \frac{1}{2}d_m - 0;54,50 - (3/12)d_m \\ &= 0;54,50 - (1/4)(0;35,20) \\ &= 0;54,50 - 0;8,50 \\ &= 0;46. \end{aligned}$$

One can verify this by comparing it to the first eclipse of the pair, in which $d_u \approx 0;45,59,40$. Ptolemy then gives the useful approximate proportion of the diameter of the moon to the diameter of the shadow:

$$1{:}2\frac{3}{5} \text{ or}$$
$$1{:}2;36,$$

which, as he rightly observes, is slightly too small.

The sizes then are the following:

	Perigee	Mean distance	Apogee
r_m	0;17,40	0;16,40	0;15,40
r_u	0;46	0;43,20	0;40,40
r_s	0;15;40	0;15,40	0;15,40

In contrast, without derivation or comment, he gives slightly more accurate values further on in VI, 7 for the shadow when he is deducing eclipse limits:

$$r_u = 0;40,44 \text{ at greatest distance,}$$
$$r_u = 0;45,56 \text{ at least distance.}$$

To conclude this section, Ptolemy states that the minimal outer eclipse limit will be when the apparent center of the moon is $0;33,20^\circ$ ($= 0;17,40 + 0;15,40$, the sum of the maximal diameters when the moon is at perigee of its epicycle) from the center of the sun.

Next, to simplify matters, in order to find the limits for eclipses, Ptolemy argues that the ecliptic and the lunar orbit "are sensibly parallel to each other, at least as far as concerns the positions [of the bodies] at the time of eclipse" (*Almagest* VI, 5). Such an assumption is frequently made, in both Greek and other contexts. For example, this is done in Indian sources, notably by Brahmagupta (see section 5.6.4). Because the observation of the magnitudes of lunar eclipses is independent of the location of the observer, the lunar eclipse limits are easily calculated (fig. 4.5).

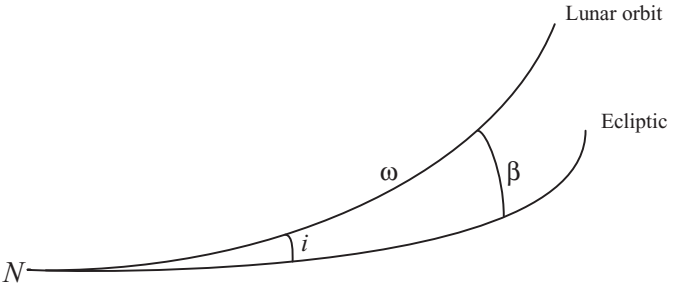


Figure 4.5 Lunar latitude (β), the argument of latitude (ω), and the inclination of the lunar orbit (i) used to determine eclipse limits.

Let ω be the distance from the node N and β be the latitude of the moon. Thus, Ptolemy assumes:

$$\beta = \omega \cdot \sin(i),$$

in which the inclination $i = 5°$. Thus he approximates $\sin(i) \approx 1/11;30$,[66] so that the relationship becomes

$$\omega = 11;30\beta.$$

The maximal latitudinal distance for which an eclipse can occur is given by the sum of the two radii at perigee:

$$|\beta| \leq r_m + r_u = 0;17,40 + 0;46 = 1;3,40°$$

and then with the above formula the distance from the node for true conjunction is given by

$$|\omega| \leq 11;30.\beta = 11;30 \times 1;3,40 = 12;12,10°.$$

However, it is necessary to take into account the deviation caused by the difference between mean and true longitudes. Ptolemy reckons the effects of the equation at its maximum to be 5;1 for the moon and 2;23 for the sun (taken in opposite directions for the maximal possible distance) and so another 3° must be included.[67] Adding this amount onto the true argument of latitude of the moon (ω') one has the maximal distance from a node at which a lunar eclipse is possible with respect to the mean argument of latitude of the moon ($\overline{\omega}'$):

$$\overline{\omega}' = 12;12 + 3 = 15;12;$$

that is, for a descending node:

$$90° - 15;12 \leq \overline{\omega}' \leq 90 + 15;12,$$

$$74;48 \leq \overline{\omega}' \leq 105;12,$$

and for an ascending node

$$270^\circ - 15;12 \leq \overline{\omega}' \leq 270 + 15;12,$$

$$254;48 \leq \overline{\omega}' \leq 285;12.$$

In the case of the sun, however, Ptolemy states outright that "solar eclipses ... are more complicated to predict because of lunar parallax" (VI,10). In the case of a solar eclipse, the simple sum of $\beta = 0;15,20 + 0;17,40 = 0;33,20$ is subject to variation because of parallax, and the boundaries at which an eclipse can occur are greatly altered. To take this into account, Ptolemy calculates the maximum effect of parallax for the seven climes,[68] which span the geographical latitudes $\phi = 16;30$ to $\phi = 48;30$. He must determine the cases in which parallax can cause the centers of the disks of the sun and the moon to be at an apparent angular distance of 0;33,20—the eclipse limit.

Ptolemy presents the values for the two extremes (IV, 5), one for each end of his prescribed region. He does not reveal how these values were derived. The values for the latitudinal (π_β) and longitudinal (π_λ) components of maximal parallax for the moon at least distance are

	π_β	π_λ	Position
Maximum northward effect	0;8	0;30	Leo/Gemini
Maximum southward effect	0;58	0;15	Scorpio/Pisces

From these values, he derives new eclipse limits:[69]

$$\omega > 8;22^\circ,$$

$$\omega > 17;41^\circ,$$

so that the following limits are established. For a descending node

$$90 - 20;41 \leq \overline{\omega}' \leq 90 + 11;22,$$

$$69;19 \leq \overline{\omega}' \leq 101;22,$$

and for an ascending node

$$270 - 11;22 \leq \overline{\omega}' \leq 270 + 20;41,$$

$$258;38 \leq \overline{\omega}' \leq 290;41,$$

for the region bounded by 16;30° and 48;30° in northern terrestrial latitudes. These numbers are then taken into account in the construction of his tables for lunar and solar eclipses (VI 8).

Using these eclipse limits, Ptolemy then proceeds to carefully show the possible intervals of months between eclipses. He firstly briefly proves that

both solar and lunar eclipses can occur at six-month intervals, showing that the mean motion of the argument of latitude of the moon over a six-month period is 184;1,25° (= 6 · 30;40,14,9), so that when an eclipse occurs at one node, six months later the moon will be at the opposite node and another eclipse will occur. He then investigates in great mathematical detail the different time intervals in which lunar and solar eclipses can occur. He sets out under what circumstances eclipses can occur at intervals of one, five, six, and seven months, taking into account parallax and geographical latitude.[70] The realization that eclipses can occur at such intervals was attributed to Hipparchus (see section 4.4.1). One can only speculate how much of this material Ptolemy simply copied from him, or whether the rigorous proofs were his own work, intended to verify the discovery first enunciated by Hipparchus.

Using the moon's position in latitude as an argument, Ptolemy has designed his eclipse tables (VI, 7–8) with the intention of determining the actual occurrence of an eclipse and its circumstances, namely, the magnitude (μ) and the duration. He has two separate tables, the first for a solar eclipse and the second for a lunar eclipse. Within each of these tables he calculates the parameters for the greatest distance of the moon (i.e., at apogee) and the least distance of the moon (i.e., at perigee). The magnitude is given in terms of linear digits—the diameter of the moon is divided into 12 such equal digits. The magnitude of an eclipse is dependent on the value of the lunar latitude. When the lunar latitude is at its maximum for an eclipse to occur ($\beta_{\max}$) then the digits of magnitude are zero, and for each one-digit increment, the lunar latitude decreases by $d/12$, where $d = d_m$ for a lunar and $d = d_s$ for a solar eclipse. When the lunar latitude has decreased from its maximum $\beta_{\max}$ by the amount of d the magnitude $\mu = 12$ is reached. All further increments in magnitude represent an eclipse with a duration of totality. The greatest possible magnitude for an eclipse is

$$\mu_{\max} = \frac{12\beta_{\max}}{d}$$

Ptolemy also computes the time from the beginning to the middle of an eclipse and (where appropriate) the time from the beginning of totality to the middle of the eclipse, using elongation and various corrections.[71] He presents these parameters in a table.

The last table, intended for those who, "observing the [weather] indications derived from eclipses measure the size of the obscuration, not by the diameters of the disks but, on the whole, by [the amount of] the total surface of the disks" (*Almagest* VI, 17), is a conversion table for obscuration (see table below). It gives the values of the digits of lunar diameter with the corresponding areas of the disks of the sun and the moon obscured. This is done by simple geometrical

rules and computations which Ptolemy carefully goes through. He explicitly states that he uses the ratio of the circumference to the diameter as 3;8,30:1, saying that this is because this ratio is about midway between 3 1/7:1 and 3 10/71:1, which Archimedes used as bounds.[72]

Linear digits	Area of solar disk (digits)	Area of lunar disk (digits)
1	0 1/3	0 1/2
2	1	1 1/6
3	1 3/4	2 1/15
4	2 2/3	3 1/6
5	3 2/3	4 1/3
6	4 2/3	5 1/2
7	5 5/6	6 3/4
8	7	8
9	8 1/3	9 1/6
10	9 2/3	10 1/3
11	10 5/6	11 1/3
12	12	12

Ptolemy's tables for solar and lunar eclipses (VI, 8) are characterized by the following parameters and assumptions. All of this material is presented in the *Handy Tables*, but values are rounded and functions simplified.

Solar eclipse table

Apogee	Perigee
$84 \le \omega \le 96$	$83;36 \le \omega \le 96;24$
$264;0 \le \omega \le 276$	$263;36 \le \omega \le 276;24$
$\beta_{\max} = r_s + r_m = 0;33,20$	$\beta_{\max} = r_s + r_m = 0;31,20$
limits: $\omega = 11;30 \cdot \beta_{\max} = \pm 6°$	limits: $\omega = 11;30 \cdot \beta_{\max} = \pm 6;24°$
Digits: 0–12	Digits: 0–12;48

Lunar eclipse table

Apogee	Perigee
$79;12 \le \omega \le 100;48$	$77;48 \le \omega \le 102;12$
$259;12 \le \omega \le 280;48$	$257;48 \le \omega \le 282;12$
$\beta_{\max} = r_u + r_m = 0;56;24$	$\beta_{\max} = r_u + r_m = 1;3,36$
limits: $\omega = 11;30 . \beta_{\max} = \pm 12;12°$	limits: $\omega = 11;30 . \beta_{\max} = \pm 10;48°$
Digits: $\max = \frac{10;48}{0;30}$	Digits: $\max = \frac{12;12}{0;34}$
bounds: 0–21;36	bounds: 0–21;32

Ptolemy recognizes that the interval between first contact and mid-eclipse and the interval between mid-eclipse and release are not always the same, citing

lunar and solar anomaly as the cause: "the effect of which is that equal distances are covered [by the bodies] in unequal times." But he concedes that "as far as the senses are concerned, no noticeable error with respect to the phenomena would result from supposing these intervals equal in time. For, even when [the luminaries] are near mean speed, where the change [in speed] resulting from an [equal] increment [in the argument] is greater [than elsewhere], the motion over the number of hours represented by the whole duration of [even] the maximum possible eclipse does not exhibit the least noticeable difference [in duration] due to the change [in speed]" (*Almagest* VI, 9; as translated by Toomer [1984]).

The last section of Ptolemy's material on eclipse computations concerns the calculation of certain angles of inclination throughout the course of an eclipse which were deemed to have astrological importance. Such angles, called *prosneuseis* by Ptolemy, derived from the verb προσνεύω (literally "to incline"), have no significance astronomically and are certainly part of a broader tradition relating to eclipse themes derived from sources as far back as the Mesopotamian tradition of *Enūma Anu Enlil* (see section 3.2). In this tradition the quadrant of the disk (the disk being divided into four quadrants named after the four cardinal directions) was an important element determining the effects of the eclipse. This, along with his table for converting the linear digits into amount of area obscured, shows that Ptolemy's audience extends beyond those interested exclusively in mathematical astronomy.[73]

The *prosneusis* is made up of two angles. The first angle ξ is the angle between the line which joins the centers of the eclipsing and eclipsed body with the ecliptic (fig. 4.6).

Ptolemy has calculated the angles for each digit of obscuration, giving an example in his text of $\mu = 6$ digits at mid-eclipse, finding 31;1° for the angle

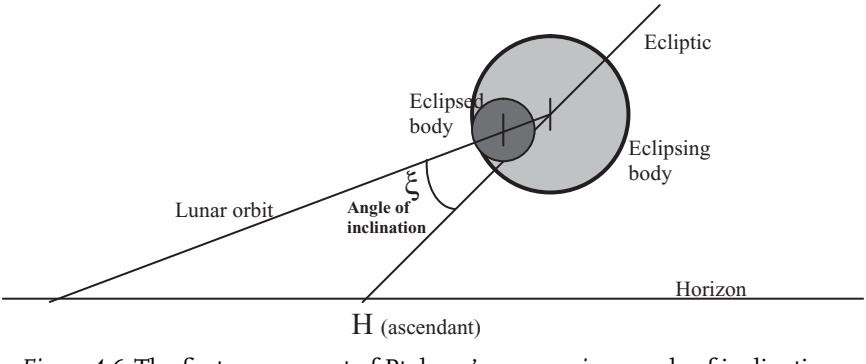


Figure 4.6 The first component of Ptolemy's *prosneusis* or angle of inclination: the angle between the line joining the centers of the eclipsed and eclipsing body and the ecliptic.

at the beginning and end of the eclipse, and 22;1° for the beginning and end of totality. His method is based on a relationship such that the angle ξ is a function of eclipse magnitude. For this angle, Ptolemy has in fact computed a table of the apparent inclinations, for each digit of obscuration. The table is tabulated from 0 to 21, the moon is calculated for mean distance, and the assumption has been made that the moon's orbit and the ecliptic are "sensibly" parallel to one another.

The second angle, the rising or setting amplitude of the ecliptic, is calculated by Ptolemy for each clime and each zodiacal sign and the results are presented in a circular table. The final inclination, the *prosneusis* itself, is then the sum or the difference of these two angles.

Ptolemy does not reveal how these angles are to be applied astrologically, but rather only presents the method with which to calculate them. He defines the inclination as being made up of two distinct angles: "the inclination of the eclipsed part [of the body] to the ecliptic and the inclination of the ecliptic itself to the horizon." The former of these is the angle formed by the ecliptic arc going through the centers of the eclipsed body and the eclipser and is thus distinct from the ordinary orbital inclination of the moon with respect to the ecliptic. As he notes "these angles ... undergo great changes ... which could not be controlled if one wanted to undertake the task of computing the inclinations throughout the whole of the duration" (*Almagest* VI, 11); he limits the problem to calculating the angles at five important phases throughout the eclipse which he specifies as the following:

i. the point of the start of obscuration
ii. the point of the completion of obscuration
iii. the point of greatest obscuration
iv. the point of the start of release
v. the point of the completion of release

One determines the time of each of the five significant phases, then from these times, according to the appropriate clime, one uses the tables in II, 13 (*Table of Zenith Distances and Ecliptic Angles*) to determine the point on the ecliptic that is rising or setting for each of the five instants. From these points, one uses the circular diagram constructed by Ptolemy, entering in with the zodiacal sign and degree, and reads off the situation of the ecliptic with respect to the horizon, for the corresponding clime. Then one uses table VI, 12 (*Table of Angles of Inclinations at Eclipses*) and selects the appropriate value(s), and adds or subtracts this to the first angle, depending on the orientation of the

moon's center with respect to the ecliptic: "we take the angles corresponding to the relevant magnitude [of the eclipse] in digits, and apply those angles to the intersection of the horizon and the ecliptic" (*Almagest* VI, 12).

Ptolemy's rules for the combination of the two preliminary angles to get the final total inclination rests on the somewhat limiting premise that mid-eclipse occurs at the zenith, and the beginning of the eclipse occurs on one side of the meridian and the end of the eclipse on the other. Such an assumption can lead to results that are not at all accurate, as eclipses more often than not occur at a considerable angular distance from the zenith and can occur at any such angle. Neugebauer observes that the triviality of such a simplification was realized in antiquity;[74] Ptolemy himself tried to introduce measures to correct for this difference in the *Handy Tables*.

4.5.2 The Handy Tables

Some time later, Ptolemy collected all the tables that he had presented in the *Almagest* and produced them in a single work with a short instructive introduction. Appropriately, he called these the *Handy Tables*; their purpose being a quick and useful reference for astronomers. It is much more convenient to have all the information needed in tabular form than to use the *Almagest*, which has long passages of description and proof throughout. This entailed that the *Handy Tables* had a profound and lasting impact. The earliest copies of the *Handy Tables* exist in an adapted version, revised by Theon of Alexandria (fl. ca. 364 CE), who wrote two commentaries on this work, one short and one lengthy. This work has survived in about forty manuscripts, not all Greek, the earliest dating to around the ninth century.[75] Indirectly this work forms the fundamental source of many Arabic and Byzantine works, in which the parameters have had only minor modifications, if any, or were recomputed for local coordinates.[76]

There is a close relationship between the *Almagest* and the *Handy Tables*; however, there are notable modifications in the latter, which are presumably intended to augment the ease of use. The *Handy Tables* are based on a more "modern" epoch.[77] The fact that the mean motion tables are given over 25-year intervals (which, as shown previously, is very nearly a whole number of synodic months) and not 18-year intervals as in the *Almagest*, the parallax tables calculated for each of the seven climes, and the tabulation of the *combined* parallax of the sun and the moon suggest that Ptolemy had in mind that his users were primarily interested in assistance for computing eclipses. In order to simplify the numbers, there has been frequent interpolation and rounding of values—in fact, the majority of the tables go no further in their accuracy than

minutes of arc. This entails, in some cases, a great loss of precision (i.e., even parameters for the moon are given only in minutes). But this practice further confirms that the tables prioritized facility and practicality over accuracy.

The following table compares the eclipse tables set out in the *Handy Tables* with the corresponding material in the *Almagest*, where [V] and [H] refer to two versions of the work.[78]

Subject	*Handy Tables*		*Almagest*
	[V]	[H]	
Prosneusis	f. 48v (col. 5–8)	I p. 146/7 II p. 94/5	VI, 12
Digits–Area magnitude	f. 49r (col. 1–3)	II p. 94/5	VI, 8
Solar eclipse	f. 49r (col. 4–6)	II p. 90/1	VI, 8
greatest distance			
least distance	f. 49r (col. 7–9)	II p. 90/1	VI, 8
Lunar eclipse	f. 49v (col. 1–4)	II p. 90/1	VI, 8
greatest distance			
least distance	f. 49v (col. 5–8)	II p. 90/1	VI, 8
Parallax–7 climata	ff. 50r–56v	II pp. 98–111	V, 18
Byzantium	ff. 7r–7v	–	
correction for parallax	ff. 48v	I p. 146/7	

The following presents the tables given in the *Handy Tables* using [V], with the differences from [H] and the *Almagest* noted beneath.

Table of the *Prosneusis*

The table for the prosneusis, originally its own table in the *Almagest* (cf. *Alm.* VI, 12), has been combined with two other tables that appear on either side: on the left a table for corrections due to anomaly (see *Alm.* VI, 8 cols. 1–3) and on the right a table for the correction due to elongation. The table is reproduced below. The values for the *prosneusis* are given below in columns 4–7. Column 4 (Digits) gives the digits of obscuration, column 5 (Sun) the angle of inclination for the sun at the beginning of the eclipse and the end of emersion, column 6 (Moon_p) for the moon at the beginning of the eclipse and the end of emersion, and column 7 (Moon_t) for the moon at the end of the partial phase and the beginning of the emersion. This begins at 12 digits because that indicates the beginning of totality when the disk of the moon (12 digits) is completely immersed in the shadow. Again, the values are taken directly from the *Almagest* (which are given in degrees and minutes) and are rounded up to the nearest degree. For comparison, the table from the *Almagest* has been included underneath.

	1	2	3	4	5	6	7	8	9	10	11
				Prosneusis							
	Corrections			Digits	Sun	Moon_p	Moon_t	Corrections			
1	6	354	0	1	67	73	0	6	354	0	0
	12	348	1	2	59	65	0	12	348	0	0
	18	342	2	3	49	59	0	18	342	0	0
	24	336	3	4	43	54	0	24	336	1	2
5	30	330	4	5	37	50	0	30	330	1	3
	36	324	5	6	31	46	0	36	324	1	4
	42	318	7	7	26	43	0	42	318	2	5
	48	312	9	8	21	39	0	48	312	2	6
	54	306	12	9	16	36	0	54	306	2	8
10	60	300	14	10	11	32	0	60	300	3	9
	66	294	17	11	6	29	0	66	294	3	11
	72	288	20	12	0	26	90	72	288	4	13
	78	282	23	13	0	23	64	78	282	4	14
	84	276	26	14	0	21	52	84	276	5	16
15	90	270	29	15	0	18	43	90	270	6	17
	96	264	32	16	0	15	36	96	264	6	19
	102	258	35	17	0	12	29	102	258	7	21
	108	252	38	18	0	10	22	108	252	8	22
	114	246	41	19	0	7	16	114	246	8	24
20	120	240	44	20	0	4	10	120	240	9	25
	126	234	47	21	0	2	4	126	234	9	27
	132	228	50	0	0	0	0	132	228	10	28
	138	222	52					128	222	10	29
	144	216	54					144	216	11	30
25	150	210	56					150	210	11	30
	156	204	57					156	204	11	31
	162	198	58					162	198	12	31
	168	192	59					168	192	12	32
	174	186	59					174	186	12	32
30	180	180	60					180	180	12	32

App. crit.
Alm. has 1st row for 0 digits 0, 90, 90
col 5:2 Alm. = 56,59
col 5:12 should be 2 not 0. (Alm. = 1,47)
col 6:4 Alm. = 54,34, here has been rounded down
col 5:23 16 εσξα. τ. αναπληρου με
col 6:23 16 εσξα. τ. αναπληρου με
col 7:23 16 πρωτ. αναπληρου με
col 5:2 H 57
col 5:12 H 2

1	2	3	4
	Sun	Moon	Moon
Digits	Beginning of eclipse End of emersion	Beginning of eclipse End of emersion	End of partial phase Beginning of emersion
0	90° 0	90° 0	
1	66 50	72 30	
2	56 59	65 10	
3	49 16	59 27	
4	42 36	54 34	
5	36 35	50 14	
6	31 1	46 15	
7	25 46	42 31	
8	20 44	39 2	
9	15 51	35 42	
10	11 6	32 29	
11	6 25	29 23	
12	1 47	26 23	90° 0
13		23 28	63 37
14		20 36	52 21
15		17 48	43 26
16		15 1	35 41
17		12 18	28 38
18		9 36	22 1
19		6 55	15 43
20		4 15	9 36
21		1 36	3 35

Table of the Magnitudes. Table of Solar Eclipses at Greatest and Least Distance

The following table is found on f. 49r. In it is included a table for digits area conversion, which is identical to that given in the *Almagest*; however, the table with which it was paired (Table of Correction) is now with the *prosneusis* material. Then the solar eclipse table at greatest and least distance is given, with some modifications from that given in the *Almagest*. The most notable is its argument, which instead of being tabulated with respect to the argument of latitude (ω) is now tabulated in latitude itself (β). Where the *Almagest* gave values both around the ascending and descending node, here only the absolute distance from the node is given. This has the consequence that the table is only half as long as that in the *Almagest*, which tabulates values for either side of each of the nodes.

	1	2	3	4	5	6	7	8	9
	Digits	Sun	Moon	Latitude	Digits	Immersion	Latitude	Digits	Immersion
1	1	0;20	0;40	32;0	0	0	34;0	0	0
	2	1;0	1;10	29;20	1	13	31;20	1	13
	3	1;45	2;15?	26;40	2	17	28;40	2	18
	4	2;40	3;10	24;0	3	21	26;0	3	21
5	5	3;40	4;20	21;20	4	23	23;20	4	24
	6	4;40	5;30	18;40	5	26	20;40	5	26
	7	5;20	6;45	16;0	6	27	18;0	6	28
	8	7;0	8;0	13;20	7	28	15;20	7	30
	9	8;20	9;10	10;40	8	30	12;40	8	31
10	10	9;40	10;20	8;0	9	30	10;0	9	32
	11	10;20	11;20	5;20	10	31	7;20	10	33
	12	12;0	12;0	2;40	11	31	4;40	11	33
	0	0	0	0;0	12	31	2;0	12	33
							0;0	12;48?	33

App.crit.
col 3:1 *Alm.* = 0;30
col 9:14 no μονη; *Alm.* = 2,0
col 7;14 H blank
col 9:10 H = 31
H has a col for μονη (duration of totality) for both greatest and least distance but it is (suprisingly) blank next to the entry 12;48 where one would expect 2,0 as in the *Alm.*
col 2:7 *H* 5 1/2th 2/3rds
col 2:11 *H* 1/2 1/3
col 3:1 *H* 0;30
col 3:3 1/5th

Rather than each value being recomputed using Ptolemy's relationship between latitude and nodal elongation ($\omega = 11;30\beta$),[79] or else by taking the difference in diameters less the digit amount,[80] the values for the immersion are taken directly from the *Almagest* (VI, 7) and rounded up or down to the nearest minute.[81] They are not computed for the new values of latitude given in the *Handy Tables* as they should be for consistency. In the *Almagest* the argument in latitude has been tabulated in 0;30 increments, and in the *Handy Tables* the latitude in increments of 2;40. To determine this increment, the maximum limit has been divided into the necessary number of digits (12 for greatest distance and 12⅘ for least distance):

Greatest distance	Least distance
$d = 0;32 \div 12$	$d = 0;34 \div 12\frac{4}{5}$
$= 0;2,40$	$= 0;2,39,22,30$
	$\approx 0;2,40$

Because of this, in general, all the values are slightly too high.

Lunar magnitudes (f. 49v)

As in the solar eclipse table, there are some significant differences between the *Almagest* and the *Handy Tables*. In the *Handy Tables* for the column of latitudes for both the greatest and least distances, the differences are based on a step functions. For entries 0–9 the difference $d = 0;2,40$ is used, and from 10 to the end $D = 2;36$ is used.

	Greatest distance		Least distance	
Maximum	0;57		0;64	
difference	Entries 0–9	$d = 0;2,40$	Entries 0–9	$d = 0;2,54$
	Entries 10 to end	$D = 0;2,36$	Entries 10 to end	$D = 0;3,1,30^{82}$

Another notable difference is the maximum value of digits in each column. In the *Almagest*, the digits run from 0 to 21, and then the final entry simply says "entire" in both cases. Technically, both values are different and are calculated as

Greatest distance	Least distance
$10;48/0;30 = 21;40$	$12;12/0;34 = 21;31,45,52, \ldots$

However in both [V] and [H] the same value for each is given. For [V], the value is 21;30 (i.e., 21½) and for [H], 21;40 (i.e., 21⅔).

Columns 3–4 and 6–8 are rounded to the nearest minutes and seconds.

	1	2	3	4	5	6	7	8
	β	Fingers	Immersion	Duration of totality	β	Fingers	Immersion	Duration of totality
1	57;0	0	0	0	64;0	0	0	0
	54;24	1	17	0	61;6	1	19	0
	51;48	2	24	0	58;12	2	27	0
	49;12	3	29	0	55;18	3	32	0
5	46;36	4	33	0	52;24	4	37	0
	44;0	5	36	0	49;30	5	41	0
	41;24	6	39	0	46;36	6	44	0
	38;48	7	42	0	43;42	7	47	0
	36;12	8	44	0	40;48	8	49	0
10	33;36	9	46	0	37;54	9	52	0
	30;56	10	48	0	34;53	10	54	0
	28;16	11	49	0	31;51	11	55	0
	25;36	12	51	0	28;49	12	57	0
	22;56	13	41	11	25;48	13	46	13
15	20;16	14	37	15	22;46	14	42	17
	17;36	15	36	18	19;45	15	40	21

continued

	1	2	3	4	5	6	7	8
	β	Fingers	Immersion	Duration of totality	β	Fingers	Immersion	Duration of totality
	14;56	16	34	20	16;43	16	38	23
	12;16	17	33	22	13;42	[1]7	37	25
	9;36	18	32	23	10;40	18	37	26
20	6;56	19	32	24	7;39	19	36	27
	4;16	20	32	25	4;37	20	36	28
	1;36	21	31	25	1;36	21	35	28
	0;0	21;30?	31	25	0;0	21;30?	35	28

App. Crit.
col 1:10 H = 30;36 28,16 etc.. in fact the whole line for 10 digits is left out.
col 2:2 H = 17 (mistake for 1, transferred the number which is supposed to be in col 3, and now col 3:2 has a zero in it)
col 2 :23 and 6:23 H = 21;40
col 7:18 H 33
col 8:20 H 47
col 8:17 H 27
col 8:14 H 15

Parallax Tables

Parallax tables have been expanded upon from those presented in the *Almagest*.[83] The values are tabulated according to the difference in parallax of the sun and the moon, which Neugebauer calls the "adjusted parallax."[84] For obvious reasons, this adjusted parallax is directly useful in the case of eclipses.[85]

The parallax tables are arranged with respect to three factors: geographical latitude ϕ, longitude of the moon λ_m, and time before or after noon beginning at sunrise and ending at sunset. Latitude has been divided into the seven different climes (cf. *Alm.* II, 13); Latitudes of the climes are given (at the top of each table) as

Clime	Region	Latitude	Longest daylight
I	Meroe	16;26°	13^h
II	Syene	23;51°	$13;30^h$
III	Lower Egypt	30;22°	14^h
IV	Rhodes	36;00°	$14;30^h$
V	Hellespont	40;56°	15^h
VI	Middle Pontos	45;01°	$15;30^h$
VII	Borythenes	48;[32]°	16^h

For each clime, the positions are tabulated for the beginning of each of the twelve zodiacal signs. For each zodiacal sign, the times are tabulated with

respect to hours before or after noon, as well as sunrise and sunset, determined by the variation of daylight for the beginning of each sign (*Alm.* II, 13).

Ptolemy's procedure for computing the components of parallax is described in Theon's *Great Commentary*. He uses information from three tables from the *Almagest* : II, 13 (Table of Zenith Distances and Ecliptical Angles), V, 18 (Vertical Parallaxes of the Sun and the Moon), and I, 11 (Table of Chords).

From *Almagest* V, 18, Ptolemy's maximum solar parallax is 0;2,51° and his maximum lunar parallax (horizontal parallax) is 0;53,34°, so that the maximum total parallax should be around

$$\begin{aligned} 0;53,32 - 0;2,51 &= 0;50,41 \\ &\approx 0;51. \end{aligned}$$

As Neugebauer observes,[86] the maximum parallax can be extracted from the *Handy Tables* by selecting a value at which the ecliptic is perpendicular to the horizon. In clime II, in which the terrestrial latitude is the same as the obliquity of the ecliptic ($\phi = \epsilon$), such a configuration occurs when Aries 0° or Libra 0° sets or rises, respectively. This value is 0;51 which confirms that the parallaxes given in the *Handy Tables* are indeed based on those from the *Almagest* (*Alm.* V, 18).[87]

Also important is the fact that these tables of parallax were calculated under the assumption that the sun and the moon are in the same position (i.e., elongation $\eta = 0$) and that the moon is at greatest distance (i.e., epicyclic anomaly $\alpha = 0$). For eclipses, the first of these conditions is met; however, for general lunar positions, as well as the aforementioned correction, allowances must be made for various values of α. Such a correction has been accounted for in another table (see the *Prosneusis* Table in [V]).

A particularly meaningful addition in [V] with respect to the tabulation of parallaxes is its inclusion of a special table constructed specifically for the latitude of Byzantium. The following values are given for latitude and longest daylight:

Clime	Region	Latitude	Longest daylight
Between V and VI	Byzantium	43;05 (or 12?)	15;15

and the tables give the latitudinal and longitudinal components. It has been suggested[88] that this table is simply derived by linear interpolation between the data of the fifth and sixth climata. Tihon and Chabás comment that the latitude of Byzantium is $\phi = 43;30$.[89] [V] clearly on several occasions writes ΜΓ ε′, which could be read as either 43;05 or 43;12.[90]

It seems that the table has been interpolated by using a conversion of 0;30—halfway between the Vth and VIth climes. The length of the longest day

for Byzantium, which is given as 15;15 equinoctial hours, is exactly half-way between the 15 hours for clime V and 15;30 hours for clime VI. However, this is not consistent with the latitude, which is a little over halfway between the two climes. Assuming that the latitude 43;05 is correct for Byzantium, and the latitudes for climes V and VI are 43;01 and 40;56, then the latitude is

$$45;01 - 40;56 = 4;05^\circ,$$
$$43;05 - 40;56 = 2;09^\circ,$$
$$2;09/4;05 = 0;31,35,30,\ldots$$

with respect to the latitudes of the two climes. So it is unclear how the interpolation was carried out, or whether or not it was carried out consistently, or, if it is a factor of 0;30, how the latitude was derived.

A small portion of each of the three tables (clime V, Byzantium, and clime VI, respectively) is laid out below, for Cancer sunrise to noon (it is symmetrical about noon, thus only the first half has been set out here):

Cancer π_λ			Cancer π_β		
V	Byzantium	VI	V	Byzantium	VI
36	34	32	36	38	38
37	36	35	35	36	36
39	37	36	31	32	33
38	36	35	27	28	29
34	32	32	23	24	26
28	27	25	20	21	23
20	19	18	17	19	20
10	10	9	16	18	19
0	0	0	15	17	19

A quick glance at the table reveals that, even with rounding taken into account, sometimes the value for Byzantium is a little higher and sometimes a little lower than halfway between those for climes V and VI. So, contrary to speculation,[91] the table for parallaxes for Byzantium is not a simple linear interpolation from the adjacent climes V and VI as presented in the *Handy Tables* but is rather computed from some more accurate source—probably recomputed from the *Almagest* itself.

Theon of Alexandria (fl. ca. 364 CE),[92] who authored several commentaries on the astronomical works of Ptolemy, gives further insight into the reception of the *Almagest* and the *Handy Tables* by early practitioners. He wrote a commentary on the *Almagest*, a *Great Commentary* on the *Handy Tables* in five books (book 5 is no longer extant), and a *Small Commentary* on the *Handy Tables* in one book. Theon's evaluation focuses largely on explanation, the

implementation of simplifying assumptions, and the working through of examples. Notably, he did not introduce any changes into the *Handy Tables*.[93]

Theon's *Small Commentary* contains material on lunar parallax (chapter 19) lunar eclipses (chapter 21), *proneusis* of the moon (chapter 22), solar eclipses (chapter 23), and *prosneusis* of the sun. His *Great Commentary* preserves this thematic order, but deals with things on a much grander scale. These topics are the subject of books II and III. Book II contains four chapters: on the setting out of the tables for lunar parallax, on the setting out of the table of correction, on the calculation of parallax, and on the calculation of conjunctions and full moons. Book III has six chapters: on the setting out of the solar and lunar eclipse tables, on the calculation of lunar eclipses, on the calculation of solar eclipses, the "inclinations," the setting out of the table of inclinations, and the calculation of the inclinations.

Theon defines parallax as "the difference between the apparent position and the true position." He notes that it is to be considered not only in longitude, but also in latitude. He begins with a brief comparison between the ways in which it is computed in the *Almagest* and the *Handy Tables*. In order to simplify things, Theon states that he considers the moon to be situated on the ecliptic and not on its inclined orbit. The difference is negligible—in the case of an eclipse, not more than one and a half minutes of arc. He gives a worked example for computing parallax for the following circumstances: three hours before noon, 0° Cancer, and in the clime that runs through Alexandria (clime III). For this situation he derives (using interpolation) the following: the zenith distance is 40°19′ and the angle of rising (angle between ecliptic) 160°30′. From this zenith distance he finds the following total parallaxes:

$$\pi_s = 0;1,50,\ldots^\circ,$$
$$\pi_m = 0;35,4,\ldots^\circ.$$

He then calculates the components of longitudinal and latitudinal parallax for each of the luminaries:

$$\pi_{\lambda,m} = 0;33,3,19,\ldots^\circ,$$
$$\pi_{\beta,m} = 0;11,42,19,\ldots^\circ.$$

and

$$\pi_{\lambda,s} = 0;1,43,41,\ldots^\circ,$$
$$\pi_{\beta,s} = 0;0,36,43,\ldots^\circ,$$

and then the total parallax in each component, combining the parallaxes of the luminaries:

$$\pi_\lambda = 0;33,3 - 0;1,43^\circ$$
$$= 0;31,20,$$

$$\pi_\beta = 0;11,42 - 0;0,37$$
$$= 0;11,5.$$

This is the adjusted parallax, that is, the components in longitude and latitude that represent the difference between the solar and lunar parallax. Obviously these values are only going to be useful in solar eclipses. If simply the lunar parallax is needed, Theon states that one can increase this original value by ⅟₂₀, which accounts for the parallax of the sun. This, of course, is only approximate.

The rest of the chapter concerns the orientation of the parallax; whether it is to be applied "forward" (that is, in the direction following the order of the zodiacal signs: εἰς τὰ ἑπόμενα) or "backward" (that is, in the opposite direction, εἰς τὰ προηγούμενα) in the case of the longitudinal component, or north (βορειοτέρον) or south (πρὸς μεσημβρίαν) in the case of the latitudinal component. Theon states that the determination of the directions is dependent on two factors: whether the zenith is north or south of the culminating point and whether the rising or setting angle is greater or less than a right angle. The following scheme results:

Zenith	Rising angle	π_λ	π_β
N	$> 90°$	Forward	S
N	$< 90°$	Backward	S
S	$> 90°$	Forward	N
S	$< 90°$	Backward	N

Theon expands his discussion of this topic with reference to other climata, not just the third. The next chapter is entitled "The Setting Out of the Tables of Correction."[94] Such a table is necessary because the general parallax tables were computed under the assumption that the moon was at maximum distance (i.e., that the epicyclic anomaly $\alpha = 0$ and the elongation $\overline{\eta} = 0$). So corrections to parallax for the varying values of anomaly and elongation were introduced. The next chapter contains a discussion of how to calculate the components of parallax using the *Handy Tables*. Pappus' explanation is straightforward. Theon then deals with lunar and solar eclipses. He compares the eclipse tables set out in the *Almagest* with those in the *Handy Tables*.

The production of the *Handy Tables* by Ptolemy, Theon, and countless later individuals represents in many senses a new direction in astronomical activity. The compilation of the practical, essential elements from the theoretically rich and comprehensive *Almagest* into an abridged, convenient handbook represents a turning away from theory and a focus on application. The inclusion of the relevant tables for Byzantium in one version of the *Handy Tables* is a valuable example of this. It shows how eclipse computation handbooks were adapted to users' individual needs, often at the expense of sophistication.

Theoretically precise results were modified and transformed by simplifying assumptions to produce local directly applicable data, which were intentionally approximate. Format and application increasingly took priority over theoretical considerations in astronomical endeavors.

Early scholars were aware of the differences between the *Almagest* and the *Handy Tables*. Theon, for example, frequently compares and contrasts the two. Ptolemy's proposals collectively represented such a comprehensive and complete system, which had so much success in modeling the phenomena, that later thinkers were unwilling to tamper with it. Ptolemy's *Handy Tables* also had an impact on astronomical inquiries. That an individual table accounted for each particular astronomical feature in many ways compartmentalized astronomical inquiry. The *Handy Tables* encouraged successive generations of scholars to update parameters of individual aspects, but dissuaded them from considering the interaction of these features. The consequences of improved parameters of particular aspects upon the broader system were not explored concurrently. Eventually, proposals to modify aspects of the fundamental workings arose, but they were a long time coming.

4.6 Omens in Greek and Hellenistic Sources

Eclipse reckoning traditions were perpetuated in other areas besides mainstream astronomical endeavors, particularly in omen literature. Substantial evidence confirms without doubt that much of the Mesopotamian omen literature was transmitted to and adopted by Greek and Hellenistic authors, and eclipses were a significant part of this movement. Much work remains to be done in both fields to learn in more detail about this transmission.

In particular, there exist several fragmentary texts from antiquity concerning astral omens which have been grouped together and ascribed to the named Egyptian Petosiris.[95] These fragments are particularly significant as they represent the Hellenistic branch of a tradition which finds its origins in Mesopotamian omen literature. Such fragments are preserved in the following authors of late antiquity: Hephaestio of Thebes (fl. ca. 415) in his *Apotelesmatiká* and John Lydus (fl. ca. 560) in his *de Ostentis*. These omens show a clear debt to Mesopotamian sources, while betraying elements of a further source, that is, of an Egyptian tradition.

Hephaestio's work covers eclipse omens as well as many other phenomena. *Apotelesmatiká* I 21[96] lists omens concerning eclipses. Omens that concern the eclipsed body are listed in combination with many different elements. These include the color of the eclipsed body, simultaneous occurrence of winds, shooting stars, halos, lightning, and rain, and the occurrence of both lunar

and solar eclipses in each of the zodiacal signs with reference to one of the four three-hour periods into which the night was divided. The division of the night into intervals of this sort was probably derived from the Mesopotamian "watches" which featured as a division of time in *Enūma Anu Enlil*. The apodoses refer to almost fifty different countries within Eurasia.

Apotelesmatiká I 22 deals with omens concerning eclipses accompanied by other phenomena such as "redness" and dimness of the disk of the luminary in question, comets, comets with sacred (ἱερός) stars, and stars that are visible during the day. In this section, the apodoses are notably restricted to Egypt and its neighboring countries, in contrast to the previous chapter.

From the protasis material, it is clear that *Apotelesmatiká* I 21 has its origins in the lunar and solar sections of *Enūma Anu Enlil* (see section 3.2), with the apodotic material replaced by geographical places contemporary with and relevant to the Hellenistic context. The first theme sets out protases that associate colors with eclipses in the following order: black (μελάς), red (ἐρυθρός), whitish (ὑπόλευκος), violet/dark (ἰοειδής), and gold (χρυσοειδής). These may be derived from the schematic representation of color used in *EAE*: white (BABBAR), black (MI), red (SA_5), yellow/green (SIG_7), and variegated (GÚN), with variegated being frequently replaced with dark (*du'um*). For example, in the context of lunar eclipse phenomena, colors appear in the following formats:[97]

DIŠ *ina* ITU.[month] UD.14.KAM AN.MI GAR-*ma* [color] . . .
[If in [month] on the 14th day, an eclipse occurs and is [color] . . .]
(*EAE* 16 III VII IX XI XII 6–10; VII 5–9; XII 8–11)

and also

DIŠ *ina* ITU.[month] UD.14.KAM EN.NUN [watch] AN.MI GAR-*ma* [color] . . .
[If in [month] on the 14th day in [watch], an eclipse occurs and is [color] . . .]
(*EAE* 17 1:1–4; 2:1–4; 3:1–4)

In the same context, Lydus' *De Ostentis* contains a similar schematization of eclipse colors; there is an association with the part of the disk colored to the region it affects:

κἂν μὲν ἐν ὅλοις τοῖς σώμασι τῶν φώτων ᾖ ἐν ὅλοις τοῖς περὶ αὐτὰ τόποις τὸ γενόμενον ἰδίωμα φαίνηται τῆς χροιᾶς, περὶ τὰ πλεῖστα μέρη τῶν χωρῶν ἔσται τὸ ἀποτέλεσμα: εἰ δὲ ἀπὸ μέρους οἱουδήποτε κατ᾽ ἐκεῖνο μόνον τὸ μέρος καθ᾽ ὃ ἂν καὶ ἡ πρόσνευσις τοῦ ἰδιώματος γένηται.
[And if in the entire bodies of the luminaries or in the entire areas around them the property of color appears, the effect will be around most parts of those (geographical) regions. And if from a certain portion, in that portion

only, at which the inclination of the property might be.]
(*de Ostentis* 9 34A lns. 4–7, p. 18)

Although this precise association, namely, between color and lunar quadrant, is not attested in *EAE*, one of the most prominent features of the omens in the Mesopotamian tradition is the association between quadrant and region affected, a relationship which is suggested in the following commentary:

DIŠ AN.MI SA_5 *šá ina* IM.KUR.RA SAR-ú AN.MI SIG_7 *šá ina* IM.MAR.TU SAR-ú AN.MI *šá ina* IM.URU.LU SAR-ú AN.MI BABBAR *šá ina* IM.SI.SÁ SAR-ú . . .
[If there is a red eclipse which begins in the East; a yellow eclipse which begins in the West; a black eclipse which begins in the South; a white eclipse which begins in the North; . . .]
(Borger *Symbolae Böhl* p. 40, as cited by Rochberg-Halton [1988] p. 57 n. 118)

The following correlations result in the Mesopotamian divination tradition:

Color	Direction
Red	East
Yellow	West
Black	South
White	North

After color, Hephaestio sets out schemes that correlate lunar and solar phenomena with the direction of the wind, and connects these protases with the regions which are affected. The following association is made:

ἐκείναις ταῖς χώραις ταπείνωσιν σημαίνουσι καθ᾽ ὧν ἐγγὺς οὐσῶν πνεύσειαν
[to those regions they indicate a "lessening" in those near one in which they blow.]
(*Apot.* I 21, 3–6)

This is probably inspired by the Mesopotamian idea that the cardinal direction in question is directly correlated with the region affected. In the lunar eclipse tablets of *EAE*, we see expressions in the following form:

(i) *šumma antalû ina* IM.[direction 1] *ušarrīma* IM.[direction 2] *illik* . . .
[If an eclipse begins in quadrant [direction 1] and the [direction 2] wind blows . . .]

and

(ii) *šumma ina* [month] UD.15.KAM *antalû iškun-ma* IM.[direction] *illik* . . .
[If an eclipse occurs on the 15th day, [month], and the [direction] wind blows . . .]

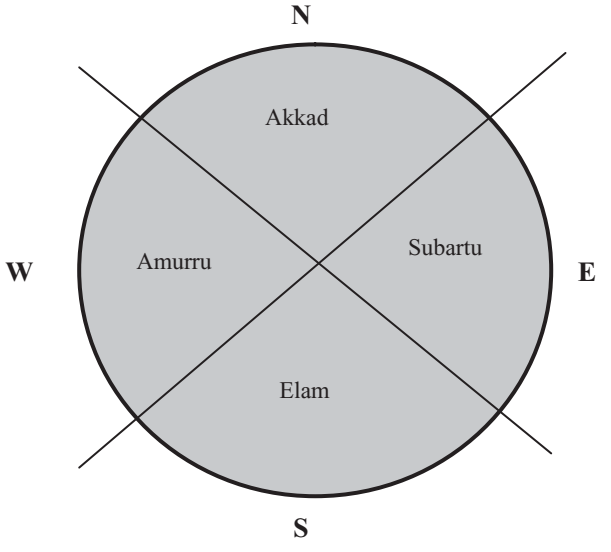


Figure 4.7 The relation between the direction of impact and the region affected in early Mesopotamian divination schemes.

The apodoses common to omens of the form (i), though by no means exclusively used, are in two parts, stating the evil consequences for the region given by "direction 1" and by "direction 2." The following locations are related to the four directions (see fig. 4.7):

Direction	Location
North	Akkad
East	Subartu
South	Elam
West	Amurru

For example,

> DIŠ AN.MI *ina* IN.I SAR-*ma* IM.MAR.TU DU HUL-*tim* NIM.MA.KI *u gu-ti-i* LUGAL MAR.TU.KI BE
> [If an eclipse begins in the South and the West wind blows. Destruction of Elam and Guti; the king of Amurru will die.]
> (*EAE* 15 col iii 4, p. 74)

Explicit statements correlating winds to countries are also attested in other cuneiform literature:

> IM.U_{18}.LU KUR NIM.KI
> IM.SI.SÁ KUR URI.KI

IM.KUR.RA KUR SU *u* GU.KI
IM.MAR KUR MAR.KI
[The South wind is for Elam
The North wind is for Akkad
The East wind is for Subartu and Guti
The West wind is for Amurru.]
(*The Great Star List*; see Koch-Westenholz [1995] Appendix B 293–294, pp. 202–3)

Also

ù šum-ma is-sa-kan
kaq-qu-ru É *'u-la-pat-an-ni*
ù š'a-a-ri a-li-ku
is-se-niš i-na-sa-ha
[And if (the eclipse) occurs, the region it afflicts and the wind blowing will be quoted together.]
(*LAS* X.26 rev. 6–9, as translated by Parpola [1993])

This theme is a prominent part of Hellenistic omen literature. Hephaestio asserts that the regions related to the winds that blow at the *start* of an eclipse will incur the evil consequences signified by the eclipse, and the regions related to the winds that blow at the *clearing* of the eclipse will benefit from the good consequences:

αἴσια γὰρ μηνύουσιν οἱ τῆς ἀναχαθάρσεως, τὰ ἐναντία δὲ οἱ τῆς ἐμπτώσεως ταῖς ὑποκειμέναις αὐτοῖς χώραις
[The (winds which blow at the time) of clearing reveal auspicious (effects), and those at the impact, the opposite (effects) for those who live in those very regions.]
(*Apot.* I 21,6)

Tablet 15 of *EAE* presents an almost identical scheme. The two important directions in the context of an eclipse in omen literature were the direction of impact (expressed by the verb SAR = *šurrû*, to begin) and the point of clearing (expressed by the verb $ZALAG_2$ = *namāru*, to clear). *EAE* sets out a scheme in which the direction of the eclipse's first contact determines the region that is affected by the evil consequences, and the direction of the clearing determines the region in which the evil consequences are bypassed. For example:

DIŠ AN.MI *ina* $IM.U_{18}$.LU SAR-*ma* IM.MAR.TU $ZALAG_2$ ŠUB-*tim* NIM.MA.KI *ana* MAR.TU.KI NU TE
[If an eclipse begins in the South and clears in the West. Downfall of Elam.

It (i.e., the evil of the eclipse) will not approach Amurru.]
(*EAE* 15 2:1–4; 3:1–4; 4:1–4; 5:1–4)

The Hellenistic authors, perhaps aware that lunar eclipses can start and clear only in two of the four directions (because the motion of the shadow is predominantly east to west) have replaced the directions of the lunar quadrant with the direction of the winds, but the underlying structure remains the same.

A rare feature of the omen literature, shooting stars, referred to as "great stars which fall" (MUL.GAL + *maqātu*), can be found in *Enūma Anu Enlil*, *Šumma Alu* and *The Assyrian Dream Book*. In *Enūma Anu Enlil*, they are combined with eclipse phenomena:

DIŠ *ina* ITI.[month] UD.14.KAM AN.MI GAR-*ma* MUL GAL ŠUB-*ut* . . .
[If in [month] on the 14th day an eclipse occurs and a great star falls . . .]
(*EAE* 16 IV)

Hephaestio and Lydus both correlate eclipses with shooting stars (διάττονες ἀστέρες), an association which is probably also derived from Mesopotamian sources:

Σελήνης δὲ οὔσης ἐν τῇ ἐκλείψει ὅταν διάττων ἀστὴρ ἐπιδραμὼν ἔλθῃ εἰς αὐτήν. . .ἐὰν δὲ ἐξέλθῃ ἀστήρ
[When the moon is in the eclipse, when an approaching shooting star enters into it . . . if the star exits it . . .]
(*Apot.* I 21 7)

ἐὰν οὖν ἐκλείψεως γενομένης ἀστέρες διάττωσιν εἰς τὸ ἔλλειμμα τῆς Σελήνης, δηλοῦσι μάχην πολεμίων κατὰ τὸ κλίμα τὸ προσαποχειούμενον τῷ ζῳδίῳ
[If, when there is an eclipse, stars shoot into the remnant of the moon, they indicate a battle of warriors at the clime which is subject to the zodiacal sign.]
(*Apot.* I 22 1).[98]

The Hellenistic tradition also adopted the Mesopotamian scheme of the divisions of the night and the concept of *adannu* (= UD.DUG$_4$.GA), the length of time that is associated with the fulfillment of the omen it portends. In the Mesopotamian tradition, the night is divided into three four-hour periods, each called a watch (*maṣṣartu*) of the night. The first was called the evening watch (*barātu* = AN.USAN$_2$), the second, the middle watch (*qablitu* = MURUB$_4$.BA), and the last, the morning watch (*šat urri* = U$_4$.ZAL.LI). Frequently within omen literature, each watch was assigned its own *adannu* period,[99] with the following scheme generally adopted:

Watch	*Adannu*
Evening watch	3 months + 10 days = 100 days
Middle watch	6 months + 20 days = 200 days
Morning watch	9 months + 30 days = 300 days

The scheme presented by Hephaestio seems to be an adaptation of this system. As in the Mesopotamian tradition, Hephaestio divides the night into intervals, but not the characteristic Mesopotamian three; instead into four.[100] In addition, following the Mesopotamian scheme, he assigns each eclipse omen a time period for the fulfillment of its consequences. In many cases he has assigned the interval of three months, an adaptation of the 100 days.[101] Other time periods were given as part of particular apodoses: 120 days,[102] 8 or 9 months,[103] 3 years,[104] and 4 years.[105]

Hephaestio includes a section, introduced by the word *holoscherōs* (literally "generally"), which takes each zodiacal sign in turn, first in relation to lunar eclipses and then to solar eclipses, and lists an apodosis for each. These apodoses are notable because of the absence of geographical qualification in them. After this, a lengthier section begins, introduced by the word *merikōs* (literally "specifically"). Again, this section runs through zodiacal sign by zodiacal sign, giving detailed protases first for a solar and then for a lunar eclipse. The fact that Hephaestio catalogues these eclipse omens zodiacal sign by zodiacal sign, rather than month by month, is a strong indication that the original text he was using was also in the sign by sign format. For, by the time Hephaestio was writing, both signs and months were available for him to use.

In contrast, a different version of Hephaestio's text (the Excerpt text), uses the month name, using Romanized forms, beginning with μηνὶ Ἀπριλλιῳ (in the month April) and ending with μηνὶ Μαρτίῳ (in the month March), as well as stating the zodiacal sign. It may initially seem puzzling that the list begins with April until one considers that April roughly corresponds to Nisannu, the first month in the Babylonian calendar.[106] Two additional scenarios were also used where appropriate. These were situations in which the moon was eclipsed as it was rising or was still eclipsed as it set.

What is the source of these omens as presented by these Hellenistic texts? The question is whether a text or a group of texts can be isolated as being the source for the Greek omen tradition. Among the lunar eclipse omens of *EAE*, there are no single tablets that contain all the information that Hephaestio's text includes. Perhaps the closest in content is tablet number sixteen. It contains (at least) thirteen sections divided into months of the year. Present in this tablet are the colors (omens 6–10), winds (13–16), shooting stars (5), thunder and

rains (meteorological section[107]) and the duration of an eclipse (until dawn [omens 11 and 17–18]; lasts the watch [12]).

Hephaestio's text contains a statement which may also confirm its Mesopotamian influence. When he covers the lunar eclipse omens in Virgo, he states:

> τῆς Σελήνης ἐκλειπούσης δακτύλους δύο πρὸς νότον. . .
> [when the moon is eclipsed two fingers to the south . . .]
> (*Apot.* I 21 20)

The use of fingers (ŠU.SI = *ubānu*) to express the magnitude of an eclipse is attested only in celestial omen commentaries and in the Letters and Reports of the neo-Assyrian period (see section 3.3), not in the omen texts themselves. This may suggest that his sources were neo-Assyrian and not *EAE* directly.

The related *Excerpt* text includes a further section dealing with solar and lunar eclipses that occur in the same month. As of yet, a similar section has not been found in the Mesopotamian omen tradition, and it most probably represents an innovation by later Hellenistic authors (although there are several instances of a lunar and a solar eclipse occurring in the same month set out in the Letters and Reports[108]).

Noticeably absent from this tablet of *EAE* are the halo (ἅλως) phenomena and partial eclipses which are in the Greek tradition. There is also material from *EAE* that does not appear in the Greek tradition, such as the direction of the beginning and clearing of an eclipse. For the latter theme in any case, it is conceivable that the Hellenistic authors, understanding that the impact and the clearing of an eclipse occur from just two fixed (east and west, respectively) directions may have discarded this element.

Another noticeable difference is that the Hellenistic authors have rightly discarded the variation of dates for the eclipse occurrences. *EAE* 16 states dates that include eclipse possibilities on the 14th, 15th, 19th, and 20th days of the month, whereas the Hellenistic texts make no mention of the date, but simply give the month. The Mesopotamian material deals with lunar and solar eclipse phenomena separately, whereas they are combined in the Hellenistic texts.[109] Other differences include a divergence from the Mesopotamian schematic order of colors. In addition, the scheme that includes the winds and directions as set out by Hephaestio is far more complicated than the scheme set out in *EAE* 16 and is closer to the scheme set out in *EAE* 15.

Thus it is difficult to ascertain what sources were transmitted to Greece, although the evidence points to sources which were commentaries or adaptations of the original omen texts. There is also evidence that some of the omen

material reached Greece via transmission through Egypt, as well as by direct transmission from Mesopotamian sources.[110]

By the close of the second century of the common era, Greek astronomy had reached a peak. Subsequent efforts by Greek scholars never strayed far from Ptolemy's designs and results. Practitioners were in a position of unprecedented strength to predict eclipses and account for the physical causes behind their occurrence. The relation of this ability to the efforts that preceded is complex. Not uninfluenced by Mesopotamian approaches to eclipse prediction (though those approaches were very different in form), the Greeks' ability to predict eclipses was of a completely new kind.

Earliest Greek thinkers had sought to detect, beyond the appearances, the physical circumstances that were responsible for the effects they observed. This new orientation toward heavenly phenomena is not to be underestimated, for the trajectories of bodies leave only intermittent or subtle traces. Eclipses are visual instances at which these trajectories intersect—they represent perhaps one of the boldest statements of heavenly alignments. As such, they drew these early thinkers into considering celestial phenomena in this more geometrical way. Observations in these early times had some serious limitations, and their worth in further refining conjectures was limited.

Instead, taking confidence in the strength and wealth of mathematical achievements in the beginning of the third century BCE, astronomical inquiry relied more and more appreciably on theoretical insight. Geometrical structures formed templates from which to assess and orient heavenly bodies. Great circles, cosmic triangles, nested spheres, and intersecting line segments were superimposed on the heavens, so that sizes, distances, and relative positions found meaning. And yet, even then, a further step had to be taken before astronomers would seek to measure such parameters, often a task requiring subtle geometrical insight, and inviting into use real practical ingenuity.

While in later stages of Greek astronomy geometrical proposals could reproduce the general patterns of heavenly phenomena, it was slow to dawn on these practitioners that they could demand quantitative significance from their designs. It was to be external stimulus that motivated such a demand, issuing from contact with Babylonian sources, whose astronomical results and preoccupations had been nurtured in an intellectual climate markedly different from that of the Greeks. The synthesis of the Babylonian numerical patterns with the Greek geometrical kinematic models as it was eventually realized proved indomitable. It was to remain at the heart of eclipse reckoning until the sixteenth century.

CHAPTER FIVE

Recipient and Remodeler
Indian Eclipse Reckoning

I shall not now speak of the knowledge of the Hindus . . . of their subtle discoveries in astronomy, discoveries that are more ingenious than those of the Greeks and the Babylonians, and of their valuable methods of calculation which surpass description . . . If those who believe, because they speak Greek, that they have arrived at the limits of science, would read the Indian texts, they would be convinced, even if a little late in the day, that there are others who know something of value.

—Severus Sebokht (b. Nisibis [Syria], ca. 575), a Syrian bishop, working in the later stages of his career at Kennesrin on the west bank of the Euphrates—a prolific scholar who played a critical role in introducing Indian concepts into Syrian intellectual culture, and ultimately, to the Islamic Near East.

It shall not be possible for an astronomer to assert his views convincingly before an assembly of scholars merely on the basis of the computation of mean planets etc. without a proper exposition of the underlying rationale. I am therefore composing this section expostulating astronomical rationale by means of astronomical models and diagrams through which they could be clear even as a myrobalan[1] placed on one's palm.

—Bhāskara II introducing his *Golādhyāya* (chapter on the "Sphere") from the *Siddhāntaśiromaṇi* (composed in 1150) 2.1.2; as translated by K. V. Sarma (1985) p. 287.

5.1 Overview

Astronomy in premodern India,[2] just like almost every member in the discipline of *jyotiḥśāstra*[3] or the "exact sciences," inherited a wealth of material from other cultures. For this reason, India has been characterized as the "recipient and remodeler of foreign elements."[4] Indian astronomy was influenced first by Mesopotamia, next by Greece, in turn by Islam, and then and most recently by modern ideas. However, sources were not adopted wholesale; each element

from the foreign scientific culture was altered in some way. It might be abbreviated, it might be expanded, improved, or misunderstood. But the fundamental modifications meant that this astronomy, albeit originally borrowed, could be considered distinctly Indian. These alterations firmly set Indian astronomy apart from the cultures whence it originated.

The very first Sanskrit astronomical works that remain preserved are the *Vedāṅgajyotiṣa* of Lagadha (composed sometime around 400 BCE)[5] and (perhaps) some Jaina texts, including the *Candapannatti*, the *Suyapannatti*, and the *Jyotiṣkaraṇḍikā* from the second half of the first millennium BCE. Some time later, in the fifth and sixth centuries CE, a new genre of astronomical literature appeared. These works were usually written in the *siddhānta* format, a type of detailed astronomical treatise characterized by its comprehensiveness and universality. In *siddhānta*s the choice of the epoch is typically the start of a major cosmic era, and the organization of their topics follows a traditional order. The earliest of these *Siddhānta*s include the *Paitāmahasiddhānta* (early fifth century[6]), the *Āryabhaṭīya* of Āryabhaṭa I (after 499), and the *Brāhmasphuṭasiddhānta* of Brahmagupta (628) which provided the basis of Indian astronomy for many centuries to come, even up until recently. But knowledge of Sanskrit astronomy precedes these texts—they imply that a developed astronomical tradition already existed prior to their composition. For, as evidenced by a treatise composed by the sixth-century astronomer Varāhamihira,[7] entitled the *Pañcasiddhāntikā* (literally the "Five *Siddhāntas*"), astronomy was developed to such a point that there was already the need for a historical survey of the discipline thus far.[8]

Perhaps the most distinctive characteristic of Indian astronomical works is the format in which they were preserved. By far the majority of principal works in the *jyotiḥśāstra* tradition were written in verse; for the exact sciences in early India as well as many other genres of intellectual inquiry were to be studied, for the most part, in an oral environment. To ensure their survival and to facilitate memorization, rules and operations were composed in metrically definite and repetitive patterns, that is, in verse.

This "scientific poetry" made heavy demands on the scientist-composer. The metrical requirements and the assimilation of mathematical concepts into such a strict framework entailed a certain measure of obscurity of expression, even forcing the composer at times to discard necessary material in order to conform to these metrical requirements. Thus, the exclusion of key words and concepts or the abbreviation of a qualification with a single key word by Indian authors was not uncommon. It was expected that the "missing" information would be supplied by the reader. Furthermore, because of this format, there could be no fixed technical vocabulary. It was necessary to accumulate many

different words with contrasting metrical rhythms which would represent the same mathematical concept or procedure, allowing the composer to select one that would fit in the metrical space remaining to him.

Accordingly, numbers had to be expressible in metrical forms. As a result, two dominant and indeed creative systems to denote numbers were devised: the *bhūtasaṅkhyā* system, in which common objects had associations with specific numbers (i.e., eye = 2, gods = 33), and the *kaṭapayādi* system, in which each number had several different letters assigned to it, so that meaningful words, whose letters represented strings of numbers, could be produced by a clever arrangement.[9]

Another consequence of this emphasis on orality was an economy of expression. Long and complicated expressions are bound to have a very short lifespan in an oral environment, with the result that conciseness and brevity were quickly adopted, sometimes, however, so thoroughly that the meaning itself was completely obscured. Furthermore, an Indian verse was not intended to provide a detailed exposition. The verses were meant for memorization, quick reference, and "memory jogs" rather than an all-inclusive mathematical explanation. Such an explanation was often supplied by an accompanying oral instruction or written commentary. Indeed the obscurity of the verses was but an encouragement to the Sanskrit paṇḍits, who further pushed their students' ability by making their verses yet more ambiguous and esoteric than necessary.

Despite its emphasis on oral transmission, India has produced more written documents than any other society. Some thirty million manuscripts are still in existence, and roughly as many as ten percent of these are of scientific texts. Indeed, the oral environment had some notable consequences for the character of particular disciplines in the *jyotiḥśāstra* tradition, especially for the nature and practice of mathematics. Because mathematics was so closely tied to economical, astronomical, and other practical needs, its emphasis was largely mensurational. Thus the intention was not to establish a body of axioms, from which all mathematical theory could be derived and expanded for its own sake, but rather to convince the student of the validity of the rule, with the consequence that a visual demonstration, or demonstration by example, was a perfectly valid form of "proof."

Thus, perhaps more difficult tasks for the historian than those of correct translation and explanation of the astronomical content are the investigation of the technical results themselves and the exploration of the possible ways in which the practitioner arrived at the results that he has presented. For Indian astronomers were little interested in revealing how they had arrived at their results, nor did they offer any proofs or justifications for validity of the result in question. Instead they simply stated the rules.

To help elucidate the text, numerous commentaries were composed. These, in contrast to the base texts, were almost always in prose. The purpose of a commentary was to explain the text, which it would attempt to do by any of the following techniques: restating the rule in prose, explaining the grammar, giving synonyms for difficult words, deriving the parameters, giving a worked example, or quoting other texts with connected material, to name a few.

The following is an example of a commentary on a particular verse (bold font) which describes rules for computing the magnitude of an eclipse, taken from an astronomical work by Brahmagupta called the *Khaṇḍakhādyaka* (see section 5.6.5). This work has had several commentaries written on it, including one by the twelfth-century scholar Āmarāja. Āmarāja introduces the verse and afterward elucidates the meaning, defining the terms used by Brahmagupta, describing the rule more fully, supplying the appropriate metrological conversions, and offering a more detailed physical interpretation of the configuration:

atha madhyagrahaṇagrāsaliptāpramāṇānayanam āha |

vikṣepaṃ saṃśodhya pramāṇayogārddhatastamaśchannam|

sarvagrahaṇaṃ grāhyād adhike khaṇḍagrahaṇam ūne ||3||

candragrahaṇe candro grāhyo bhūcchāyā grāhikā | ravigrahaṇe ravir grāhyaścandro grāhaka iti | itaragrahāṇāṃ tu yo yasmād ūnabhuktiḥ sa tasmāduparyata ūrddhvastho grāhyaḥ adhaḥ stho grāhaka iti | tato grāhyapramāṇādadhike sarvagrahaṇaṃ jñeyamūne tu khaṇḍagrahaṇam | yadā mānaikyārddhādvikṣepo na śuddhyati tadā grahaṇābhāvaḥ | viparītaśodhane kṛte yadavaśiṣyate tattayor bimbakendrāntaraṃ kalātmakaṃ bhavati | kalānāṃ sārddhadvayenāṅgulaṃ jñeyam iti |

atra vāsanā | apamaṇḍale raveḥ sadaivārddhacakrāntaritā bhūcchāyā bhramati | candro 'pi madhyagrahaṇakāle raverarddhacakrāntarita eva vimaṇḍale 'tastayoścandrabhūcchāyayorapamaṇḍalapūrvāparayā tulyatvaṃ kevalaṃ | vimaṇḍalāpamaṇḍalāntarakṛto dakṣiṇottarabhedo vikṣepaśca tāvānevocyate | tena yadā grāhyagrāhakabimbamānaikyārddhatulyo vikṣepo bhavati tadā chādyacchādakabimbayoḥ paridhisamparka eva | yatas tayorbimbakendrāntaraṃ tadā tāvadeva bhavati | yadā mānaikyārddhādadhiko vikṣepas tadā tatparidhyor api samparko na bhavet | yadā tu mānaikyārddhādūno vikṣepas tadā tatparidhyoḥ parasparam anupraveśo bhavet | sa ca kiyān iti vikṣepamānaikyārddhayor antareṇa tatpramāṇamānīyate | sa grāsa ityabhidhīyate | vṛttadvayaśarayoḥ sampāta ityarthaḥ | yadā tvanupraveśo grāhyabimbādadhikas tadā grāhako grāhyabimbaṃ sakalamapi saṃchādyāvatiṣṭhate | tatsarvagrahaṇam ucyate | nyūne cānupraveśe khaṇḍagrahaṇam iti |

[Here, he describes the computation of the amount, in liptas, of the eclipse magnitude at mid-eclipse.

Subtracting the (lunar) latitude from half the sum of the measures (of the diameters), that is (the amount) covered in darkness. When (this) is greater than the obscured body, there is a total eclipse. When less, a partial eclipse.

In an eclipse of the moon, the moon is the eclipsed body and the shadow of the earth is the eclipsing body. In an eclipse of the sun, the sun is the eclipsed body and the moon is the eclipsing body. But whichever of the two luminaries has the smaller daily velocity than the other, it is (placed) after it (and) the eclipsed body is situated above and the eclipsing body is situated below. When (it) is greater than the measure of the eclipsed body, a total eclipse is to be understood, but when less, a partial eclipse. If the latitude can not be subtracted from half the sum of the measures, then there is no eclipse. When the reverse subtraction is computed, whatever remains is the difference of the centers of the disks of the two, having the characteristic of minutes. An aṅgula is to be known as 2½ minutes.

Here is the explanation. The shadow of the sun moves at a distance of half a circle from the sun always on the ecliptic. Also, the moon at the time of mid-eclipse (moves) on the lunar orbit in the same way at a distance of half a circle from the sun, because of the equality of the east-west (direction) of the ecliptic with respect to the shadow of the earth and the moon. The difference between the south and the north, caused by the difference between the lunar orbit and the ecliptic, is just called the (lunar) latitude. Whenever the (lunar) latitude is equal to half the sum of the measures of the disks of the eclipsed body and the eclipsing body, then there is a touching of the circumferences of the disks of the shadowed and the shadower. Whenever there is a difference in the centers of their disks, then (there is a difference) by that much. Whenever the (lunar) latitude is greater than half the sum of the measures, then there is not a touching of their circumferences. But whenever the (lunar) latitude is less than half the sum of the measures, then there is an overlapping of their circumferences, one to the other. And how much is this? This amount is computed by means of the difference between the (lunar) latitude and half the sum of the measures. This is known as the magnitude. The meaning is this: there is a coincidence of the diameters of the two circles. Whenever the body to be entered is greater than the disk of the eclipsed body, then the eclipsing body stands there covering the whole disk of the eclipsed body. This is called a total eclipse. And when the body to be entered is less, a partial eclipse.]
(*Khaṇḍakhādyaka* of Brahmagupta IV, 3 with commentary of Āmarāja)

Āmarāja is an exceptional commentator, in that he always attempts to give a full description of the base text. However the existence of a commentary is

not always helpful in clearing up the difficulties presented in the verses. Most commentators had little interest in explaining the theoretical bases or the rules, or in deriving the formulas. Most commonly, they would simply provide an obvious restatement of the rules and perhaps give a trivial example to show that the rule worked. Many failed in even this task, perhaps a testament to the obscurity of the base text or else an insight into the somewhat questionable abilities of the commentator himself. However, some commentaries are of an exceptionally high quality, and greatly clarify the text; without these modern historians would have been at a loss to explain the meanings of the verses.

Beyond their attempts to explain the texts, commentaries, both good and bad, are valuable for a number of other reasons. They offer the historian a degree of insight into the workings of an astronomer, revealing the ways in which astronomical material was presented and explained to students. They also indicate the status of the text in the period contemporary with the commentator. They give insight into how the practitioners thought about mathematics and astronomy—perhaps the closest thing to a consideration of the philosophy of mathematics and astronomy. Furthermore, some authors we know about simply because a handful of their verses have been included in commentaries on other texts.

The emphasis on oral tradition combined with the belief that the basic texts were attributed to gods and ṛṣis[10] could have contributed to the fact that Indian astronomers in many cases neither valued nor attempted innovation in their discipline. The desire to maintain the tradition intact meant that repetition of the basic parameters was highly esteemed and their improvement was not. Indian astronomical works are thus highly repetitive. Rather than develop new methods or establish new theories that would improve the correspondence between their rules and observation, the Indian astronomers focused their attention on tinkering with the computational techniques themselves; original theories were for the most part embellished with either trivializing simplifications or pointless elaborations. The documentation then of the history of this discipline often lapses into a description of the preservation of the tradition, an accounting of the distribution and rearrangement of material inherited from previous authors, and an explanation of the elaborations of or approximations to some "original" computations and parameters.

In this intellectual climate, it is significant that Indian astronomy did not remain entirely unchanged. It was inspiration from external forces that motivated fundamental change and development. Five major "intrusions" from foreign cultures have been identified[11] that were to impact the Indian scientific climate and to motivate scientists to adjust and adapt their native traditions. These foreign techniques and methods were not adopted wholesale but were

adjusted so that they might fit less awkwardly within the existing Indian material.

Source	Time
Mesopotamia via Iran	ca. Fifth century BCE
Mesopotamia via Greece	ca. Second and third centuries CE
Greece	ca. Fourth century CE
Iran	ca. Tenth to eighteenth centuries CE
Europe	ca. Eighteenth to twentieth centuries CE

Indeed, the very beginnings of Indian culture were formed through the influx and blending of foreigners, and India has remained in continuous contact with its Eurasian neighbors from classical antiquity to the modern period. Throughout this time, its contacts with adjacent cultures were both peaceful and aggressive. In the sixth century BCE, the Persian empire had expanded to the northwestern Gandhara area, a region which was taken over by Alexander the Great in the late fourth century. In his wake, a new cultural group, the Indo-Greeks, the product of the amalgamation of Greek and Indian cultures, flourished in this region for several centuries, and, following them, the Śakas, also known as Indo-Scythians, in about the first century BCE, who originated from central Asia. Interactions with Mongolia and the Roman Empire, the emergence and spread of Buddhism, particularly from China, and, by the eighth century, contact with the Islamic Near East, meant that all parts of India enjoyed large and diverse trading relations with many different cultures. The continuity of active trade networks and the influx and settlement of foreigners ensured an abundant and continual flow of foreign ideas and concepts into the Indian intellectual environment.

The astronomical ideas that the Indians inherited from foreign cultures, particularly in the early periods, were not always the most developed or leading astronomical theories of the time from the culture in question. What was regarded by the Indian practitioners as some progressive foreign advancement that must be incorporated into their own system was all too often outdated and/or second rate. Perhaps this is most easily seen from the fact that, despite substantial transmission from Greece, the Indian astronomical environment was largely oblivious to most of the ideas and theoretical models of Ptolemy, as expounded in his seminal work, the *Almagest*, until around the eighteenth century! Similarly, despite the powerful predictive tools and periods used by the Mesopotamian astronomers to determine eclipse possibilities, Indian astronomers remained largely ignorant of such aspects.

In the process of amalgamating the indigenous traditions with foreign imports, conflict and contradiction invariably manifested themselves. This had

the result that overriding consistency within the system was not necessarily a priority and contradiction was tolerated. This is no doubt another factor helping to explain why innovation in the Indian scientific climate was not a priority. Because of the inherent contradictions in their systems, Indian astronomers mostly avoided considering the logical presuppositions or the foundations of their science. Things were not to be examined too closely.[12]

What then were their criteria of validation? What would convince the astronomers of the validity of a rule or model? Did they even deem it necessary to put their rules under such tests? There were certainly in existence various schools of philosophy that dealt with the examination of the various means of acquiring valid knowledge. Such knowledge, in Sanskrit, is conveyed by the word *pramā*. Its derivative noun, *pramāṇa* was a technical term, having the sense of "the means of acquiring *pramā*," or in the right contexts, "proof" or "authority." This was subdivided into various categories by the different philosophical schools and included *pratyakṣa*, perception by the senses, *upamāna*, analogy or comparison, *anumāna*, inference, *śabda*, verbal authority, *anupalabdhi*, nonperception or negative proof, and *arthāpatti*, inference from circumstances.[13]

Āmarāja, a late twelfth–early thirteenth century commentator, reveals some of this sentiment in a scientific setting in his commentary on Brahmagupta's *Khaṇḍakhādyaka*. During a discussion about precession and trepidation, he states:

> ato duravabodhatvena tadvida eva pramāṇam iti |
> [Because it is difficult to understand, the "experts" are thus the authority (*pramāṇa*).]
> (KKD p. 108, lines 13–14)

Soon after, he also mentions:

> ataḥ pratyakṣopalabdhir evāyanāṃśotpattau śreyasī |
> [So the understanding by visual apprehension (*pratyakṣa*) is thus better in the production of the degrees of precession.]
> (KKD p. 109 line 14)

Indeed, in India, epistemological criteria were somewhat distinct from those used in the early Greek culture. Claims could be validated in a number of different ways, and Indian scholars did not regard themselves as limited to, say, the formal deductive steps that Greek mathematicians viewed as essential and fundamental to proving a mathematical statement. However, just because Indian scholars regarded a variety of justifications as valid, it did not mean that they held no respect for deduction or logic, nor did the justification from authority

take ultimate priority over other forms of validation. In the Indian discipline of the exact sciences, there was no explicit hierarchy or organization with respect to the means of acquiring valid knowledge considered fundamental in every case. This brought both restrictions and benefits to practitioners. Moreover, it was expected that verbal authority, sense-perception, inference, comparison, and other forms of justification would stand ultimately as complementary to one another, and that each had its own role in the generation, development, and justification of scientific knowledge.

The role observation played in Indian astronomy was somewhat distinctive. The emphasis on preserving the tradition discouraged astronomers from looking too carefully at the sky and comparing their results with what they saw. Perhaps that was because a large disparity between the two dissuaded them from subjecting their results to any close physical scrutiny. Another contributing factor might be the quality of the observations made throughout India. In order to use observations to determine or correct parameters, very accurate tools need to be employed as well as careful systematic observations. While the Indian astronomers were aware that there were many causes for the perturbations in planetary positions, and proposed a system with two primary variables, they had no methodology for distinguishing the effect of one from the other. They did not seem to recognize that one could observe the position of the planet when one of the two variables was zero, hence eliminating it.

In addition, very small effects, such as those of lunar and solar parallax, could not be measured directly to any degree of accuracy. Although the Indian astronomers comprehended the theoretical effects of parallax, the direct detection of it was beyond their observational capabilities. They did not make the effort, as was done in Greece, to measure it indirectly, by combining a number of carefully selected observations at which it was maximized and minimized so as to ascertain its effects with any degree of accuracy. In addition, they did not have a device, such as the Greeks had, to accurately take zenith distances (and directly convert the chord measurements to their corresponding arc). As a result their parallax calculations were based upon concepts that could be computed (such as the nonagesimal) and minimized reliance on observations.

Despite this, the Indian practitioners constantly referred to the importance of observations. Brahmagupta criticizes Āryabhaṭa's exposition on solar eclipses, asserting the need to reconcile observation with computations:

dṛggaṇitaikyaṃ na bhavati yasmāt pañcajyayā ravigrahaṇe |
tasmād yathā tadaikyaṃ tathā pravakṣyāmi tithyante ||
[Because there is no unity of observation and computation in a solar eclipse

by means of the five sines (used in computing eclipses), therefore I shall tell how the unity of these at the end of the (syzygy) tithi (is obtained).]
(BSS V, 1)

In a similar sentiment, Āmarāja comments:

yathā ca varāhamihirācāryaḥ
āśleṣārddhād dakṣiṇam uttaram ayanaṃ raver dhaniṣṭhādyam |
āsīt kadācid evaṃ yenoktaṃ pūrvaśāstreṣu ||
sāmpratam ayanaṃ savituḥ karkaṭakādyaṃ mṛgāditaś cānyat |
uktābhāve vikṛtiḥ pratyakṣaparīkṣaṇair vyaktiḥ ||
vikṛtau tu tatraiva phalam
[And so, the teacher Varāhamihira (says): Once the southern *ayana*[14] of the sun was from the middle of the constellation Āśleṣā, the northern from the beginning of the constellation Dhaniṣṭhā since it is said thus in the ancient texts. Now, the (same) *ayana* of the sun begins in Cancer the other begins in Capricorn. There has been a change, since what was said does not exist (any more). It is a manifestation of truth by means of seeing (it) with your own eyes. There is a result in this change.]
(KKD p. 109 lns. 14–19)

Despite the repeated claims of many Indian astronomers about the importance of the conformity of their rules with observation, it is obvious that observational astronomy was contributing very little or even nothing at all to the development of their theories. Furthermore, it is not yet known how the dimensions of epicycles, longitudes of apogees, and so on were initially determined for various planets. The so-called *bīja* corrections[15] for the most part simply converted parameters from those of one tradition to those of another. Lamentably, the absence of a tradition of observation meant that rules and parameters that failed to successfully predict and model phenomena continued to be employed and repeated.

It is not until the late fourteenth century with the works of the Kerala astronomer Parameśvara that observations seem to play any overt role in modifying Indian astronomy. In his treatise called the *Siddhāntadīpikā*[16] (see section 5.7.4), Parameśvara has included an account of 13 eclipse observations, observed over a period of thirty-four years.

However, despite the fact that Parameśvara presented a list of real observations, it is difficult to ascertain to what extent these observations actually influenced his theories. There is no attempt by him to reveal to the reader what factors contributed to the parameters and positions that he has given in his texts. His four works devoted solely to eclipse calculations, the *Grahaṇāṣṭaka*, the *Grahaṇamaṇḍana*, the *Siddhāntadīpikā*, and the *Grahaṇanyāyadīpikā*,

clearly represent distinct phases of a developing eclipse theory. However, in none of these works does he reveal exactly how or to what extent his observations have influenced the development of the theories. As a contrast, it is clear that the second century Greek astronomer Ptolemy used eclipses to determine parameters in lunar theory. Notably however, Ptolemy only refers to those eclipses in his works that have been used to construct his theories.

Nīlakaṇṭha Somayājin (1444–1545), who was taught by Parameśvara's son Dāmodara (fl. ca. second half of fifteenth century), discusses the conflict between the desire to preserve the original authority and the need to reconcile the rules with observation, most notably in his work *Jyotirmīmāṁsā* (literally "Analysis of Astronomy"). He argues that the models enunciated by the divinities were accurate at the time at which they revealed them, but that the universe is constantly changing. Thus, the astronomer must keep observing and constantly correcting the original system:

> pañcasiddhāntās tāvat kvacit kāle pramāṇam evety avagantavyam | api ca yaḥ siddhānto darśanāvisaṃvādī bhavati so 'nveṣaṇīyaḥ | darśanasaṃvādaś ca tadānīntanaiḥ parīkṣakair grahaṇādau vijñātavyaḥ | ye punar anyathā prāktanasiddhāntasya bhede sati yantraiḥ parīkṣya grahāṇāṃ bhagaṇādi saṃkhyāṃ jñātvā abhinavasiddhāntaḥ praṇeya ity arthāt | tat ta iha loke 'hasanīyāḥ paraloke 'daṇḍanīyaś ca iti |
> [One has to accept that (each of) the five siddhāntas had been authoritative at one time (or other), (though they might not be so, now). Therefore, one has to look for a system which tallies with observation. The said tallying has to be verified by contemporary experimenters at the time of eclipses. When the two vary, i.e., when an earlier system does not agree (with observation), experiments have to be conducted with instruments, and revolution-numbers of the planets calculated therefrom. A new system has thus to be expounded. Nobody should be ridiculed for this in this world nor punished in the next.]
> (*Jyotirmīmāṃsā* p. xvii and p. 6, as translated by Sarma [1977], with modification)

This sentiment is seen already in Brahmagupta, who asserts at the beginning of the *Brāhmasphuṭasiddhānta*:

> *brahmoktaṃ grahagaṇitaṃ mahatā kālena yat ślathībhūtam |*
> *abhidhīyate sphuṭaṃ taj jiṣṇusutabrahmaguptena ||*
> [The extent to which the calculation of the planets proclaimed by Brahmā has been weakened by (the passing of) a great period of time, that is explained accurately by Brahamagupta, the son of Jiṣṇu(gupta).]
> (BSS I, 2)

5.1.1 Schools

Beginning in about the fifth century CE, five different "schools" (*pakṣa*s) of astronomical thought defined themselves.[17] The members of each *pakṣa* are characterized by having parameters, models, and dominant features in common. While adherents to a particular *pakṣa* asserted its uniqueness and superiority over the others, more often than not their foundational theories, rules, and techniques were borrowed, adapted, or simply repeated from one *pakṣa* to another. Frequently, texts by members of one *pakṣa* would be grounded in that particular *pakṣa*, but could have elements from other *pakṣa*s incorporated into them. The five different *pakṣa*s and their emergence are classified as follows:

Pakṣa	Emergence	Region
1. Brāhmapakṣa	ca. 400	Western and Northwestern India
2. Āryapakṣa	ca. 500	Southern India
3. Ārdharātrikapakṣa	ca. 500	Rājasthan, Kashmir, Nepal, and Assam
4. Saurapakṣa	ca. 800	Northern, Northeastern, and Southern India
5. Gaṇeśapakṣa	ca. 1500	Western and Northwestern India

As can be seen, the *pakṣa*s were not a chronological division of astronomy, but largely a regional one.[18] Initial studies of the internal development of the five *pakṣa*s and their interconnection suggest that they were ultimately derived from at most two (and possibly only one) Sanskrit copies of Greek planetary tables.[19] Astronomical knowledge in each *pakṣa* was maintained and preserved by means of lineages of scholars connected by family, as was typical among all Indian disciplines. The promulgation of knowledge beyond immediate kin was rare. Learning was passed from one generation to the next—and each family adhered to an individual *pakṣa* and remained faithful to it. Private libraries would contain works dedicated to the *pakṣa* of choice, and reflection and research rarely went beyond the scope of that *pakṣa*.

5.1.2 Significance of Indian Astronomy

Astronomy as a discipline was practiced by Indian scholars largely without any significant interaction with other fields of inquiry. It was at no point incorporated or rationalized into a larger program of knowledge-seeking. The goals of astronomy were not to search for the physical reality beyond the appearances of celestial phenomena and the apparent motions of the heavens; the

emphasis was primarily on the instruction of individuals who functioned in society for consultation regarding the correct times for the numerous religious requirements, the management of the various facets of astrological practices, the organization of the calendar, and related tasks. Efforts on the part of these professionals were primarily directed toward practical considerations and astronomy therefore remained essentially a practical endeavor. For the most part, this attitude was collectively sustained, to the detriment of deeper theoretical inquiry and organization.

Central to astronomy was the insistence on the divine origin of this discipline; subscribers to particular *pakṣa*s maintained a fierce loyalty to the particular deity from whom astronomical knowledge had been obtained. Thus suggestions for fundamental alterations to the model amounted to disagreements with the divine. Consequently, modifications were unpopular and therefore rarely initiated. Change, when it was admitted, was largely limited to mathematical modifications of the techniques of computation—elaborations, simplifications, approximations, or alternatives—rather than adjustments of the basic model or parameters. In this respect, Indian scholars became renowned for their ingenuity and impressive facility in mathematics.

Change also impressed itself on Indian astronomy in another significant way: the importation and integration of foreign material. India, throughout its history, has never been culturally isolated, but has had significant and continual interaction with other cultures, through trade, conquest, the spread of religious traditions, or peaceful cultural amalgamation. Exposure to foreign ideas and influences was therefore inescapable. However, these external influences were evaluated and incorporated for the ways in which they corroborated and ratified the divinely revealed indigenous content, and not for the inspiration they might provide to consider things in another way. For these reasons, Indian astronomy is reasonably characterized as "the recipient and remodeler of foreign elements."

Competitive astronomers often invoked improved correspondence between observation and computed results, confirming the superiority of their system over those of their rivals. However, such claims were generally unsubstantiated. Observations were rarely made, and those that were suffered from a lack of suitable equipment, technique, and precision. Not until late in the course of Indian astronomy was any series of systematic observations recorded with a view to improving fundamental astronomical parameters. For the most part, observations were isolated, haphazard, and arbitrary. Astronomers were not accountable for the accuracy of their predictions, and the minimal attention afforded to observation reflected this. Furthermore, physical astronomy was never widely pursued as illuminating or relevant.

There were advantages in not being bound by physical considerations or locked into a single geometrical perspective. Greek inquirers, who had committed themselves to explaining the apparently irregular trajectories of the celestial phenomena by means of regular, uniform, circular (and for some, concentric) motions as a result often required very complex mechanisms to account for certain aspects of the phenomena. Mechanisms with these prerequisites, for the most part, were commanding, but, with respect to certain details, unsatisfactory. On the other hand, Indian hybrid models could account for many aspects of nonuniform motions much more simply. Unbound by any particular geometrical requirements or commitment to a single physical perspective, Indian practitioners were able to modify and revamp existing models imaginatively and without restriction, and address computational or observational inconsistencies in ways not permitted before.

This had some striking results. For example, the fifteenth-century astronomer Nīlakaṇṭha radically modified the existing planetary model that had been used for centuries by the *Āryapakṣa* system, rearranging the planetary configurations in ways that would have been unacceptable to those who subscribed to concentricity. Among his numerous modifications, he changed the standard relations of deferents and epicycles, so that the central point of the orbit of a planet rotated on its own *manda* epicycle.[20] In turn, he situated the center of rotation of this *manda* epicycle at the longitude of the sun as reckoned on the *śighra* epicycle, the center of which was the earth. This and other adjustments resulted in what amounted to a "helio-referential" account of planetary motion, which was in many respects equivalent to a heliocentric system. An astronomical model of this sort in which most of the planets revolve around the sun, which itself revolves around the earth, was put forward by Tycho Brahe, some time later.[21] Indeed, this example shows the advantages to Indian astronomers from being unconstrained by strict geometrical prerequisites in their accounting of heavenly phenomena. Outstanding discrepancies in earlier models often could be addressed only by drastic reconfigurations—the sorts of reconsiderations available to those who were not committed to existing fundamental geometrical or physical conditions.

Overall, astronomy in India was a technical subject and had little place in wider intellectual pursuits. It was only through its applications (astrological pursuits, ritual requirements, and the like) that the wider scholarly community had any exposure to its methods, suppositions, and scope. For this reason, it was a tradition largely sustained independent of the general intellectual setting. Astronomy was never confronted with the need to clarify its connection to physics and natural science, nor was it influenced by considerations being promoted in philosophy. In terms of methodology, no attention was given to

rigorizing or standardizing practice, no consistent scientific methodology was furthered. The demands of an oral environment also had their impact. For these reasons, Indian astronomy progressed on a trajectory markedly different from that in other cultures.

5.2 General Features of Eclipse Reckoning in India

There are many words and phrases that were used to express the occurrence of an eclipse in Sanskrit (*uparāga* [the act of coloring, dyeing, darkening], *tamas* [darkness, gloom, obscuration] *upasargaḥ* [misfortune, trouble], etc.). But by far the most popular Sanskrit word to express the phenomenon of an eclipse is *grahaṇa*, a noun which is derived from the root √*grah/grabh* (to seize, grasp). Such a term immediately calls to mind the fantastic cosmological conception devised to explain the mechanics behind eclipses. This is that Rāhu, an *asura* (demon), is responsible for eclipses. He devors the moon or the sun on full-moon or new-moon days, causing them to be obscured.

> svarbhānur vāsura ādityaṃ tamasāvidhyat | taṃ devā na vyajānan
> | te atrim upādhāvan| tasya atrir bhāsena tamo 'pāhanyat |
> prathamam apāhan sā kṛṣṇāvir abhavat | dvitīyaṃ sā rajatā yat
> tṛtīyaṃ sā lohitī yathā varṇam abhyatṛṇat sā śuklāsī ||
> [The asura Svarbhānu (Rāhu) struck the Sun with darkness. The Gods did not notice him (the sun). They turned to Atri. Atri repelled its darkness by means of the light. Repelling the first part, it became a black sheep, the second a silvery (sheep), whatever was third a red one, just as a white sheep procurs color.]
> (*Tāṇḍya Paṅcaviṃśa Brāhmaṇa*, VI, 6.8)

Rāhu[22] and the other half of his body, Ketu, sliced off by a vengeful stroke of Viṣṇu's sword,[23] are also known as the shadowy planets[24] that are placed at the two nodal points of the lunar orbit. Tradition argued that the luminaries would reappear after being devored by Rāhu by simply falling out of his gaping neck-hole! The epic *Mahābhārata* somewhat elusively gives more detail to the form of Rāhu and his interaction with the sun and the moon:

> [It cannot be distinctly seen how Rāhu approaches and leaves the moon. After the same manner, the Soul cannot be seen how it leaves one body and enters another. Rāhu becomes visible only when it exists with the sun or the moon. Similarly, the Soul becomes an object of apprehension only when it exists with the body. When liberated from the sun or the moon, Rāhu can no longer be seen.
> (*Mahābhārata* 12.196 [20–23])

The famous astrologer Varāhamihira boldly dismisses the cosmological interpretation of Rāhu as the cause of eclipses, and in addition criticizes the connection between eclipses and certain meteorological and geological events. It seems likely that the target of this criticism was the eclipse omen literature originally inspired by the Mesopotamian *Enūma Anu Enlil* (see section 3.2). Varāhamihira would not have known *EAE* directly, but rather the Sanskrit adaptations of this material.[25]

> evam uparāgakāraṇam uktam idaṃ divyadṛgbhir ācāryaiḥ |
> rāhur akāraṇam asminn ity uktaḥ śāstrasadbhāvaḥ ||13||
> na kathañcid api nimittair grahaṇaṃ vijñāyate nimittāni |
> anyasminn api kāle bhavanty athotpātarūpāṇi ||16||
> [In this way, the teachers endowed with divine insight have explained the cause of an eclipse. Hence the scientific fact is mentioned that Rāhu is not at all the cause of this.
> An eclipse can by no means be ascertained through omens. For, the portents such as the fall of meteors occur at any other time as well.]
> (Varāhamihira, *Bṛhatsaṃhitā*, V 13 and 16 [pp. 45–46], as translated by Ramakrishna Bhat [1996])

An interesting presentation of the Rāhu debate, in which the tension between traditional views and the astronomical model is clearly manifested, is in chapter XXI of Brahmagupta's *Brāhmasphuṭasiddhānta* and the commentary written on it by the ninth-century commentator Caturveda Pṛthūdakasvāmin.[26] Brahmagupta has several verses (XXI, 36–38) that give different "physical" refutations of the opinion that Rāhu causes eclipses. All the arguments focus on the differences in quality between lunar and solar eclipses, which, Pṛthūdakasvāmin infers, means that the same thing cannot be causing both. Pṛthūdakasvāmin calls these differences *dūṣaṇa*s (literally "defects," namely, flaws in the idea that an eclipse is caused by Rāhu).

> mahad indor āvaraṇaṃ kuṇṭhaviṣāṇo yato 'rdhasaṃchannaḥ |
> ardhacchanno bhānus tīkṣṇaviṣāṇas tato 'syālpam ||
> [The obscurer of the moon is large because the half-covered (moon) has blunt horns. The half-covered sun has sharp horns. Therefore its (obscurer) is small.]
> (BSS XXI, 36, as translated by Ikeyama [2003])

Pṛthūdakasvāmin explains that what Brahmagupta has intended here is that two separate things are responsible for the eclipses. He comments:

> anyena raviś chādyate 'nyena ca śaśī . . . tau ca bhūchāyācandramasau muktvā nānyau niyuktikau kalpayituṃ śakyete iti
> [The sun is covered by one thing and the moon by another . . . It is impossible

to assume any other two things are suitable except the shadow of the earth and the moon.]
(BSS *com.* XXI, 36, as translated by Ikeyama [2003])

The next argument against Rāhu presented by Brahmagupta (XXI, 37) employs the difference in the direction of impact (from the east in the case of a lunar eclipse and from the west for a solar eclipse) as evidence against the notion that one thing is responsible for both. He also points out the fact that the (greatest possible) half duration of a solar eclipse is shorter than that of a lunar eclipse. Again Pṛthūdakasvāmin supports Brahmagupta's critique by alluding to the motions of the sun, the moon, and the shadow on their orbits, physically describing the direction of impact. By this he shows that the explanation that Rāhu is responsible for eclipses brings one into difficulty. He also states that the reason the half durations are different is a result of the comparative sizes of the shadow and the moon. The last defect is the fact that the magnitudes of solar eclipses differ. This is again backed up by Pṛthūdakasvāmin in a similar way.

However, Brahmagupta's sentiment changes almost instantly as next he criticizes those who assert that Rāhu is *not* the cause of eclipses as being sacrilegious—outside of the two śāstras of dharma (*smṛti*) and the Vedas (*śruti*).[27] In stark contrast to his previous refutations, he composes verses that state the direct opposite:

rāhukṛtaṃ grahaṇadvayam
[The two (solar and lunar) eclipses are caused by Rāhu.]
(BSS XXI, 41, as translated by Ikeyama [2003])

Again the final verse of the chapter:

bhūchāyā nendum ato grahaṇe chādayati nārkam indur vā |
tatsthas tadvyāsasamo rāhuś chādayati śaśisūryau ||
[Therefore the shadow of the earth does not cover the moon in an eclipse nor does the moon (cover) the sun; Rāhu which stands there and is equal (in size) to their diameters covers the sun and the moon.]
(BSS XXI, 48, as translated by Ikeyama [2003])

As Brahmagupta mentions, these opinions are those expressed in the Vedas, the *smṛtis* and the *saṃhitās,* views that are widely believed and accepted. Thus in order to generate and maintain a positive reception of his work in such an audience, he too must seem to conform to these views. Pṛthūdakasvāmin tacitly supports Brahmagupta for this move, and gives some insight into this seemingly radical swing of opinion:

lokaviruddho 'pi na vaktavya ity asyārtham āryārdham āha
[(Considering) that what is repugnant to the people should not be mentioned, he says half a verse for this purpose.]
(BSS *com.* XXI, 43cd, as translated by Ikeyama [2003])

Thus he confirms that it is indeed popular opinion that causes Brahmagupta to express such sentiments, even though they conflict with his "scientific" reasoning.

This kind of contradiction is not just confined to the work of Brahmagupta; it was a very common challenge that astronomers grappled with throughout the history of Indian astronomy. Indeed the eighth-century astronomer Lalla,[28] in the *Śiṣyadhīvṛddhidatantra*, has many valid scientific refutations (XX, 17–27) of the existence of Rāhu as the cause of eclipses, such as the fact that eclipses can be predicted by calculation and that such regularity would not be the result of the actions of an impetuous hungry demon:

asuro yadi māyayā yuto niyato 'tigrasatīti te matam |
gaṇitena kathaṃ sa labhyate grahakṛtparva vinā kathañcana ||
savituś ca yad anyathā 'nyathā pratideśaṃ sakalaṃ samīkṣyate |
na ca kutracid ity avetya kaḥ kurute rāhukṛte grahe graham ||
[If you are of the opinion that an artful demon is always the cause of an eclipse by swallowing (the sun or moon), then how is it that an eclipse can be determined by means of calculation? Moreover, why is there not an eclipse on a day other than the day of new or full moon?
In a solar eclipse, people at different parts (of the earth) see different portions of the Sun eclipse. Some do not see (the eclipse) at all. Knowing this, who can maintain that an eclipse is caused by Rāhu?]
(ŚDV XX, 22 and 26, as translated by Chatterjee [1981])

But again in the very next verse Lalla connects Rāhu back to eclipses:

grahaṇe kamalāsanānubhāvād dhutadattāṃśabhujo 'sya sannidhānam |
yad ataḥ smṛtivedasaṃhitāsu grahaṇaṃ rāhukṛtaṃ gataṃ prasiddham ||
[Because of the great authority of Brahmā, at the time of an eclipse the Sun is near Rāhu. So, in the Vedas, smṛtis, and saṃhitās, it has come to be known that Rāhu is the cause of eclipses.]
(ŚDV XX, 27, as translated by Chatterjee [1981])

Again, it is clear that the authority of the ancient texts supersedes the scientific response—that authority is so embedded that even the astronomers, despite their obvious and vocal objections, will in the end submit to the pressure to conform. Furthermore, Lalla, in verse 22 above, hints at something quite revealing about the nature of the scientific endeavor. His mention of

"calculation" is an allusion to the belief that the universe is ordered, rational, and explainable, and can be modeled and represented (fairly well) by mathematical rules and models—something which is not the case should one concede that demons and other irrational, unpredictable forces cause celestial events.

5.2.1 Conventions and Technical Details Related to Eclipse Reckoning

Elements of eclipse reckoning can be found in a variety of different sources. The most important sources are the *siddhānta* texts, detailed astronomical treatises characterized by comprehensiveness and universality. Works of this type typically make the epoch the start of a major cosmic era, and the order of astronomical and mathematical topics follows a traditional sequence.[29] Another important format was the *karaṇa* (literally "making"). This type of work is related to the *siddhānta*, but is purposely simplified and bases its parameters on a more recent epoch, often within the lifetime of the composer, rather than the start of a large cosmic era. Commentaries (*ṭīkā* or *bhaṣya*) almost always accompany these texts of various sorts and provide further insight into the ways in which astronomers understood and developed their work. Inscriptions represent another important source for Indian eclipse inquiry. These are permanent, (usually) unmovable records of gifts and grants made by a noted member of society, often on the occasion of a specific celestial event, such as an eclipse.

These sources use various astronomical and mathematical conventions in the course of recording, predicting, and developing eclipse reckoning techniques. Traditional Indian conceptions of time centered around the notion of immense cycles—these were adopted by the astronomers, who used them to calculate position and motion using period relations consisting of such temporal intervals. The largest of these was the *kalpa*, which lasted 4,320,000,000 years. This was the duration of the creation, flourishing, and destruction of the universe. A smaller division of the *kalpa* was the *mahāyuga*, lasting 4,320,000 years, which itself was further subdivided into intervals known as *yuga*s, most commonly by the ratio 4:3:2:1. The last of these *yuga*s, the *kaliyuga*, is the phase the world is in presently. It began on the 17/18th February 3102 BCE—this instant is commonly selected as an epoch for many *siddhānta* texts. The date was often described in the number of days elapsed or accumulated since this epoch—these are known as *ahargaṇa* or "accumulated" days. Other traditional epochs of Indian chronology include the Vikrama era, whose beginning falls in 58 BCE, named after a (yet unidentified) king Vikramāditya; the Śaka era whose beginning falls in 78 CE, after the Scythians or Śakas who invaded western India, and the Kalachuri era, whose beginning falls in 248 CE.

On a more local level, time was delimited as follows: years (*saṃvatsaras*), usually taken to be 360 days; half years (*ayanas*), one of which is when the sun travels north (between the vernal equinox and the autumnal equinox) and the other when the sun travels south (between the autumnal equinox and the vernal equinox); seasons (*ṛtus*); months (*māsas*) and intercalary months (*adhimāsas*); fortnights (*pakṣas*), intervals of 14 or 15 days—the interval from new moon to full was commonly referred to as the "bright half" (*śuklapakṣa*), or waxing, and that between full moon and new moon, the "dark half" (*kṛṣṇapakṣa*) or waning; and days (the so-called nychthemeron in Sanskrit *rātri*). Smaller important divisions of the 24-hour period are *ghaṭikā*s or *nāḍī*s. There are 60 of these in one 24-hour period, so that one *ghaṭikā* is 24 minutes. These could be further subdivided to produce *vināḍīs*, which were a sixtieth of a *ghaṭikā*, that is, 24 seconds.

By the third century CE, Indian astronomers differentiated between four types of time reckoning: solar (*saura*), lunar (*cāndra*), sidereal (*nakṣatra*), and civil (*sāvana*). In particular, the synodic month was of fundamental importance to astronomers, and was measured from the new moon (*amāntamāsa*) with the full moon (*pūrṇimāntamāsa*) in the middle. The synodic month was divided into 30 equal *tithi*s, or lunar days. The *tithi* is the interval at which the difference in the eastward motions of the sun and the moon accumulates to 12°.

Eclipses occur in various places in the sky, and there were many ways in which their positions were referenced. Celestial coordinates were commonly given with respect to celestial longitude and latitude. There were other important reference points on the celestial sphere (fig. 5.1). The nonagesimal (V) is the point 90° from the ascendent along the ecliptic (the Sanskrit word for nonagesimal is *vitribha(lagna)* (literally "the ascendant less three zodiacal signs"; hence the letter V is used to represent it in the formula). The meridian ecliptic point (M) is the point at which the ecliptic intersects the prime meridian. The zenith (Z) is the point directly above the observer. The luminaries (S) were often imagined to be on the ecliptic at the time of an eclipse, as they were sufficiently close. The ascendant (H) is the point on the ecliptic that is rising on the eastern horizon. The terrestrial latitude is given by the arc ZQ and the longitudinal distance between the sun and the nonagesimal is VS. The hour angle is one of the pair of components used in the equatorial coordinate system. It is the angle ($\Delta\tau = \angle MZS$) between the plane of the prime meridian (the plane given by the poles of rotation of the earth and the local zenith) and the plane through the poles of rotation of the earth and the position of the celestial body. Various constellations (27 or 28 roughly equally spaced) along the lunar orbit (*nakṣatra*s) or more specifically zodiacal signs (*rāśi*s) were also often used to locate these celestial events. A common measure of terrestrial

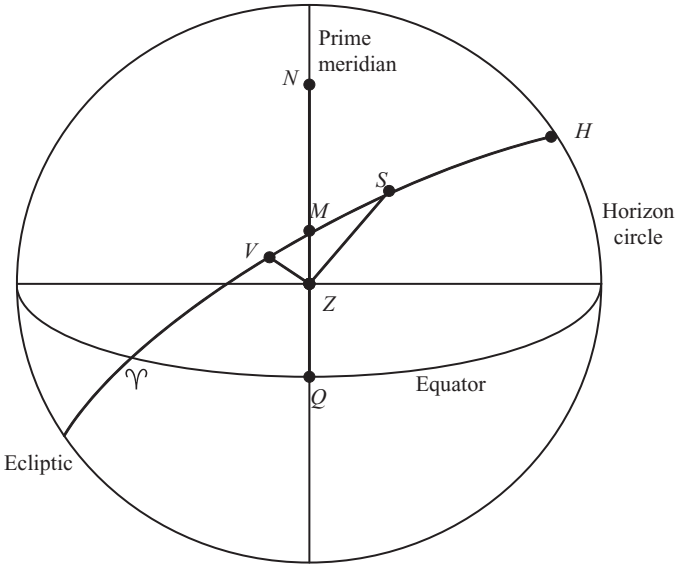


Figure 5.1 Various points on the celestial sphere.

distance was the *yojana*, the metric equivalent of a length in the order of about 10 kilometers.

There was a group of measures that were considered central by some astronomers to the computation of solar eclipses, especially parallax. In particular, Āryabhaṭa[30] identified these as the "five sines," and they included:

Sine of the altitude of the sun	$\mathrm{Sin}\alpha(S)$
Sine of the rising amplitude	$\mathrm{Sin}\eta$
Sine of the zenith distance of the meridian ecliptic point	$\mathrm{Sin}z(M)$
Sine of the zenith distance of the nonagesimal	$\mathrm{Sin}z(V)$
Sine of the depression from the nonagesimal	$\mathrm{Sin}VS$

Various mathematical techniques were used by Indian scholars in their eclipse inquiries. The sine function (Sanskrit *jyārdha* or *jīvā*) was used by Indian scholars, replacing the chord function, along with its related trigonometric function, the versine (*śara* [literally "the arrow"] or *utkramajyā* [literally "the reversed sine"]), which is $1-\cos\theta$. Various values for the radius of the base circle were used by scholars, including $R = 120$, 150, 3438, 3270, 3415, among others. Those values of the radius which are close to 3000 are chosen because they provide good integer values for the trigonometric functions. A selection of the various values for R are presented in the following table:

Author	Work	Value
Varāhamihira	*Pañcasiddhāntikā*	120
Āryabhaṭa	*Āryabhaṭīya*	3438
—	*Paitāmahasiddhānta*	3438
Brahmagupta	*Brāhmasphuṭasiddhānta*	3270
Brahmagupta	*Khaṇḍakhādyaka*	150
Śrīpati	*Siddhāntaśekhara*	3415
Parameśvara		3438, 239, 240

To distinguish between the modern sine function, which has a unit radius, and these early functions, capitalization of the first letter of the function designation has been used—the initial letter in lower case refers to the modern trigonometric function, the upper case, the function with a nonunit radius (see section 2.5 "Early Trigonometry).

Another frequently used mathematical relation is the ubiquitous Indian rule of three (*trairāśika*), which is based on the equality of two proportions.[31] These relations, in combination with the Pythagorean theorem and simple ratios between angles and arcs on spherical triangles, were used in the construction and articulation of the rules and procedures of Indian astronomy.

One other important aspect of eclipse computations was the reckoning of parallax. Convenient estimates for the maximum parallax were frequently given by Indian scholars as "1/15 of the motion of the planet in a day" or the motion in four *nāḍī*s ($^{60}/_{15} = 4$), a convenient approximation that was systematically adopted by generations of Indian astronomers. This approximation is found uniquely in India.

There are two components of parallax in the Indian tradition: longitudinal and latitudinal parallax, which combine to make one vector of total parallax. Interestingly enough, there is no word in Sanskrit, as there is in Greek, used to represent total parallax, just terms for each of the two components. The longitudinal parallax is usually calculated first because of its direct impact on the time of conjunction, so that it affects the time of the eclipse. The latitudinal parallax corrected the computed lunar latitude. Typically, the components of parallax are calculated by taking the extremes of parallax and varying this sinusoidally for intermediary values. To do this, an arc of 90° was sought, and in the Indian tradition this was generally a measure that was easy to compute, such as the zenith distance of the nonagesimal, or the angular difference between the luminary in question and the nonagesimal, rather than a measure that was derived from direct observation.

One of the challenges in applying parallax to the positions of the luminaries is to establish the precise moment of conjunction. Parallax adjusts the true position of a planet to the apparent position. A parallax correction is made all

the more complex in the case of a conjunction, as it requires that two sets of parallactic corrections, one for each body, preserve the conjunction of the two bodies. To determine the time of apparent conjunction, the distance between the two positions, true and apparent, needs to be determined, and the time it takes for the luminaries to cover this distance can be added (or subtracted) onto the time of true conjunction to give the estimate time of apparent conjunction. However, as this small time interval elapses, so too do the other circumstances of the heavens with respect to the observer, and thus the parallactic correction will have changed as well. So the determination of the time of apparent conjunction is complicated. A variety of solutions were proposed, including iterative procedures.

As well as being included in general astronomical works, records of eclipses are found in inscriptions and were described with many details. The basic element in recording the eclipses in inscriptions was the date. The type of eclipse (solar or lunar) was specified along with the year (given in a pertinent era, such as Śaka, Vikrama, Kalachuri, etc.) whether the year was elapsed or current, the month (in the *paurṇimānta* or *amānta* system), the fortnight of the lunar month (*śuklapakṣa* "bright half" or *kṛṣṇapakṣa* "dark half"), and the lunar day (*tithi*) of the fortnight. *amāvāsyā* refers to the new moon and *purṇima* the full moon.

To this basic format, additional information could be added. This could include the weekday, the constellation, the zodiacal sign, the time of the night or day, and the cyclical year of Jupiter.[32] Other information was very occasionally supplied, such as whether the eclipse was total or partial. One inscription refers to an eclipse occurring within a specified interval of the night, using the term *yāma*,[33] which means in this context a night watch or a period or watch of three hours.[34]

5.3 Eclipses as Omens

Indian culture valued omens, and the Mesopotamian tradition which found its way into the Indian intellectual scene greatly influenced both the content and format of texts on this subject.[35] The *saṃhitā* texts from the *jyotiḥśāstra* tradition set out both terrestrial and celestial omens. It may be that the initial interest in eclipses in India in fact goes back to the emphasis given to them in the Mesopotamian omen tradition.

There exist several texts,[36] including the *Kauśikasūtra* of the *Atharvaveda*, which show the earliest Indian attempts to arrange and classify omens and their counteractive rituals (*śānti*). They consist of celestial, atmospheric, and terrestrial phenomena, but eclipses and related phenomena are not among

them. While it is impossible to confirm that the content of these lists originated from Mesopotamian prototypes and their corresponding *namburbi* rituals (the Sanskrit omens do not include any apodotic material), it seems likely that they are in some way inspired by material transmitted from Mesopotamia around the sixth century BCE. The major Mesopotamian source of celestial omens, *Enūma Anu Enlil*, was probably not directly transmitted to India but rather through an Aramaic intermediary.[37]

In the fifth and fourth centuries BCE, however, transmission is directly discernible. Both Sanskrit and Prakrit (a Sanskrit vernacular) texts of this time reveal a knowledge of at least two main Mesopotamian omen catalogs, that of *Enūma Anu Enlil* and *Šumma ālu* (terrestial omens). The early Buddhist canonical texts, in particular the *Dīghanikāya* (Sermons of the Buddha), include a section entitled the *Brahmajālasutta*, in which the Buddha condemns certain activities of the *Brāhmaṇa*s (scholars) and the *Śramaṇa*s (mendicants), among which are included their knowledge and interpretation of omens. With respect to this criticism, he gives a list of the omens that he states the *Brāhmaṇa*s are familiar with. He commences his discussion of omens by the Pali (a member of the Prakrit language family) word *uppāda* (Sanskrit *utpāta*) which refers to "phenomena contrary to nature."[38] He classifies these omens into three sections; those occurring on earth (for which compare *Šumma ālu*), those produced in the atmosphere (compare the *Adad* [weather] section of *EAE*), and those occurring in the heavens (compare the other sections of *EAE*). The terrestial omens are listed by the Buddha in essentially the same order as they appear in *Šumma ālu*. The Buddha also gives some of the general themes which are part of the protasis material in *EAE*, beginning with lunar and solar eclipses.

By means of such Buddhist accounts and adaptations,[39] the Mesopotamian omen tradition spread through Central Asia. The earliest extant Sanskrit omen text is the *Gargasaṃhitā* (likely written some time between the first centuries BCE and CE in North India) on which Varāhamihira's work of the same name is partly based. The *Gargasaṃhitā* is obviously inspired by the Mesopotamian tradition, and much of its content is derived from the two omens series *Enūma Anu Enlil* and *Šumma ālu*, as well as Varāhamihira's influential text, the *Bṛhatsaṃhitā*.

Chapter 5 of the *Bṛhatsaṃhitā* contains eclipse omens, and many of the protasis themes from earlier Mesopotamian eclipse omens are echoed here. Eclipses were almost always portents of bad outcomes:

candrārkayorekamāse grahaṇaṃ na praśasyate |
parasparaṃ badhaṃ kuryuḥ svabalakṣubhitā nṛpāḥ ||
grastāvuditāstamitau śāradadhānyāvanīśvarakṣayadau |
sarvagrastau durmikṣamarakadau pāpasandṛṣṭau ||

[Should the luminaries rise or set being eclipsed, there will be danger to autumnal crops, as well as to kings. On the other hand, if the eclipse should also be total and the eclipsed orb be aspected by malefics as well, there would be famine and pestilence all over the country.]
(Varāhamihira *Bṛhatsaṃhitā* V, 27, as translated by Bhat [1996])

Another excerpt from Varāhamihira reveals a connection back to the "ancients" and an interesting interpretation of the protasis material. The omen Varāhamihira cites refers to an eclipse happening a little before or after its calculated time:

velāhīne parvaṇi garbhavipattiśca śastrakopaśca |
ativele kusumaphalakṣayo bhayaṃ sasyanāśaśca ||
[Should an eclipse take place a little earlier than the time arrived at by astronomical calculation, there would be abortions and clash of arms. On the other hand, if it should commence a little later than the calculated time, there would be destruction of flowers and fruits, panic would grip the people, and loss of crops would ensue.]
(Varāhamihira *Bṛhatsaṃhitā* V, 25, as translated by Bhat [1996])

However, Varāhamihira somewhat dismisses this possibility, claiming that good astrologers of his day are in possession of techniques that ensure their predictions will be accurate, and neither late nor early. In other words, the difference between a predicted time and the actual time would not be ominous, but rather a reflection of the inexperience of the one performing the computation!

hīnātiraktakāle phalamuktaṃ pūrvaśāstradṛṣṭatvāt |
sphuṭagaṇitavidaḥ kālaḥ kathañcadapi nānyathā bhavati ||
[I have mentioned the effects of eclipses taking place a little earlier or later than the calculated time for the sole reason that the ancients have stated so in their works. However, if the astrologer knows the correct method of calculating eclipses, by no means can there by any difference whatsoever in the time of their occurrence.]
(Varāhamihira *Bṛhatsaṃhitā* V, 26, as translated by Bhat [1996])

Indeed, "progress" entailed that some omen protases had become obsolete, but were nonetheless retained, probably for historical interest.

Omens that included eclipse phenomena are found in Jain texts as well. Perhaps contemporary with Varāhamihira was an anonymous Jain author who wrote an enormous work on omens, called the *Aṅgavijjā*, which treats every conceivable event as ominous and catalogs it as such. Another Jain author, Bhadrabāhu, composed the *Bhadrabāhusaṃhitā* or *Nimitta*[40] on which much of the material included in Varāhamihira's *Bṛhatsaṃhitā* is based.

5.4 Eclipse Observation Records in Inscriptions

India has nothing comparable to the substantial records of celestial phenomena contained in the Mesopotamian Diaries, but it does have many eclipse observations or predictions that have been recorded in ancient Indian inscriptions. Such inscriptions, found on copper plates, rocks, temples, pillars, and the like, were, for the most part, recordings of grants and gifts given by kings and other important members of society. Very often, these grants were given on the occasion of a certain celestial event, a recognized religious custom:

> grahoparāge yad dānaṃ sūryasaṃkramaṇeṣu ca |
> dvādaśyādau ca yad dānaṃ pūrtam ity abhidhīyate ||
> [It is considered that whatever gift (is given) at an eclipse and at *saṃkrāntis*[41] of the Sun, (and) whatever gift (is given) at the beginning of the twelfth (lunar month), (it) is meritorious.]
> (*CII* VI p. liii, as cited from the verse of Jātūkarṇya quoted in the ṭīkā of Aparārka)

Eclipses were dangerous occasions and thus often a reason for grants to be made. There was the belief that the adverse effects of the eclipse would be neutralized by the gesture of making a donation.[42] Such documentation bears witness to the fact that eclipses must have been considered very ominous events. Commonly (especially in the case of those given by the Śilāhāra kings; see CII VI), grants given on the occasion of a solar eclipse were for the maintenance of the donors' religious duties and security of their families. Kings would bathe in a *tīrtha* (sacred bathing place), the waters of the ocean, or some other body of water, offer garlands of flowers to the sun, and then make the grant. The language used is often formulaic, as in the following example of a grant by the eleventh-century ruler Nāgārjuna at Ṭhāṇā:

> iti dharmmādharmmavicāracaturacirantanamunivacanāny avadhārya mātāpitror ātmanaś ca śreyorttthinā mayā śakanṛpakālātītasamvatsaraśateṣu navasv ekaṣaṣṭyadikeṣu pramāthināmasamvatsarāntarggataśrāvaṇavādi paṃcadaśyāṃ yatrāṃkato 'pi samvat 961 śrāmvaṇa vadi 15 budhe saṃjātasūryagrahaṇaparvvaṇi sutīrtthe snātvā gaganaikacakracūḍāmaṇaye kamalinīkāmukāya bhagavate savitre nānāvidhakusumaślāghyam arghyaṃ dattvā sakalasurāsuraguruṃ trailokyasvāminaṃ bhagavantam umāpatim abhyarccya yajanayājanādhyayanādhyāpanādi ṣaṭkarmaniratāya kratukriyākāṇḍaśauṇḍāya paramabrahmaṇe mahābrahmaṇāya hastigrāmavinirggatāya madhyadeśāntaḥ pāti pārāśaragotrāya yajurvvedaśākhine mādhavapaṇḍitāya . . .
> [Having thought over these sayings of old sages adept in discriminating between *dharma* and *adharma* and having bathed at an excellent *tīrtha* on the

> holy occasion of a solar eclipse which occurred on Wednesday, the fifteenth tithi of the dark fortnight of Śrāvaṇa in the (cyclic) year *Pramāthin*, when nine hundred years increased by sixty-one had expired in the era of the Śaka king—in figures the year 961—Śrāvaṇa, the dark fortnight and the 15th (tithi), and having offered an *arghya*, beautiful with flowers of various kinds to the divine Sun, the sole jewel-crest of the sky and the lover of the lotus plant and having worshipped the divine Śiva, the lord of the three worlds and the guru of all gods and demons, I have given, for the spiritual welfare of my mother, and father and of myself, as a gift free from all taxes with great devotion (and) with the pouring out of water to the great Brāhmaṇa Mādhava Paṇḍita . . . of the Pārāśara gotra and the Yajurveda śākhā who has emigrated from Hastigrāma situated in the Madhyadeśa, for the performance of the six duties such as sacrificing for himself and for others, . . . the village Muñjavali . . .]
> (Ṭhāṇā Plates of Nāgārjuna, Number 13, *CII* VI, p. 75ff, esp. 79 and 81 l.54ff, as translated by Mirashi).

For the geographical coordinates 73° E, 19° N (an approximation to the location of Ṭhāṇā), a solar eclipse was visible on 22 August 1039 (= 961 Śrāvaṇa 15th day of dark fortnight) where a little over half of the disk was obscured. First contact occurred at 17:26, fourth contact at 19:10, and the instant of syzygy occurred at 18:20.

The various details in records of eclipse observations and predictions have been discussed above. The following inscription, the Janjirā Plates of Aparājita, gives a good example of how the various features of an eclipse observation can be described:

> śakanṛpakālātītasaṃvatsaraśateṣu navasu paṃcadaśottareṣv aṅkataḥ saṃvat 915 vijayasaṃvatsarāntarggataśrāvaṇabahula 15 ravau saṃjātasūryagrahaṇa-mahāparvvaṇi rāhugrahagrāsīkṛtatīvradyutimaṇḍale siṃharāśigate ca sūrye sati . . .
> [On Sunday, the fifteenth tithi in the dark fortnight of Śrāvaṇa in the cyclic year Vijaya in the expired year nine hundred and fifteen (915) of the era of the Śaka king on the very holy occasion of a solar eclipse when the disc of the hot-rayed one (the sun) was swallowed by the planet Rāhu, and when the sun was in the rāśi Siṃha.]
> (*CII* VI 5 p. 17 lns 57–59)

All details about this eclipse are correct and correspond to the Julian date Sunday 20 August 993 (= Śaka 915). A total solar eclipse was visible at this locality (tentatively assuming that Janjirā [73° E and 18° N] was close to the place of observation). First contact occurred at 12:01, fourth contact at 14:50, and the instant of syzygy was at 13:30. The eclipse indeed occurred in the zodiacal sign (rāśi) of *Siṃha*, which is Leo. The sun being in the rāśi *Siṃha* is not an event

that can be observed—the stars and the sun cannot be observed at the same time! Therefore, there must be an element of computation in this record.[43]

There exists one inscription with a "prediction" of a total lunar eclipse by an astronomer named Padmanābha, which, in fact, actually came to pass. The inscription is on the Sarkho Plates of Ratnadeva II, Kalachūri year 880 or 1128 CE.[44] This feat so impressed the king and his court that Padmanābha was richly praised and granted a whole village and its tax revenue, a testament, perhaps, to the infrequency of such a success!

> śrīpadmanābha . . . yaḥ siddhāntadvayaṃ vetti horāsāgarapāragaḥ | saṃhitāśāstratattvajño varāhamihiropamaḥ ||
> tenāśītyadhikāṣṭavatsaraśate yāte dine gīḥpate kārttikyām atha rohiṇībhasamaye rātreś ca yāmatraye | śrīmadratnanareśvarasya sadasi jyotirvvidām agrataḥ sarvvagrāsam anuṣṇagoḥ pravadatā tīrṇṇā pratijñānadī ||
> iṃdor muktiṃ kurvvatāyaṃ tadānīṃ sarvvādāyair mmaṇḍale 'narghavallyāṃ rājñā tuṣṭenātha ciṃcātalāīgrāmas tasmai śāsanīkṛtya dattaḥ ||
> [the illustrious Padmanābha . . . who knows two siddhāntas, has crossed the ocean of astrology, and being versed in the tenets of the Saṁhitās and sacred writings is like Varāhamihira. He (Padmanābha), declaring in the assembly of the illustrious Ratnadeva (II), in the presence of all astronomers, that when the year eight hundred increased by eighty had passed, on the day of the Lord of Speech (i.e., Thursday), on the full moon day of Kārttika, during the third quarter of the night when (the moon would be in) the constellation Rohiṇī, there would be a complete eclipse of the moon, crossed the river of assertion (i.e., vindicated his prediction). For releasing the moon, the king, who was pleased, gave him as a grant the village Ciñcātalāī in the maṇḍala of Anarghavallī with all taxes.]
> (Sarkho Plates of Ratnadeva II year 880, verses 17–20, as translated by Mirashi)

This predicted eclipse did in fact occur. If we tentatively take the geographical coordinates of Chiñcātalāī (modern Chichōlā), 82° 39E, 22° 10N as (close to) the locality of observation, a lunar eclipse was visible on 8 November 1128 CE. The eclipse, as the inscription remarks, was total (magnitude computed as 1.74), with the phases occurring at 0:40, 1:49, 3:34, and 4:43 respectively. The eclipse indeed began in what would have been the third quarter (*yāma*) of the night.

5.4.1 List of Inscriptions that Mention Solar Eclipses

Below is a list of selected inscriptions that mention solar eclipses. There exist a huge number of inscriptions with eclipses in them, and to cover all of them is beyond the scope of this work. Presented here are those inscriptions excerpted

from the six-volume work *Corpus Inscriptionum Indicarum* (henceforth CII) that are datable. The rationale behind focusing on solar eclipses exclusively is because they are less straightforward to compute and predict than lunar eclipses, and thus are more revealing.

Inscriptional evidence is vital in the greater field of eclipse reckoning. It shows both directly and indirectly the ways in which practitioners actually used the developing techniques and methods presented in the textual sources. Furthermore, it is important as evidence of the accuracy of eclipse reckoning techniques in India.

In order to recompute the circumstances of the eclipses, the approximate locality of where the inscription was found has been used, rather than the village that was given as a gift as a result of the eclipse. Although CII gives exact locations of small towns, the coordinates from the closest major town as recorded in the *Times Atlas* have been used. There is a mixture of predictions and observations in the list. By comparing them against modern calculations, it is possible to determine whether they have been computed or observed in many cases.

Nagardhan Plates of Svāmirāja (*CII* IV, 120, p. 611 l. 13ff.)

Kalachuri 322
Solar eclipse
Caitra, new-moon day, (cyclic?) year Āṣāḍha

This corresponds to the date 19 March 573 in which the month, the cyclic year, and the occurrence of a solar eclipse visible in this region are in agreement. The coordinates used are for Nagpur (79.12E, 21.10N) in Madhya Pradesh and the eclipse was almost total (magnitude 0.921). In comparison, the date of the inscription is given as the 5th tithi of the bright fortnight of Kārtikā Kalachuri year 322, eight months later.

Kasare Plates of Allasakti (*CII* IV, 25, p. 110 l. 31–33)

Kalachuri 404 (current)
Solar eclipse
Āṣāḍha, dark fortnight, new-moon day

This corresponds to the date 1 June 653 when a partial solar eclipse occurred (magnitude 0.546). The coordinates used are for Dhulia (74.50E, 20.52N).

Janjirā Plates of Aparājita (*CII* VI, 5, p. 17)

Śaka 915 (elapsed)
Solar eclipse

Śrāvana, dark fortnight, cyclic year Vijaya, fifteenth tithi, Sunday, the Sun in the rāśi of Siṁha

See the previous section for analysis.

Janjirā Plates of Aparājita (*CII* VI, 6, p. 28)

Śaka 915

This refers to the same eclipse as the previous inscription, but here it is described in less detail.

Kundeśvara Copper-Plate inscription of the time of Vidyādharadeva (*CII* VII, 201, p. 651)

Vikrama 1060
Solar eclipse
Śrāvaṇa, Amāvāsyā

This inscription refers to the partial solar eclipse that occurred on 20 July 1004 (magnitude 0.297). The coordinates used are for Tikamargh (78.50E, 24.44N).

Bhāṇḍup Plates of Chittrarāja (*CII* VI, 9, p. 54)

Śaka 948 (elapsed)
Solar eclipse
Kārttika bright fortnight, fifteenth tithi, cyclic year Kṣaya

There is an immediate problem with this inscription: a solar eclipse cannot occur in the bright fortnight! This error is repeated twice within the inscription. It may be that the author intends to refer instead to a partial *lunar* eclipse (magnitude 0.86) which occurred on Sunday 30 October 1026, two days after the date of the inscription. The coordinates used are for Bombay (72.51E, 18.56N).

Ṭhāṇā Plates of Nāgārjuna (*CII* VI, 13, p. 75 l. 54ff.)

Śaka 961 (elapsed)
Solar eclipse
Śrāvaṇa, dark fortnight, fifteenth tithi, Wednesday, cyclic year Pramāthin

The date of the inscription corresponds to 27 August 1039; however, it seems to refer to a solar eclipse that occurred on 22 August 1039 (magnitude 0.563). The coordinates used are for Thāṇā (73.02E, 19.14N).

Rājpur Copper-Plate Charter of the Paramāraraṇadhavala
(*CII* VII, 198, p. 640 l. 16–18)

Vikrama 1148
Solar eclipse
Pauṣa, dark fortnight, fifteenth tithi, year 1148

The date of this inscription corresponds to 25 December 1091, but no eclipse was visible. The coordinates used are for Rājpur (75.09E, 21.58N) in Madhya Pradesh.

Paragaon Plates of Ratnadeva II (*CII* IV, 122, p. 622 l. 15ff.)

Kalachuri 885
Solar eclipse
Kārttika, Āśvina, bright fortnight, first lunar day, Wednesday

The inscription may be referring to the solar eclipse that occurred on 23 July 1134 which was almost total (magnitude 0.843) and the reference to the bright fortnight must be a mistake. The coordinates used are for Baloda Bazar (82.13E, 21.41N).

Chānje Stone Inscription of Aparāditya I (*CII* VI, 22, p. 130 l. 14ff.)

Śaka 1060
Solar eclipse
Māgha, bright fortnight

The date of this inscription corresponds to 3 January 1139. There was no solar eclipse close to this date, even taking into account that the reference to the bright fortnight must be a mistake. The closest one is on 15 November 1137, when a partial solar eclipse occurred (magnitude 0.690). The coordinates used are for Uraṇ (72.59E, 18.51N).

Bilaigarh Plates of Pṛthvīdeva II (*CII* IV, 89, p. 458 l. 16ff)

Kalachuri 896
Solar eclipse
No more information on the date is given

In Kalachuri 896 (= 1144/45 CE) there occurred two solar eclipses: 26 December 1144 and 22 June 1145. The coordinates used are for Bilaigarh (82.47E, 21.37N). The first was not visible for the locality. The second was visible and was almost total (magnitude 0.906).

Koni Stone Inscription of Pṛthīdeva II (*CII* IV, 90, p. 463 l. 35ff.)

Kalachuri 900
Solar eclipse

No solar eclipse was visible in the year of the inscription. The coordinates used are for Bilaspur (82.12E, 22.03N).

Bassein Stone Inscription of Mallikārjuna (*CII* VI, 29, p. 153 l. 1ff.)

Śaka 1083
Solar eclipse
Pushya (Pausha) dark fortnight, fifteenth tithi, Monday, cyclic year Vṛṣa

This eclipse probably corresponds to the one occurring on 17 January 1162 (magnitude 0.803). The coordinates used are for Bassein (72.52E, 19.21N). The weekday was Wednesday and not Monday, however.

Semrā Copper-Plate Grant of Paramādideva (*CII* VII, 126, p. 418)

Vikrama 1223
Solar eclipse
Māgha, dark fortnight, fifteenth tithi, Thursday

The date of the inscription refers to Thursday 15 February 1162. However, a solar eclipse did not occur on this date; one did occur at the preceding conjunction, Wednesday 17 January 1162 (magnitude 0.882). The coordinates used are for Chatarpur (79.35E, 25.54N).

Jabalpur Stone Inscription of Jayasiṃha (*CII* IV, 64, p. 331)

Kalachuri 926
Solar eclipse

There was a solar eclipse in the year of this inscription on 26 November 1174 (magnitude 0.956). The coordinates for the locality used are for Jabalpur (79.59E, 23.10N).

Pipliānagar Copper-Plate Inscription of Mahākumāra Hariścandra (*CII* VII, 45, p. 152)

Vikrama 1235–6 (elapsed)
Solar eclipse
Pauṣa, new-moon day

There were two eclipses in this year, 20 January and 11 December 1178, but neither of them was visible at this locality. The coordinates used are for Shājāpur (76.46E, 23.21N).

Lōnāḍ Stone Inscription of Aparāditya II (*CII* VI, 30, p. 156 l. 2ff. [date] l. 8ff. [eclipse])

Śaka 1106
Solar eclipse
Kārttika, dark fortnight, fifteenth tithi, Monday, cyclic year Krodhin

This corresponds to the solar eclipse that occurred on 5 November 1184 (magnitude 0.827). The coordinates used are for Biwaṇḍi (73.08E, 23.21N).

Kutāpur Grant of Bhoja II (*CII* VI, 64, p. 282)

Śaka 1113 (elapsed)
Solar eclipse
Mārgaśīrṣa, new-moon tithi, cyclic year Virodhakṛt

This corresponds to the solar eclipse that occurred on 18 December 1191, but was not visible at this locality. The coordinates used are for Kolhapur (74.19E, 16.42N).

Khidrāpur Stone Inscription of the Yādava King Siṅghaṇa (*CII* VI, 65, p. 287 l. 8)

Śaka 1136
Solar eclipse
Caitra, Monday, cyclic year Śrīmukha

This corresponds to the solar eclipse that occurred on 22 April 1213 (magnitude 0.971). The coordinates used are for Kolhapur (74.19E, 16.42N).

Sehore Copper-Plate Inscription of Arjunavarman (*CII* VII, 48, p. 166)

Vikrama 1270
Solar eclipse
Vaiśākha, new-moon tithi of the dark fortnight

This corresponds to the solar eclipse that occurred on 22 April 1213 (magnitude 0.814). The coordinates used are for Sehore (77.08E, 23.12N).

Tahankāpār Plate of Pamparājadeva (*CII* IV, 117, p. 599 l. 9ff.)

Solar eclipse
Kārttika, in nakṣatra Citrā, in cyclic year Iśvara

This corresponds to the solar eclipse that occurred on 5 October 1214 (magnitude 0.333) but the cyclic year is not in agreement. The coordinates used are for Kanker (81.30E, 20.17N).

Chānje Stone Inscription of Someśvara (*CII* VI, 39, p. 175 l. 8ff.)

Śaka 1182
Solar eclipse
Caitra, dark fortnight, fifteenth tithi, Monday, cyclic year Raudra

This corresponds to the solar eclipse that occurred on Monday 12 April 1260 (magnitude 0.369). The coordinates for the locality used are for Uraṇ (72.59E, 18.51N).

5.5 Eclipse Reckoning in Indian Texts with Greco-Babylonian Influence: The *Pañcasiddhāntikā* of Varāhamihira

In the *Bṛhatsaṃhitā*,[45] Varāhamihira advises astrologers to study "the five siddhāntas" (treatises), namely, the *Pauliśa*, the *Romaka*, the *Vāsiṣṭha*, the *Sūrya*, and the *Paitāmaha*. Perhaps with this purpose in mind, he compiled these five *siddhānta*s into one work, entitled the *Pañcasiddhāntikā*, composed some time in the sixth century of the common era.

Varāhamihira's work is important not only because of its relationship with Babylonian and Greek sources, but also because it contains various parameters and techniques from several different systems, all within the same composition. While there is some overlap between the parameters and techniques from the various *siddhānta*s, there also exist some very fundamental differences, such as the measures of the diameters of the luminaries and the obliquity of the lunar orbit. The intention, then, was not to create a work presenting a single unified treatment of astronomical themes, but rather to document several different approaches, set out side by side. Rather than a prescriptive single-authority treatise, the text offers many options.

Varāhamihira (I,3) reveals that his knowledge of the first two *siddhānta*s, the *Pauliśa* and the *Romaka*, was through recensions by a pupil of Āryabhaṭa, Lāṭadeva (fl. ca. 505 CE), who knew of them from an unidentified earlier source, and had corrected both of them to a common contemporary epoch (21 March 505), among other changes which are impossible to determine. The origin of these two *siddhānta*s is, as has been demonstrated,[46] undoubtedly from Greek and Babylonian sources.

The treatment of eclipse theory in these two *siddhānta*s reveals a fairly simplistic method of predicting eclipses. Computations concerning lunar eclipses

in the *Pauliśa* (VI, 1–15) and solar eclipses in the *Pauliśa* (VII 1–6)[47] and in the *Romaka* VIII (1–18) point to a Hellenistic origin with Babylonian elements. Because these two *siddhānta*s include rules for the computation of parallax, they are already based on the spherical model, but do not yet employ the more sophisticated spherical trigonometrical rules that were developed by the Greeks and seen in the *Sūryasiddhānta*. What this suggests is that there were (at least) two transmissions of Greek material to India, one in the second or third century CE, which included pre-Ptolemaic astronomy, and later a fuller transmission of the spherical universe, still pre-Ptolemaic, but much more sophisticated, and used by Indian authors in the fifth century CE.

The following thirteen subjects[48] are covered in the various chapters: opposition and conjunction (1), diameters (2), geocentric distances (3), eclipse limits (4), longitudinal (5) and latitudinal (6) parallax and their combination (7), duration (8), magnitude (9), angle of inclination (10), colors (11), direction of impact (12), and methods of graphical projection (13).

5.5.1 *The Pauliśasiddhānta*

The *Pauliśasiddhānta* begins its section on lunar eclipses with a rule to compute the moment of opposition of the sun and the moon (VI, 1). Their longitudes are to be computed for the time of the sunrise *after* the opposition occurred. Then the distance between the moon and the center of the shadow is sought. This distance ($\Delta\lambda$) is computed from the longitudes of the moon (λ_m) and the sun (λ_s):

$$\Delta\lambda = (\lambda_m - 180^\circ) - \lambda_s.$$

This elongation is converted into a time difference measured in *nāḍī*s so it can be directly applied to the time of sunrise. The elongation is to be multiplied by 5, i.e., the equivalent of dividing by 0;12. The value 0;12° is an approximation of the increase in the elongation between the sun and the moon per *nāḍī*, that is, the difference in velocities ($\approx [\nu_m - \nu_s] = [0;13 - 0;01]^\circ$ per *nāḍī*). The relationship between velocity, distance, and time is used to convert the distance of the moon from the center of the shadow into a time difference Δt measured in *nāḍī*s:

$$\Delta t = 5\Delta\lambda = \frac{1}{0;12}\Delta\lambda.$$

In this way, the longitude of the moon at opposition can be found at Δt *nāḍī*s before sunrise.

The eclipse limits are given by the second verse of chapter VI, in which the position of the node is increased by 0;36° or 0;26°. This ambiguity derives directly from the reading of the Sanskrit compound *ṣaṭkṛtikalāṃ*, which can

equally be translated as the "square of six minutes," which is 36, or equally "two-six minutes" (=26).

> rāhoḥ saṣaṭkṛtikalāṃ hitvāṃśaṃ tacchaśāṅkavivarāṃśaiḥ |
> grahaṇaṃ trayodaśāntaḥ paṃcadaśān tas tamas tasya ||
> [Put down the degree of the ascending node increased by 36 (or 26) minutes. (Operate) with the degrees of difference between this and (the longitude of) the Moon; if they are within 13°, there is an eclipse, and if within 15°, a darkening of it.]
> (Varāhamihara *Pañcasiddhāntikā* VI, 2, as translated by Pingree [1970])

From this modified nodal position ($\lambda_n{}' = \lambda_n + 0;36^\circ$ or $0;26^\circ$), the following eclipse limits can be derived:

$$|\lambda_n{}' - \lambda_m| < 13^\circ.$$

In other words,

$$|\lambda_n + 0;36 \text{ or } 0;26 - \lambda_m| < 13^\circ,$$
$$|\lambda_n - \lambda_m| < 13^\circ \pm 0;36 \text{ or } 0;26.$$

The purpose of this shift in the nodal position is unclear. An eclipse limit should be symmetrical about the node and the addition of an increment skews the boundaries to one side of the node. Perhaps this value is not meant to be applied to the eclipse limit but to account for some other correction with respect to the node. Indeed, $13;36^\circ$ is a poor eclipse limit—$12;24^\circ$ is better. Furthermore, there is no allowance for the fact that the limits are somewhat dependent on lunar and solar anomaly.

Lastly, the verse states that if the nodal elongation is between 13° and 15° the moon is said only to be "darkened" (*tamas*). It seems only natural to assume that a penumbral eclipse is meant. It has been observed[49] that this type of eclipse is seemingly otherwise unknown in ancient astronomy. In his *Bṛhatsaṃhitā*, Varāhamihira enumerates ten different types of eclipses:[50]

Type	Literal translation
Savya	Southern
Apasavya	Northern
Leha	"Licking"
Grasana	"Swallowing"
Nirodha	"Blocking"
Avamardana	"Crushing"
Āroha	"Mounting"
Āghrāta	"Smelling" or "satiated"
Madhyatamas	Darkness in the center
Tamontya	Darkness at the end (or edge?)

Most of these are fantasies, such as *āroha* (mounting), which is described as a phenomenon in which, after the eclipse ends, the luminary is once again plunged into darkness. However, several seem to indicate true observable phenomena, such as *madhyatamas* (darkness in the center), which in all probability refers to an annular solar eclipse. Among these a penumbral eclipse does not seem to be included.

These limits allow one to determine whether a particular syzygy will produce an eclipse, but beyond this give no further information about the eclipse. In fact, in all the different systems presented in the *Pañcasiddhāntikā*, these limits are the only items given with respect to determining eclipse possibilities. Nowhere in this text is there any mention of a pattern of eclipse possibilities in the calendar.

Despite the fact that necessary parameters (radii) or concepts (lunar latitude) have *not* been introduced, nonetheless a rule to compute the duration of the eclipse (Δt), which uses this information, is given next. The rule relies on the assumption that the lunar orbit is parallel to the ecliptic. It is based on applying the Pythagorean theorem to an approximating plane triangle, the hypotenuse of which is the sum of the radii of the moon and the shadow, and one of the sides the lunar latitude (fig. 5.2):

$$PQ = \sqrt{(r_m + r_u)^2 - \beta^2}.$$

This is multiplied by 2 to give the total duration and converted into time by dividing by the increase in elongation of the sun and the moon (0;12 *nāḍīs*).

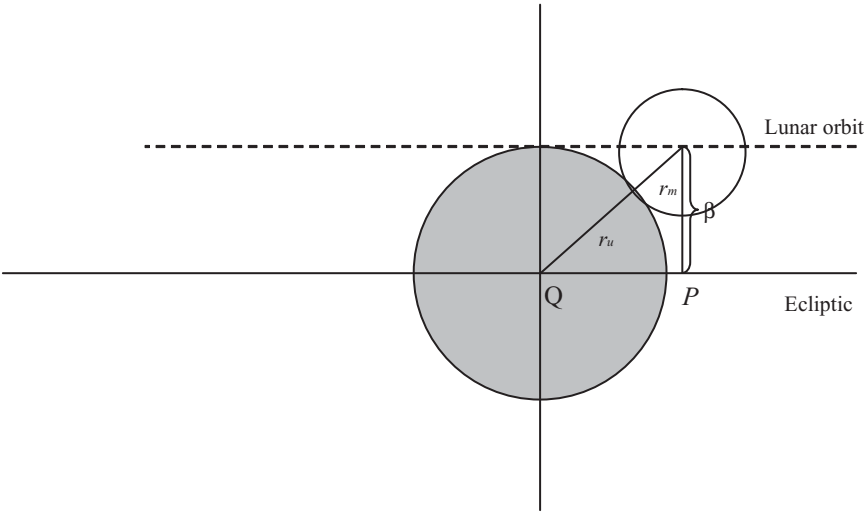


Figure 5.2 The rule for the duration of an eclipse.

The verse gives a rule, then, that can be mathematically expressed as

$$\Delta t = \frac{2}{(v_m - v_s)}\sqrt{(0;55)^2 - \beta^2}.$$

The individual values of the radii of the moon and the shadow are not given until later (VI, 12). Here simply their sum (0;55) is seen.

There may be an alternative way to understand this rule, where the lunar orbit is not considered parallel to the ecliptic (as in fig. 5.2), a simplifying assumption that was frequently made, but rather is inclined to the ecliptic (fig. 5.3). If the lunar latitude (β) is taken to be that at mid-eclipse, instead of at the beginning of the eclipse, and if that latitude (β) is considered as the distance perpendicular to the lunar orbit and *not* to the ecliptic, it can be argued that the lunar orbit was considered as being inclined to the ecliptic for the computation of the duration.[51]

The next verse presents challenges with respect to its interpretation.

> śaśitimiravivarabhāgais trayodaśonāḥ śarāhatāḥ kṣepyāḥ |
> sthityā vināḍikāstā rāhāv adhike 'nyathā hāniḥ ||
> [Subtract the degrees of the difference between (the longitudes of) the moon and the node from 13° and multiply (the result) by 5. The (resulting) *nāḍīkā*s are to be added to the duration of the eclipse if the ascending node is greater (in longitude), otherwise subtracted.]
> (Varāhamihira *Pañcasiddhāntikā* VI, 4, as translated by Pingree)

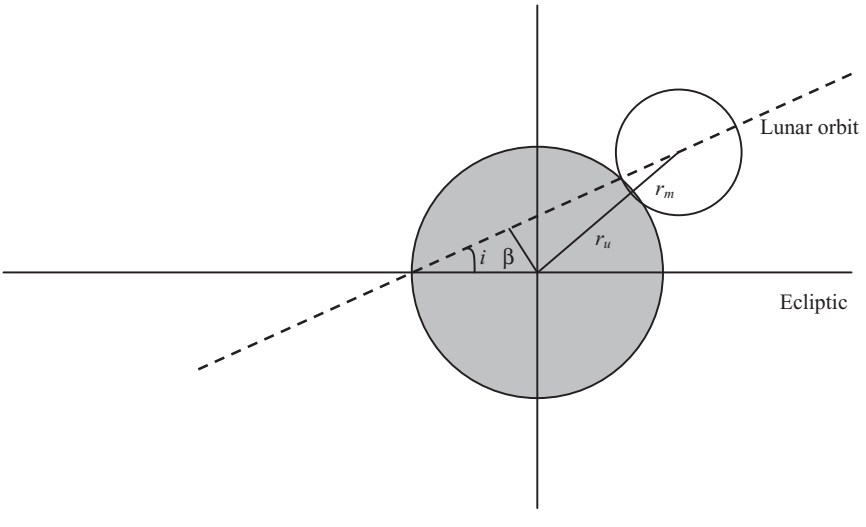


Figure 5.3 An alternative way to consider the computation of the duration of an eclipse.

This seems to imply a calculation of longitudinal difference $\Delta\lambda$:

$$\Delta\lambda = 13^\circ - (\lambda_n - \lambda_m).$$

The multiplication by 5 (equivalent to division by 0;12) converts this distance into a time interval:

$$\Delta t = 5.\Delta\lambda = 5(13^\circ - (\lambda_n - \lambda_m)).$$

The time interval Δt is to be added to or subtracted from the duration of the eclipse. It is unclear what this is intended to capture.

A rule for the duration of totality (Δt_{tot}) is given next (VI, 5). The quantity represented here by $\Delta\lambda^*$ is simply referred to in the verse as "difference"—it may make best sense to interpret this as the difference in longitude between the center of the shadow and the node.[52] The verse describes the following arithmetical operations:

$$\Delta t_{\text{tot}} = \frac{21}{5}\sqrt{4(5 - \Delta\lambda^*)[10 - (5 - \Delta\lambda^*)]}.$$

The form of the above expression is somewhat involved, considering that it can be simplified to

$$\Delta t_{\text{tot}} = \frac{2 \times 21}{5}\sqrt{5^2 - \Delta\lambda^{*2}}.$$

The original statement of this rule in such a form may be an indication of how it was initially derived. What is clear, though, is that the simplifying assumption that the lunar orbit is parallel to the ecliptic has *not* been made here.

The half-duration of the partial phase is next defined as the difference between half of the above computed duration Δt and the half-duration of totality $\frac{\Delta t_{\text{tot}}}{2}$. In this same verse the "direction" of first and last contact is said to be found from the difference between the (longitudes of) the moon and the node. Perhaps what is intended here is the relation between the positions of the moon and the node. Since it is always the eastern side of the disk of the moon that is eclipsed first, in the case of a descending node, if the first contact occurs *before* the moon has crossed the node, then the direction of impact is more southerly, and if the first contact occurs *after* the moon has crossed the node, the direction of impact is more northerly.

The next verse describes the division of the disk of the moon by means of thirteen lines. Thirteen lines will give twelve spaces which may be related to the Mesopotamian division of the (whole) disk of the moon into twelve fingers, and the Greek tradition which measures the magnitude of an eclipse in twelfths of the lunar diameter.

Following this, the calculation of the angle of "deflection" (Sanskrit *valana*) of an eclipse is given. The *valana* is an astrological concept which most likely has

its origins in the Mesopotamian omen tradition (see section 3.2), in which the quadrant of the moon or sun that is impacted is significant for the outcome of the prognostication. Such a concept is also seen in Ptolemy's theory of eclipses in the more advanced form of two separate angle measurements, the so-called *prosneúseis* (see section 4.6). In the Indian system, the *valana* is the angle (γ) between the ecliptic and a set of orthogonal vectors, the "north-south" and "east-west" lines. These vectors are projections through the eclipsed body of the great meridian circle going through the north and south points on the horizon and their east-west complements. Such an angle has no usefulness astronomically.

This angle, for computational purposes, was divided into two separate angles: the first, called the *akṣavalana*, is the component of deflection due to the latitude of the place of observation and the other, the *ayanavalana*, is the component due to the position of the bodies on the ecliptic. In the *Pauliśa* (VI, 8), the *akṣavalana* is computed as follows:[53]

> śaśiparidhidalārdhatthe svendvantarabhāgasaṅguṇe cākṣe |
> svasvarūpāṣṭahute prāgvalanaṃ vāmaṃ pare savyam ||
> [Multiply a quadrant of the Moon (i.e., 90°) by the terrestrial latitude and multiply (the product) by the degrees of difference between (the longitudes of) (mid)-heaven and the Moon; divide (the product) by 8100. The (resulting) deflection is to the north (if the Moon is) in the east, to the south (if the Moon is) in the west.]
> (Varāhamihira *Pañcasiddhāntikā* VI, 8, as translated by Pingree)

The following formula can be extracted from the verse:

$$\gamma_1 = \frac{90^\circ \times \phi \times (\lambda_M - \lambda_m)}{8100},$$

where λ_M is the longitude of the point at which the ecliptic crosses the meridian (the meridian ecliptic point). The *Pauliśa* here mentions only the moon, since chapter VI deals with lunar eclipses; for solar eclipses, one would use λ_s instead of λ_m.

This formulation gives two extremes. One extreme is when the eclipse occurs at the meridian ecliptic point, so that the difference of λ_M and λ_m is zero, and hence $\gamma_1 = 0$. The other extreme is when the eclipse occurs at the horizon, and $\gamma_1 = \phi$. In this latter case, the *akṣavalana* is the component of the total deflection that is dependent on the particular latitude of the place of observation.

Next, the colors related to a lunar eclipse are given in VI, 9–10. The idea of ascribing different colors to eclipses can be found as early as Mesopotamian omen texts, such as the Babylonian series *Enūma Anu Enlil* (Tablets 15–18;

(see section 3.2). In such texts, the colors of the eclipse were restricted to four, each of which had a specific impact on the outcome of the omen.[54] But in the *Pauliśa* the color variations are curiously attributed to the *position* of the eclipse in the night sky. This is in contrast to later traditions in which the colors are dependent on the *phase* of the eclipse. Here, the criteria are the closeness of the eclipse to the ascendant or descendant, or its altitude, or its magnitude.

Sanskrit	Color	Criteria
rakta	Bloodred	"Up" especially in west
kapila	Reddish brown (literally monkey colored)	"Down" especially in west
dhūmra	Smoky	Partial
varṇaviśeṣa	Diversity of colors	Total

The last color, the "diversity of colors" may be related to the Mesopotamian "variegated" (GÚN). No colors are given for solar eclipses in this or any of the other *siddhānta*s contained in the *Pañcasiddhāntikā*.

The rule following this is unclear. It purports to give the direction of impact of the eclipse, reckoning this to be related to the longitudinal difference between the moon and the node, multiplied by a constant:

> rāhuṃ mukhonacakraṃ triyamadviguṇaṃ śaśi<*hīna*> saṃyuktam |
> abhikleśo 'yamuccaḥ kriyādiḥ kanyāntago nīcaḥ ||
> [(The longitude of) the ascending node, (or) 360° diminished by (the longitude of) the ascending node, multiplied by 223, is diminished or increased by (the longitude of) the Moon; the result is (the direction of) impact (of the eclipse). It is high (if the Moon is) at the beginning of Aries, low if it is at the end of Virgo.]
> (Varāhamihira *Pañcasiddhāntikā* VI, 11, as translated by Pingree).

This procedure appears to attempt to quantify direction of impact as dependent on the positions of the node and the moon on the ecliptic and relates this direction to the presence of the moon in various zodiacal signs. It may be that this is a rule for the *ayanavalana*, the component of deflection due to the position of the bodies on the ecliptic. From descriptions in other texts, this component is maximized at the equinoxes and minimized at the solstices, which is in keeping with the description given above. However, the parameters and constants used in the above description do not appear to fit. 223^{55} is the number of synodic months in 18 years, the length of the Saros eclipse cycle, but its inclusion in the present context serves no evident purpose.

The next four verses (VI 12–15) give details for the projection or graphical representation of an eclipse. Three circles concentric with radii equal to that

of the moon, that of the shadow, and their sum respectively are to be drawn. Here, the first two radii are explicitly given (VI, 12) as

$$r_m = 0;17^\circ,$$
$$r_u = 0;38^\circ.$$

Verse 13 again makes reference to the disk of the moon being divided by thirteen lines "having equal intervals (between them)" which again suggests the interpretation of the disk being divided into twelve sections. Alternatively, it may somehow represent the thirteen degrees that are the eclipse limit as given in VI, 2 and VI, 4. Finally, the last verse of this chapter sets out the fact that at first contact the eastern part of the moon's disk is obscured first, and likewise in a solar eclipse.

The next chapter, chapter VII, gives the rules concerning computation of solar eclipses according to the *Pauliśa* in just six verses. Included in these rules are those for the computation of parallax. The first rule in this section of the *Pauliśa* is a method for calculating longitudinal parallax. It is referred to here by the phrase "displacement of the tithi" (*tither nāmaḥ*) which refers to the fact that the longitudinal parallax affects the time of the eclipse.[56] For this reason, this component of parallax is measured in *nāḍī*s and is a function of the hour-angle ($\Delta\tau$) that is, the depression from the meridian (fig. 5.4). The general principle behind this rule is that the maximum parallax is taken to be 4 *nāḍī*s and occurs when the moon is at its ascendant on the horizon, and the minimum value, $\pi_\lambda = 0$, occurs when the moon is on the meridian (as the hour-angle

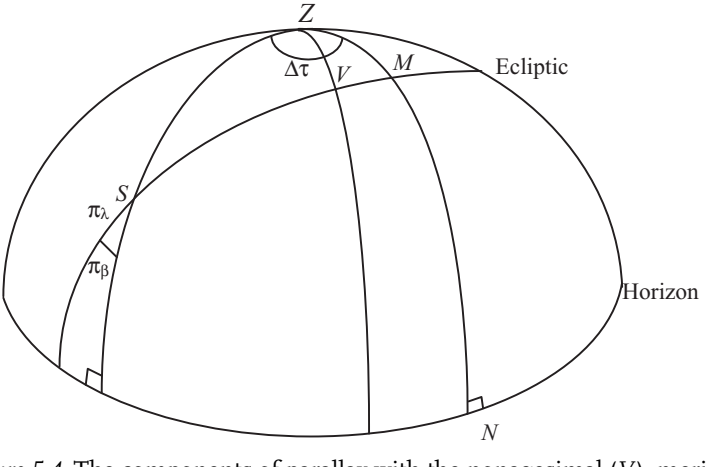


Figure 5.4 The components of parallax with the nonagesimal (V), meridian ecliptic point (M), position of the eclipse (S), and the hour-angle ($\Delta\tau$).

[$\Delta\tau$] is measured from the meridian). For the intermediary values, the longitudinal parallax is varied sinusoidally (1 *nāḍī* = 6°):

$$\pi_\lambda = 4\,\sin(\Delta\tau^\circ) = \frac{\mathrm{Sin}(\Delta\tau^\circ)}{30} = \frac{\mathrm{Sin}(6\Delta\tau^n)}{30} n\bar{a}ḍ\bar{\imath}\mathrm{s}.$$

This procedure assumes that the meridian ecliptic point (the point where the meridian intersects with the ecliptic) is 90° from the horizon along the ecliptic, which is not always the case. In order to vary this component of parallax sinusoidally, a 90° arc is needed. The meridian ecliptic point is not usually 90° from the ascendant; the nonagesimal is always 90° away from it, by definition. In later texts, this measure was replaced by the nonagesimal.

At any rate, the significance of this formulation of longitudinal parallax in the *Pauliśa* is that it is not a function of the distance of the body from the zenith. More conveniently, longitudinal parallax is taken as a function of the moon's distance from the meridian. This is far more straightforward to find.

Next the latitudinal component of parallax is calculated. As with longitudinal parallax, it is computed according to the following basic relation in which parallax varies between its two extremes sinusoidally:

$$\pi_\beta = 4 \cdot \sin z(V).$$

Here $z(V)$ is the zenith distance of the nonagesimal. The use of the zenith distance of the nonagesimal and not that of the moon is significant. As is apparent from this rule, the maximum latitudinal parallax occurs when the nonagesmial is on the horizon, and parallax decreases as the nonagesimal gets closer to the zenith distance. In other words, the latitudinal parallax is *independent* of where exactly the moon is with respect to the zenith. It is determined by the position of the nonagesimal, and not the position of the body, which could be anywhere along the ecliptic. In every *siddhānta* in the *Pañcasiddhāntikā*, this basic assumption is the same, but the parameters each *siddhānta* uses to compute this basic relation and the methods utilized to determine the value $z(V)$ are different.

According to the *Pauliśa* (VII, 2), latitudinal parallax, in *nāḍī*s, is given by the rule:

$$\pi_\beta = 4^{n\bar{a}ḍ\bar{\imath}\mathrm{s}} \cdot \frac{5}{23} \cdot \frac{\mathrm{Sin}(z(V))}{2},$$

where 4 *nāḍī*s is the maximum parallax and $^5/_{23}$ is an approximation to 0;13° (i.e., $0;4^{\mathrm{days}} \times 13 = 4 \times 0;13 = 4 \times {}^5/_{23}$; 13° is an approximation of the mean daily lunar velocity). The maximum parallax according to this is therefore $\approx 0;52°$.

The zenith distance $z(V)$ of the nonagesimal according to the *Pauliśa* is to be calculated by taking the sum of the lunar latitude, the terrestrial latitude,

and the declination of the nonagesimal (here in VII, 2 indicated by the phrase "a fourth of the zodiacal signs" which refers to the fact that the nonagesimal is 90° from the ascendant along the ecliptic) (fig. 5.5):

$$z(V) = \beta_m \pm \phi \pm \delta(V).$$

This is not technically the zenith distance of the nonagesimal. First the declination of the nonagesimal has been approximated by the declination of the meridian ecliptic point (M):

$$\delta(V) \approx \delta(M),$$

where $\delta(M)$ is to be found presumably from a table of declinations. Second, by addition or subtraction of the lunar latitude an amount closer to the zenith distance of the moon is being found.

Rules for applying parallax given the orientation of the celestial bodies are stated next (VII, 3). At the ascending node,

In northern *ayana*	"Positive" in the east
	"Negative" in the west
In southern *ayana*	"Negative" in the east
	"Positive" in the west

and the inverse at the descending node.

The author does not state which component of parallax (or even their combination) he had in mind. However, the proximity of this scheme to the verse on the component of latitudinal parallax combined with the reference to the nodes may suggest that the author had only the component of latitudinal parallax in mind.

Unfortunately, no matter which way it is interpreted, the verse presents a system that is incorrect. First of all, the reference to the *ayana* is meaningless.

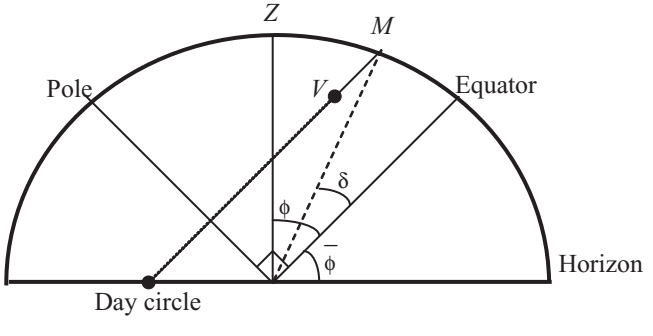


Figure 5.5 An analemma showing the computation of the zenith distance of the nonagesimal.

Parallax is independent of where the body is oriented with reference to this division north or south of the celestial equator. Furthermore, the net effect of the longitudinal and latitudinal parallax is to push the position of the body always toward the horizon. As discussed in more detail previously (in the discussion of VII, 1), a convention particular to the Indian system is that it is *not* the meridian which is the dividing line (the midpoint) with respect to the direction of the parallax, but rather the nonagesimal (as will be seen below). The above system is right insofar as the parallax in longitude is negative toward the west of the nonagesimal and positive toward the east. However, the parallax in latitude will always have a negative effect, contrary to the text.

Ptolemy has a section on the application of parallaxes (V, 19), which seems similar to the above:

> For the latitudinal parallax, when the zenith is to the north of the point of the ecliptic then culminating on the meridian, the [effect of the] parallax will be towards the south of it [the ecliptic]; but when the zenith is to the south of the culminating point, [the effect of] the parallax in latitude will be towards the north. For the longitudinal parallax: the angles tabulated in the Table represent the northernmost of the two angles cut off to the rear of the intersection of the ecliptic [and altitude circle]. Therefore, when the latitudinal parallax is to the north, if the angle in question is greater than a right angle, the effect of the longitudinal parallax will be in advance [i.e., in reverse order] of the signs, but if the angle is less than a right angle, the effect will be towards the rear [i.e., in the order of the signs]. However, when the latitudinal parallax is to the south the reverse will be true: if the angle in question is greater than a right angle, the longitudinal parallax will be towards the rear [i.e., in the order] of the signs, but if it is less than a right angle, the longitudinal parallax will be in advance.
>
> (Ptolemy *Almagest* V, 19, as translated by Toomer [1984], pp. 266–67)

It may be that Indian scholars were attempting to reproduce such trends.

The purpose of the next verse in the *Pauliśa* is somewhat unclear. It sets out the following formula:

$$\frac{\Delta t \times \delta_m}{80},$$

where Δt is the time interval between syzygy and sunrise or sunset, measured in *nāḍī*s, and δ_m is the declination of the moon. This result is said to be positive in the zodiacal signs beginning with Aries, and negative in those beginning with Libra, with the inverse to be applied at a descending node. The purpose of this rule and exactly what it quantifies remain mysterious.

The following verse gives rules for the limits of both lunar and solar eclipses. As in VI, 2, the longitude of the node is to be increased by either 36 or 26 minutes to give the modified nodal position λ'_n, and the limits for a lunar and a solar eclipse are, respectively,

$$|\lambda'_n - \lambda_m| < 13^\circ,$$
$$|\lambda'_n - \lambda_s| < 8^\circ.$$

Nothing is said about how parallax might affect these limits.

Finally, the chapter closes with the times of duration for both lunar and solar eclipses, Δt_m and Δt_s respectively (in which $\Delta\lambda$ in the case of a lunar eclipse is the distance between the center of the shadow and the node, and in the case of a solar, the distance between the center of the sun and the node):

$$\Delta t_m = \frac{3}{4}\sqrt{(13)^2 - \Delta\lambda^2},$$
$$\Delta t_s = \frac{3}{4}\sqrt{(8)^2 - \Delta\lambda^2}.$$

Notably, the equation for the duration of the lunar eclipse here is different from the one given in VI, 3, and the factor ¾ in both formulations remains a mystery. Again, nothing is specified as to how parallax might affect these limits.

5.5.2 *The* Romakasiddhānta

The treatment of eclipse theory in the *Romaka* system is very similar to that of the *Pauliśa*, especially the theory of parallax. In fact, several verses are identical. This similarity may suggest that they both originated from a single source or identical set of sources, probably Greek. Chapter VIII of the *Pañcasiddhāntikā* presents the solar eclipse theory of the *Romaka*. Verses 1–8 deal with longitudes and mean motions. Verse 9 is on the parallax in longitude and is identical to the one given in the *Pauliśa* (VII, 1).

However, the *Romaka* (VIII, 10–14) has a more complicated method to determine the component of latitudinal parallax. One is first instructed to determine the longitude of the ascendant [$\lambda(H)$], and from that, by the addition of 270° (the same as the subtraction of 90° in a circle), the nonagesimal [$\lambda(V)$]:

$$\begin{aligned}\lambda(V) &= \lambda(H) + 9 \times 30^\circ \\ &= \lambda(H) + 270^\circ \\ &= \lambda(H) - 90^\circ.\end{aligned}$$

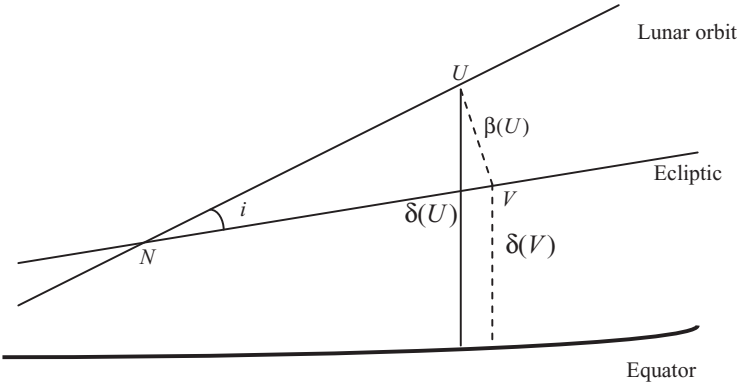


Figure 5.6 An analemma showing the computation of the zenith distance of the nonagesimal.

Once the longitude of the nonagesimal point V is determined, it is trivial to find the declination $\delta(V)$. Then, a point U on the lunar orbit corresponding to V is set up, such that

$$\lambda(V) = \lambda(U),$$

i.e., they are equal in longitude.

The declination of U, $\delta(U)$, is then sought (fig. 5.6). Its computation is based on the following approximation:

$$\delta(U) \approx \delta(V) \pm \beta(U),$$

where the latitude of the point U $[\beta(U)]$ is computed from

$$\beta_U \approx \frac{2}{60}\mathrm{Sin}[\lambda(V) - \lambda_{\mathrm{node}}].$$

With these values determined, the zenith distance of U, $z(U)$, can be found through

$$\begin{aligned} z(U) &\approx \phi \pm \delta(U) \\ &= \phi \pm [\delta(V) \pm \beta(U)]. \end{aligned}$$

This assumes that the nonagesimal V and indeed U lie on the meridian, which more often than not is untrue. However, this approximation is based on the assumption that V is sufficiently close to the meridian.

The remaining parts of VIII, 11 and VIII, 12 give the rules for whether to add (subtract) $\beta(U)$ to (from) $\delta(V)$ and $\delta(U)$ to (from) ϕ depending on the orientation of the lunar orbit with respect to the ecliptic at U and of the ecliptic with respect to the value of ϕ.

Once $z(U)$ has been determined, then the following formula is given for the calculation of latitudinal parallax (VIII, 13):

$$\pi_\beta = \frac{\nu_m \cdot \mathrm{Sin}(z(U))}{1800}.$$

Again this is based on the assumption that parallax is varied sinusoidally between two extremes. Here the scaling factors are explained as follows: $1/1800 = 1/(15 \times 120)$, such that $\frac{\nu_m}{15}$ is the assumed maximum parallax (1/15 of the motion of the planet in a day) and $120 = R$. Here, the latitudinal component of parallax is related to the distance the moon is from the ecliptic, rather than where it is with relation to the horizon.

The true latitude (*sphuṭāvanatiḥ*) of the moon is given (VIII, 14) by the following rule:

$$\beta = \frac{21}{9}\mathrm{Sin}(\lambda_m - \lambda_{\mathrm{node}}).$$

This formula implies that $i \approx 4;30°$ and not $i \approx 4$.[57] The apparent latitude of the moon is then calculated as

$$\beta_a = \beta \pm \pi_\beta$$

Verse 13 gives the mean radii of the moon and the sun as (VIII, 13)

$$\overline{r}_m = 0;17°,$$

$$\overline{r}_s = 0;15°.$$

In addition, the *Romaka* gives the following ratios (VIII, 15) to determine the true diameters:

$$d_m = \overline{d_m} \cdot \frac{\nu_m}{\overline{\nu_m}}$$

and

$$d_s = \overline{d_s} \cdot \frac{\nu_s}{\overline{\nu_s}},$$

a ratio which was used in Greek works[58] for determining the distances of the planets:

$$\frac{d}{\overline{d}} = \frac{\nu}{\overline{\nu}}.$$

This is the earliest Indian use of this ratio.

The next rule gives the duration (Δt) of a solar eclipse:

$$\Delta t = 2\sqrt{(r_s + r_m)^2 - {\beta_a}^2},$$

in which the latitude β_a of the moon is corrected for parallax.

Finally, the *Romaka* section on solar eclipses ends with a method to determine the magnitude of a solar eclipse. VIII, 17 gives a rule to find the "digits" of magnitude at mid-eclipse:

$$\mu = (r_m + r_s) - \beta_a.$$

Thus lunar parallax is taken into account. Verse 18 instructs that the disks of the sun and the moon be drawn in order to compute the magnitude.

Additional material related to eclipses is found in other sections of the *Pañcasiddhāntikā* as well. Chapter XI contains six verses that include graphical means to represent the *valana* or angle of deflection that was mentioned in the *Pauliśa* section (VI, 8). The formulas for both the *akṣavalana* (XI, 2) and the *ayanavalana* (XI, 3) are given.

Chapter 15, the so-called "*Upaniṣad* of Astronomy," also contains material related to eclipses (XV, 1–10), among other phenomena. The tone of this chapter is more cosmological than mathematical. It opens with the following statement:

> sūryendubhagaṇadhātrīsaṃsthānavido 'dhikṛtya kathayāmi |
> grahaṇaṃ sadaiva bhānoḥ sthānaviśeṣāt kacid dṛśyam ||
> [Placing in advance those who know the relative positions of the sun, moon, zodiac, and earth I speak. There is always an eclipse of the Sun; somewhere it is visible because of the difference of localities.]
> (Varāhamihira *Pañcasiddhāntikā* XV, 1, as translated by Pingree)

Although this verse reveals a comprehension of the fact that the observation of a solar eclipse is dependent on the locality of the observer, there is not *always* an eclipse of the sun, if "always" is to be interpreted as "at every moment" and "is visible" refers to somewhere on the earth. A solar eclipse is possible only when the longitudes of the sun and the moon are close to each other, which happens only once a month from the vantage point of the earth. Furthermore, the moon must be close to the ecliptic at this time, which is not always the case at every conjunction because of the lunar latitude (although it is less easy to determine these limits due to parallax). At the least, though, this verse contains the recognition that for solar eclipses, the position of the observer on the earth is crucial to its being seen. A curious statement is given further in the chapter (XV, 3), where the author specifically says that "this locality is somewhere everyday." There is also the statement that solar eclipses are caused by the moon being in front of the sun (XV, 7). This is a correct description of the physical alignment required for a solar eclipse.

5.6 Eclipse Reckoning from the Fifth to the Tenth Century

A new infusion of astronomical ideas, traceable back to Greece, changed the character of Indian mathematical astronomy, probably in the late fourth or early fifth century[59] (see section 5.1). This astronomy was characterized by a fusion of several main elements: Greek spherical models (epicycles and eccenters, trigonometry, and analemmata) and the Indian concept of *Kalpa*s (4,320,000,000-year intervals) and their various subdivisions, a concept that may in fact be Babylonian in origin.[60] The planetary models were pre-Ptolemaic, dominated by the Aristotelian prerequisite of concentricity,[61] and were substantially adapted to fit in with particular Sanskrit traditions and scientific practices. The occurrence within the Greek material of several different planetary models and parameters for the same elements indicates that more than one text was transmitted from Greece.

5.6.1 *The* Paitāmahasiddhānta

The *Paitāmahasiddhānta* (Pitāmaha is one of the many alternative names of Brahmā) is an early fifth-century *siddhānta*. It is the foundational text for the *Brāhmapakṣa*; it is said to have been "revealed" by the God Brahmā to the sage Bhṛgu. The original text is lost, but a recension of it has been included, albeit imperfectly, in a large collection of many works called *Viṣṇudharmottarapurāṇa* (ca. seventh century).[62]

The material pertaining to eclipse theory in the *Paitāmahasiddhānta* is unfortunately scattered, fragmentary, and obviously corrupt in places. Relevant material can be found in III, 8 and V, 2–4 (diameters), as well as IX, 5–7 (parallax). Except for the last two sentences, the whole text is in prose, which also accounts for the fact that the received text is so corrupt. All that remains of lunar eclipse reckoning are rules for the diameters of the sun and the moon, and methods to determine their relative distances with respect to the shadow cone. Fragmentary rules for the computation of parallax cover an obviously incomplete solar eclipse section. In general, though, aspects of these techniques show a certain degree of development from the originally Greek ideas.

The diameters of the disks of the sun and the moon are given in *yojanas* (the metric equivalent of a length on the order of about 10 kilometers). These are (III, 8)

$$d_s = 6500 \text{ } yojanas,$$

$$d_m = 480 \text{ } yojanas.$$

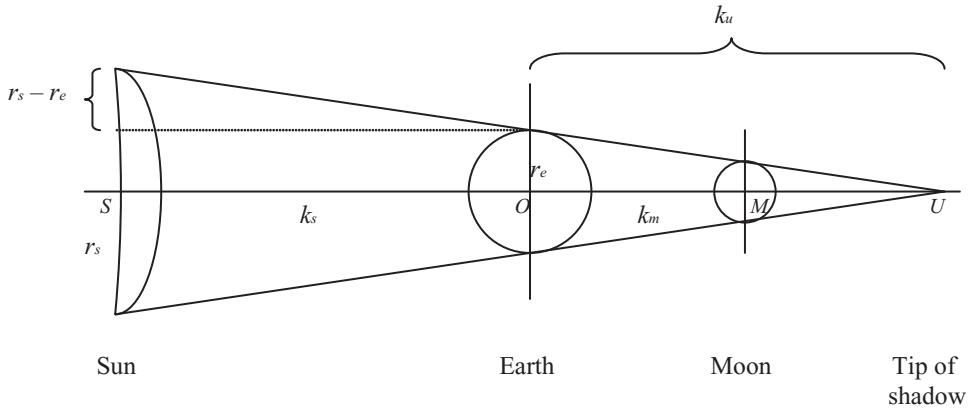


Figure 5.7 The shadow cone; $k_s = OS, k_m = OM, k_u = OU$.

A rule is given later to convert these into minutes of arc (V, 2). The resulting mean orbital radii (in *yojanas*) and diameters (in minutes) are

$$k_m = 512{,}28.89809,$$
$$k_s = 684{,}869.889,$$
$$d_m = 32;12{,}47, \ldots,$$
$$d_s = 32;37{,}46, \ldots.$$

The proportion of the diameters in *yojanas* of the moon and the sun, namely,

$$\frac{6500}{480} \approx \frac{325}{24} \approx 13\frac{1}{2},$$

led the composers of the *Paitāmahasiddhānta* to infer that the sun is about 13 ½ times further away from the earth than is the moon. This ratio is consistent with the rule that the ratio of the diameters is equal to the ratio of the velocities (namely, 13;10/0;59, which is a little under 13½).

The formula for calculating the length (k_u) of the cone of the earth's shadow (that is, the distance between the center of the earth and the tip of its shadow) is given as follows (V, 3) (fig. 5.7) where d_s and d_e ($= 2 \cdot r_e$) are the diameters of the sun and the earth, respectively:

$$k_u = (k_s - r_e) \cdot \frac{d_e}{d_s - d_e}.$$

As has been pointed out by Pingree,[63] this formula is in error. One would expect from similar triangles

$$\frac{k_u}{r_e} = \frac{k_s}{r_s - r_e},$$

$$\frac{k_u}{\frac{1}{2}d_e} = \frac{k_s}{\frac{1}{2}(d_s - d_e)},$$

$$k_u = k_s \cdot \frac{d_e}{d_s - d_e}.$$

He concludes, probably correctly, that the term $(k_s - r_e)$ gives the distance of the shadow cone from the *surface* of the earth and not the center, and this term somehow got injected into the formula.

Following this, a rule for computing the diameter of the earth's shadow at the distance of the moon in minutes is given (V, 4):

$$d_u = \frac{k_m \cdot d_e}{k_u} \cdot \frac{R}{k_m}.$$

Again, this rule seems incomplete. Assuming that the apparent diameter in minutes is inversely proportional to its orbital distance, converting the diameter from the true (in *yojanas*) to the apparent (in minutes) can be achieved as follows:

$$d^{\text{mins}} = d^{yojanas} \cdot \frac{R}{k^{yojanas}}$$

where $R = 3438$. Again, by similar triangles,

$$\frac{d_m}{k_u - k_m} = \frac{d_e}{k_u}.$$

Accordingly, it seems that the correct formulation for the diameter of the earth's shadow at the moon's distance in minutes should be

$$d_m^{\text{mins}} = \frac{(k_u - k_m)}{k_u} \cdot \frac{R}{k_m}.$$

Finally, there is a section on the application of parallax. Some fragmentary sentences give the longitudinal parallax. The verse given here is carefully reconstructed by Pingree[64]:

> Having determined [the longitude of] the Sun, then, as [the longitude of] the Moon [is found] from [that of] the Sun, so [should be found] . . . Add the bhuja[65]; at the tip of this; facing in its own direction is the koṭi.[66]
> Then one should cause this again to end on the eastern [horizon] by means of [the rising-times] till the ascendant; as much as it is, all of it, is diminished by the quotient [?]. Having thus determined the longitudinal parallax, [the result] is [the longitude of] the Moon at the end of the tithi at mid-eclipse.

The first half of this verse was probably a description of the epicycle model (compare with IV, 10). The second half appears to give the longitudinal component of parallax, which is probably equivalent to that given in

the *Pauliśasiddhānta* of Varāhamihira in which the longitudinal component of parallax is a function of the hour-angle. At the end of this passage, we see the realization that it is the longitudinal parallax alone that causes a difference in the conjunction time.

Then the latitudinal parallax is to be calculated as follows (IX, 6):

> From what is obtained with respect to [the longitude of] the ascendant, subtract 90° and find its declination; if it is northern, subtract this from the terrestrial latitude; if southern, add. Multiply the sine of the result by the difference between the mean velocities of the sun and the moon; one should divide this by 15*R*. The quotient is the latitudinal parallax. If the latitude was added to the declination, it is to the north; otherwise to the south.

This gives the formula

$$\pi_\beta = \frac{\sin[\delta(V) \pm \phi] \cdot (v_m - v_s)}{15R},$$

where the zenith distance of the nonagesimal is found by combining its declination with the latitude, $\sin(\delta(V) \pm \phi) = \sin z(V)$ and the $\frac{(v_m - v_s)}{15}$ is the standard value for the maximum parallax. This expression of latitudinal parallax seems also to be equivalent to that of the *Pauliśa*, with the relative velocity of the moon as compared to the sun being taken instead of simply v_m, and the zenith distance of the nonagesimal being the sum or the difference of the declination of the nonagesimal and the terrestrial latitude only. As the latitudinal component of parallax corrects the computed lunar latitude and the longitudinal component affects the eclipse time, the total parallax is to be applied as follows:

> If the latitude of the Moon at that time and the parallax in latitude are in the same direction, add them; if in opposite directions, take their difference. [The result is] the corrected latitude. When an eclipse has been established by means of this, calculate the longitudinal parallaxes with the luminaries' [longitudes] increased or decreased by 180° [?] at the times of first contact, the beginning of totality, the end of totality, and the last contact. If the mean and the desired longitudinal parallaxes are in the same direction, take their difference; if in opposite directions, their sum. [The longitudes of] the Sun and the Moon at mid-eclipse increased by this [amount] are correct at that time. The half-duration of the eclipse and that of totality are to be diminished by the longitudinal parallax. In this the projection and so on is as in a lunar eclipse.

The last statement of this section implies that there was a separate lunar eclipse section, located further back in the work. However, this section is not in the manuscripts that still exist. The theory of parallax is therefore appropriately in the solar eclipse section, although the instruction to take the longitudes

"increased or decreased by 180°" sits uncomfortably in this section. This would seem to refer to a lunar eclipse, in which the bodies, being at opposition, are 180° apart, but of course parallax is not necessary in the computation of a lunar eclipse. This section also reveals that its author rightly understands that the parallax is changing throughout the eclipse. The desire to compute it for the five significant phases of the eclipse suggests that the second estimates were to be used to correct the first estimate in order to achieve better accuracy for half-durations and so on by (presumably) some sort of iterative procedure.

5.6.2 Ārdharātrikapakṣa: *The* Sūryasiddhānta

The *Ārdharātrikapakṣa*, also known as the midnight system (*ārdharātrika* means midnight), differs from the other *pakṣa*s primarily in the fact that its epoch begins, as its name implies, at midnight, rather than dawn. The beginning of the Kaliyuga according to this *pakṣa* is thus 17/18 February -3101 at midnight. This shift of epoch has its advantages, as it is much easier to calculate from midnight than from dawn. Accordingly, this *pakṣa* has its own parameters and is closest to the *Āryapakṣa*. This is because, like the *Āryapakṣa*, it was conceived by Āryabhata. Unfortunately, however, the only record of the work by him from which this school was derived is lost.[67] The only clues we have for what this work resembled are later texts and commentaries which were composed in response to it (such as Brahmagupta's *Khaṇḍakhādyaka*—see section 5.6.5).

The earliest details of this *pakṣa* can be recovered indirectly from a work called the *Sūryasiddhānta*, a now lost treatise composed in the early centuries of the common era. The *Sūryasiddhānta* was revised by Lāṭadeva (fl. ca. 505 CE), a pupil of Āryabhaṭa, who made it conform to the fundamentals of the *Ārdharātrikapakṣa*. This revision is included by Varāhamihira in his *Pañcasiddhāntikā* and given a contemporary epoch (midnight 20/21 March 505), the same epoch as the *Pauliśa* and the *Romaka*.

The chapter that contains the *Sūryasiddhānta*'s treatment of solar eclipses begins by setting down parameters and rules for mean motions, longitudes, anomalies, sizes, and distances (IX, 1–16). In particular, the mean daily motions of the sun and the moon are given as (IX, 11)

$$\nu_s = 0;59,8^\circ,$$
$$\nu_m = 13;10,34^\circ.$$

Their geocentric distances in *yojanas* can be derived as

$$k_s = 4,27,21,$$
$$k_m = 20,0,$$

as well as their apparent diameters (measured in minutes of arc):

$$d_s = 32;14,6,\ldots,$$
$$d_m = 32;12.$$

From these parameters and relationships, it has been shown[68] that the relative sizes are as follows: the radius of the sun is about four times the radius of the earth, and the radius of the earth is about three times that of the moon.

The next several verses (IX, 17–23) contain the calculation of the longitudinal component of parallax. The formulas are more complex (but the degree to which this improves the accuracy is somewhat questionable!) and involve more sophisticated trigonometrical assumptions than the methods set out in the *Pauliśa* and *Romaka* schemes, in which the longitudinal parallax is a function of the magnitude of the (approximate) hour-angle of the moon. In the *Sūrya* scheme, longitudinal parallax is a function of the moon's distance from the nonagesimal and the distance of the nonagesimal from the zenith. The rule gives longitudinal parallax for the "end of the tithi," i.e., the sun and the moon at conjunction. An iterative procedure is set out in IX, 23 to determine the time of apparent conjunction from that of true conjunction.

> bhāgaviśeṣāt tithivat tithyanto 'taḥ punaḥ punas tat syāt |
> evaṃ mṛgyaḥ kālas tūtpanno yāvad aviśeṣaḥ ||
> [From the difference in the degrees (of the two arcs is found) the end of the tithi, in the same way as (is found what has passed, or is to come of) a tithi; from that (the procedure) is (to be iterated) again and again. The resulting time is to be investigated thus, until there is no remainder.]
> (*Sūryasiddhāntā* IX, 23, as translated by Pingree)

The iterative procedure seems to be as follows:

1. Compute the positions at a conjunction according to the rules already given
2. Apply the parallax, which gives two new positions in longitude
3. Compute the new conjunction time
4. Recompute the positions at conjunction
5. Apply parallax which gives two new positions.
6. Repeat from 2 until the conjunction time is fixed.

This brings to mind similar concerns seen in Ptolemy and his concept of *epiparallax*.

The formula to calculate longitudinal parallax is somewhat complicated and given in several steps which take four whole verses to cover.[69] The ultimate goal is to determine the amounts $z(S)$ and $z(V)$, the zenith distances of the sun and

the nonagesimal, respectively (fig. 5.8). This is because the assumption here is that the longitudinal component of parallax is proportional to the distance of the body from the zenith as well as the distance of the body from the nonagesimal.

To find $z(V)$ (in Sanskrit, the *dṛkkṣepa*), the following rule is given. It is a crude approximation, based on assuming that the spherical triangle ZVM is plane (fig. 5.9), and, as it is a right-angled triangle, applying the Pythagorean theorem, so that

$$z(V) = \sqrt{\mathrm{Sin}^2 z(M) - \mathrm{Sin}^2(VM)}.$$

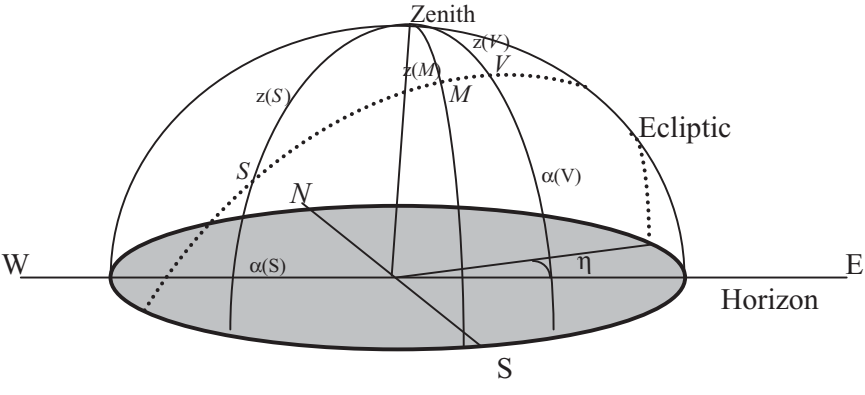


Figure 5.8 Computation of parallax; V is the nonagesimal, M is the meridian ecliptic point, and S shows the positions of the luminaries.

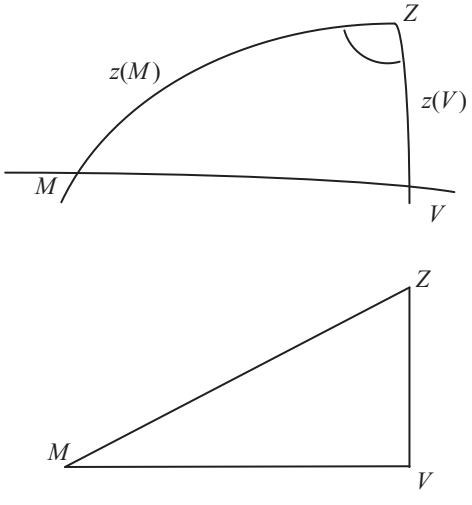


Figure 5.9 Triangle ZVM.

The arc $z(M)$ is simply found by the equation (from IX, 17–18)

$$z(M) = \phi \pm \delta(M),$$

where $\delta(M)$ is found from a table of declinations. Because $z(M)$ is an arc along the meridian, the fact that the depression of the point M from the zenith Z can be found by adding or subtracting the declination to or from the terrestrial latitude is easily derived from an analemma projection (see fig. 5.1; $MZ = QZ \pm MQ$). The sine of the arc $z(M)$ is known as the *madhyajyā* (literally "sine of the middle").

The arc VM is the arc of the ecliptic between the nonagesimal (V) and the point at which the ecliptic intersects the meridian (M). It is found by the relation

$$\mathrm{Sin}(VM) = \frac{\mathrm{Sin}\eta \cdot \mathrm{Sin}z(M)}{R}.$$

This formula invokes properties of spherical trigonometry, in particular the relation known as the law of sines for two arcs a and b in a spherical triangle and their opposite angles A and B:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}.$$

Thus, the two previous *siddhāntas* in the *Pañcasiddhāntikā* use plane triangles, but the *Sūrya* scheme employs spherical trigonometrical techniques, implying that there was an additional transmission from Greece of slightly more advanced techniques to deal with triangles on the sphere, methods that were not available to the composers of the *Pauliśa* and the *Romaka siddhāntas*.

Sinη, the Sine of the rising amplitude, also called the *udayajyā* (literally "Sine of the rising") is the arc of the horizon between the due east point on the horizon and the point where the body rises. It is calculated through the relation (IX, 19) (see fig. 5.8)[70]

$$\mathrm{Sin}\eta = \frac{\mathrm{Sin}\epsilon \mathrm{Sin}\lambda(M)}{\mathrm{Sin}\overline{\phi}}.$$

Then $\alpha(V)$, the altitude of the nonagesimal, is computed. As $z(V)$ and $\alpha(V)$ are complementary:

$$z(V) + \alpha(V) = 90^\circ \text{ as } z(V) = \overline{\alpha}(V),$$

it follows that

$$\mathrm{Sin}\alpha(V) = \sqrt{R^2 - \mathrm{Sin}^2 z(V)}.$$

Once $\alpha(V)$ has been found, $z(S)$ is computed in a similar way from similar triangles, where $\Delta\lambda$ is the longitudinal difference between the sun and the

rising or setting point of the ecliptic and ΔV is the longitudinal difference between the nonagesimal and the ascendant (which is 90° by definition):

$$\frac{\mathrm{Sin}\Delta\lambda}{\mathrm{Sin}\alpha(S)} = \frac{\mathrm{Sin}\Delta V}{\mathrm{Sin}\alpha(V)},$$

$$\mathrm{Sin}\alpha(S) = \frac{\mathrm{Sin}\Delta\lambda \cdot \mathrm{Sin}\alpha(V)}{R}$$

as $\mathrm{arc}\Delta V = 90°$. Again, because $\alpha(S)$ and $z(S)$ are complementary, the following relation holds:

$$\mathrm{Sin}z(S) = \sqrt{R^2 - \mathrm{Sin}^2\alpha(S)}.$$

Once these parameters have been established, the longitudinal difference between the sun and the nonagesimal is found:

$$\mathrm{Sin}(VS) = \sqrt{\mathrm{Sin}^2 z(S) - \mathrm{Sin}^2 z(V)}.$$

Accordingly, the rule for the total parallax in longitude is given as

$$\pi_\lambda = \frac{18}{\rho R}\sqrt{\mathrm{Sin}^2 z(S) - \mathrm{Sin}^2 z(V)},$$

where ρ is the distance of the sun or the moon from the earth and 18 a standardized or normalized measure for the radius of the earth.[71] Effectively, the factor of $\frac{18}{\rho}$ is the maximum parallax; in the case of the moon, where $k_m = 20{,}0 = 1200$ *yojanas*, the resulting maximum parallax is 0;54°, which is reasonable.

In the *Sūrya* system, the latitudinal parallax is a function of the zenith distance of the nonagesimal (IX, 24–25), given by the rule

$$\pi_\beta = \frac{18}{\rho R}\mathrm{Sin}z(V).$$

This verse also states the direction of the vector of latitudinal parallax *madhyajyādig* (literally "[whose] direction is that of the sine of the middle"), which refers to the point of the ecliptic that crosses the meridian.

The latitudinal parallax is then added to the latitude of the moon to get the apparent latitude of the moon. With this corrected latitude, one is then instructed to determine the magnitude and duration of the eclipse (IX, 25cd).

The next verse gives a rule for the duration of the eclipse and is identical to the one given in the *Romaka* (VIII, 16). The final verse in this chapter (IX, 27) asserts that the longitudinal component of parallax will have an effect on the beginning and the end of the eclipse, and states that it will be either additive or subtractive as it is in the west or east, respectively.

The next chapter of the *Pañcasiddhāntikā* concerns lunar eclipses, and although it does not specifically attribute the material to the *Sūrya*, its identification as such is confident, as many of the parameters are similar to those of the *Sūrya* mentioned in other parts of the work.

As with the solar eclipse section, in general the calculation of the various aspects related to lunar eclipses is slightly more complex than in the two other relevant *siddhānta*s included in the *Pañcasiddhāntikā*. In the *Sūrya* (X, 1–2), a more elaborate formula is expounded for the calculation of the size of the shadow of the earth. This computation is based on the idea of the shadow cone, which takes into account the varying positions of the moon with respect to the earth and the geocentric distances of the sun and the moon.[72] The following formula is stated for the diameter of the shadow at the moon's (mean) distance:

$$2d_u = \left(36 - \frac{36k_m}{90k_s/286}\right)\frac{120}{k_m}.$$

Using the values for the mean orbital radii in IX, 15–16, $k_s = 16,041$, $k_m = 1200$, the mean diameter of the shadow cone at the moon's distance is

$$d_u = 2{,}44;39{,}5,\ldots \textit{ minutes of arc}.$$

If this is compared with the value given for the apparent diameter of the moon,

$$d_m = 32;12 \textit{ minutes of arc},$$

the ratio of the diameter of the shadow to the moon can be derived. It is a little over 5:1, a value that is significantly too high.

A simple formula for the half-duration of the eclipse is given (X, 3):

$$\Delta t = \frac{120}{v_m - v_s}\sqrt{(r_u + r_m)^2 - \beta^2}.$$

The validity of this formula is dependent on approximating the latitude of the moon to be the same at the beginning as at the middle of the eclipse. In fact, it is changing, and this seems to be accounted for in the next verse, which mentions an iterative procedure, such that a new latitude (β_1) is calculated for the time $t_m - \Delta t$, where t_m is the instant of mid-eclipse. Substitution of this new latitude into the equation gives a more accurate value for the half-duration. The process ends when there is "no more remainder."

X, 5–6 gives a rule for calculating the magnitude of a lunar eclipse at a desired time, which is equivalent to the procedure given earlier in the work. One can simply exchange the radius of the shadow for that of the sun in order to find the magnitude of a solar eclipse. The rule for computing the half-duration

of totality that ends the chapter on eclipses is based on computing the distance between first contact and mid-eclipse by means of the difference of radii and the lunar latitude:

$$\frac{\Delta t_{\text{tot}}}{2} = \sqrt{(r_u - r_m)^2 - \beta^2}$$

Chapter XI may be derived from the *Sūrya*. An expression for the *valana* also occurs in this chapter, which contains instructions for the graphical descriptions of lunar eclipses. The *akṣavalana* is to be computed by the following rule (XI, 2):

$$\gamma_1 = \frac{\sin\phi \cdot \sin(\lambda_M - \lambda_m)}{120},$$

where λ_M is the longitude of the meridian ecliptic point. An expression for the second component, the *ayanavalana*, is given as follows (XI, 3):

$$\gamma_2 = \delta(\lambda_m + 90^\circ).$$

5.6.3 *The* Āryapakṣa: *Āryabhaṭa's* Āryabhaṭīya

The *Āryapakṣa*,[73] as its name implies, was first set out by Āryabhaṭa I, in his seminal work the *Āryabhaṭīya*.[74] Like the *Brāhmapakṣa*, it has similarities with the *Paitāmahasiddhānta*, but the *Āryabhaṭīya* and its associated works following the *Āryapakṣa* are distinct with respect to stylistic features, such as layout and form, as well as having theoretical and computational differences. One important difference is Āryabhaṭa's division of the kalpa with respect to the four yugas. He does not split them up into four yugas with the ratio 4:3:2:1, but rather into four equal yugas of 1,080,000 years each (see section 5.2.1).

Āryabhaṭa was so heavily criticized for introducing this digression from the tradition that several centuries later followers of the *Āryapakṣa* were forced to abandon the contentious division and revert back to the system presented in the *Brāhmapakṣa*. The *Āryapakṣa* is also linked to the so-called *parahita* system of astronomy introduced by Haridatta in the late seventh century, a system that was widely used in southern India for many centuries. This system is characterized by the representation of mean motions of the planets as period relations.[75]

The material related to eclipses (*Gola*, 32–47) in the *Āryabhaṭīya* arranges its "divisions" in an order quite distinct from the various *siddhānta*s in the *Pañcasiddhāntikā* or works within the *Brāhmapakṣa*. Notably, rules for parallax precede the more general rules for different aspects of eclipses. Furthermore, the presentation of eclipse theory deals with solar and lunar eclipses side by side, rather than in two separate sections.

Verse in *Gola* section	Content
28–34	The "five" sines and "parallax"
35–36	Visibility conditions
37–38	General eclipse material
39–40	Diameters
40(?)	Geometric distances
41–42	Duration
43–44	Magnitude
45	Valana
46	Colors
47	Prediction and limits

Āryabhaṭas's mean daily motions (obtained by dividing the number of revolutions in the yuga by the days) and orbital radii are as follows:

Sun	0;59,8,10,13,3,31 . . .°
Moon	13;10,34,52,39,18,56 . . .°
Manda	0;6,40,59,30,7,38 . . .°
Node	-0;3,10,44,7,49,44 . . .°
k_m	$216{,}000 = 60^3$
k_s	$2{,}887{,}666\frac{4}{5}$

The diameters of the sun, moon, and shadow are given early in the work (*Gītikā* 7), in a separate section on parameters:

$$d_e = 1050 \text{ } yojanas,^{76}$$
$$d_s = 4410 \text{ } yojanas,$$
$$d_m = 315 \text{ } yojanas.$$

As, for Āryabhaṭa, ten *yojanas* is equal to one minute of arc at the distance of the moon,[77] the diameter of the moon in minutes is

$$d_m = \frac{315'}{10} = 0;31;30°.$$

Āryabhaṭa gives rules for the computation of five key parameters, which are fundamental to the computation of parallax. These are the sine of the altitude of the sun (28), the sine of the rising amplitude (30), the sine of the zenith distance of the meridian ecliptic point (32), the sine of the zenith distance of the nonagesimal (33), and the sine of the longitudinal difference between the sun and the nonagesimal (34).

The sine of the altitude of the sun (*śaṅku*) is

$$\text{Sin}\alpha(S) = \frac{\text{Sin}\lambda(S) \times \text{Sin}\overline{\phi}}{R}.$$

The sine of the rising amplitude (*udayajyā*) is:

$$\mathrm{Sin}\eta = \frac{\mathrm{Sin}\epsilon \times \mathrm{Sin}\Delta\lambda}{\mathrm{Sin}\overline{\phi}}.$$

The sine of the zenith distance of the meridian ecliptic point (*madhyajyā*) is

$$\mathrm{Sin}z(M) = \mathrm{Sin}\alpha(S) \quad \text{at noon.}$$

The sine of the zenith distance of the nonagesimal (*dṛkkṣepajyā*) is

$$\mathrm{Sin}z(V) = \sqrt{\mathrm{Sin}^2 z(M) - \mathrm{Sin}^2(VM)},$$

where

$$\mathrm{Sin}(VM) = \frac{\mathrm{Sin}\eta \times \mathrm{Sin}z(M)}{R}.$$

The sine of the longitudinal difference between the nonagesimal and the sun is

$$\mathrm{Sin}(VS) = \sqrt{\mathrm{Sin}^2 z(S) - \mathrm{Sin}^2 z(V)}.$$

All rules are based on treating spherical triangles as plane and are similar to those given in the *Sūryasiddhānta*. But notably Āryabhaṭa does not give the reader any further indication as to how to apply these values, nor even that they relate to the components of latitudinal and longitudinal parallax. Presumably, it is intended that the reader recognize that these values are the bases for the computation of parallax, and will know the rest of the rules for multiplying and dividing the appropriate quantities to find the proper values.

To finish his section on parallax, Āryabhaṭa gives the following verse, in which he introduces the word *dṛkchāyā* (*dṛk* = direction, *chāyā* = shadow) to indicate the total effect of parallax:

> kṣitije svā dṛkchāyā bhūvyāsārdhaṃ nabhomadhyāt
> [Parallax increases from zero at the zenith to the maximum value equal to the earth's radius at the horizon.]
> (*Āry. Gola*, 34, second half)

Āryabhaṭa's value for the earth's radius is $1050/2 = 525$ *yojanas* which leads to a value of

$$\pi_{\max} = 0; 52, 30^\circ$$

on the assumption that each minute of the orbit of the moon equals ten *yojanas*. Āryabhaṭa does not give any indication of how or whether solar parallax is to be computed. But what is more important about this verse is the recognition that parallax is minimized at the zenith. Previous formulations, as well as Āryabhaṭa's, relate parallax to the nonagesimal and the distance of the

luminaries from it—according to these points, the parallax was minimized, whether or not these points were at the zenith or not.

After two verses on visibility corrections for the moon and the sun, Āryabhaṭa announces the following relations:[78]

> candro jalam arko 'gnir mṛd bhūś chāyāpi yā tamas tad dhi |
> [The Moon is water (*jalam*), the Sun is fire (*agni*), the Earth is "earth" (*bhū*) and what is called the Shadow is darkness (*tamas*).]
> (*Āry. Gola*, 37 first half)

These identifications, in particular the identification of the moon with water, could be derived from Vedic references.

The causes of a lunar and solar eclipse and their occurrence in the middle and end of the month, respectively, are correctly stated (37cd and 38). The length of the shadow cone (k_u) is given by the formula

$$k_u = \frac{k_s.d_e}{d_s - d_e},$$

which is a simple approximation derived from similar plane triangles (see fig. 5.7).[79] Similarly, the diameter of the earth's shadow at the moon's distance is given by the rule

$$d_u = \frac{d_e(k_u - k_m)}{k_u},$$

again a simple approximation derived from similar plane triangles. The resulting mean diameter of the shadow at the moon's mean distance can be calculated as

$$d_u = 798;40 \textit{ yojanas}.$$

The diameter in minutes (using ten *yojanas* equal to one minute of arc at the moon's distance) is

$$d_u = 1;19,52 \textit{ minutes}.$$

Standard rules for the half-duration and half-duration of totality are given next. With respect to the computation of obscuration, Āryabhaṭa's rule calculates the part of the moon *not* obscured, as measured along the diameter (43), although in the next verse (44), in his rule for the magnitude of the eclipse at the given (*iṣṭa*) time, it is the *obscured* part he calculates.

Next, Āryabhaṭa sets out the calculation of the "inclination" of the eclipse, the *valana*. First, Āryabhaṭa (45, first half) gives a formula for the first of the two components that make up the *akṣavalana*:

$$\gamma_1 = \frac{\text{Versin}(\Delta\tau) \times \text{Sin}\phi}{R},$$

where $\Delta\tau$ is the hour-angle, and this component of the *valana* is a function of the depression of the body from the prime meridian. This is identical to the formulation in the *Sūryasiddhānta*. The second component, the *ayanavalana* (45, second half) is to be computed as follows:

$$\gamma_2 = \delta(\lambda_m + 90^\circ).$$

Āryabhaṭa states that one must use it for the time of first contact. The colors of the eclipse are given with respect to the phases of the eclipse (46):

Phase	Sanskrit term	Translation
Beginning	*dhūmra*	Smoky
Half obscured	*kṛṣṇa*	Black
Just totally eclipsed	*kapila*	Reddish-brown (literally monkey colored)
Well inside shadow	*sakṛṣṇatāmras*	Coppery-black
End	*dhūmra*	Smoky

Āryabhaṭa concludes with the eclipse limits, stating that a solar eclipse cannot be observed when it amounts to 1/8 of the sun's diameter or less. He attributes this to the brightness of the sun. Such concern with practical observational difficulties is unusual.

5.6.4 *The* Brāhmapakṣa: *Brahmagupta's* Brāhmasphuṭasiddhānta

The most influential of the texts belonging to the *Brāhmapakṣa* is the *Brāhmasphuṭasiddhānta*[80] (BSS) composed in 628 by Brahmagupta (b. 589, died after 665). A later commentator on Brahmagupta's works, the ninth-century scholar Pṛthūdakasvāmin, referred to Brahmagupta as "the teacher from Bhillamāla," a region which may be the modern Bhinmal in Rajasthan. It is also speculated from a remark by Pṛthūdakasvāmin that he may have been a royal astronomer, under the ruler Vyāghramukha.[81] The *Brāhmasphuṭasiddhānta* and his other works were hugely influential, and their impact spread beyond Rajasthan, to Gujarat, Madhya Pradesh, Uttar Pradesha, Bihar, Nepal, Panjab, and Kashmir. The *Brāhmasphuṭasiddhānta* in particular is one of the most significant works in the history of Indian astronomy.[82]

The *Brāhmasphuṭasiddhānta* contains 24 chapters.[83] The first ten chapters are in part a summary of the *Paitāmahasiddhānta* supplemented by Brahmagupta's improvements. This method of composition explains why the work contains conflicting rules for solving the same problem. The remaining chapters are Brahmagupta's further additions, revisions, and innovations. Because of the importance of this text for all later Indian astronomy, it will be discussed here in detail. Furthermore, Brahmagupta is explicitly and demonstrably

opposed to his predecessor Āryabhaṭa and the systems he presents. Brahmagupta's critiques of his rival's work and the ways in which suggests "improvements" gives a fascinating insight into the practices of professional scholars.

In the *Brāhmasphuṭasiddhānta*, Brahmagupta deals with lunar eclipse theory in chapter IV and solar eclipse theory in chapter V; however, additions and innovations to eclipse theory are found interspersed in other sections of the work (in particular chapter XVI, 1–47). At the outset of chapter IV he describes the elements of eclipse theory that he has seen fit to cover, enumerating fourteen of them:

> kālajñānaṃ prāyaḥ parvajñānārtham iṣyate sadbhiḥ |
> śaśibhāskaragrahaṇayor vāstavabhedāḥ sphuṭā bhedaiḥ ||
> digvarṇavalanavelānimīlanonmīlanasthitivimardāḥ |
> sparśacchāyāmokṣagrāseṣṭagrāsaparilekhāḥ ||
> bhedāś caturdaśa tayor indvarkagrahaṇayoḥ parijñānāt |
> yasmād bhedajñānaṃ tasmād grahaṇe pravakṣyāmi ||
> [The understanding of time is generally desired by good (astronomers) for the sake of the understanding of the syzygies. The true parts of the two eclipses, the lunar and the solar, are (made) clear by these divisions: the direction, the color, the inclination, the time, the total immersion, the reappearance (after total immersion), the duration, the duration of totality, the (first) contact, the shadow, the release, the magnitude, the magnitude at a given (time), and the graphical projection. (Such) are the fourteen divisions of these two eclipses, the lunar and the solar. From that knowledge whence (comes) the knowledge of (these fourteen) divisions, I will explain the two (types of) eclipses.]
> (BSS IV, 1–3, as translated by Pingree and Montelle)

All of these fourteen elements, except for that of color, which is part of the omen tradition, are found in Greek eclipse theory.

The majority of the rules in this chapter apply equally to solar and to lunar eclipses; however, the rules in the case of a solar eclipse, must take parallax into account, and the treatment of parallax is not set out until chapter V.

Brahmagupta identifies five significant phases of the eclipse: first contact (*sparśa*), beginning of totality (*nimīlana*), mid-eclipse (*madhyagrahaṇa*),[84] end of totality (*unmīlana*), and release (*mokṣa*). He begins with a rule for determining the longitudes of the sun, the moon and the node at syzygy:

> tithigatagamye bhuktiguṇe bhuktyantarahṛte phalonayutau |
> raviśaśinau samaliptau pātas tātkāliko bhavati ||
> [What has passed or remains of the tithi is multiplied by the daily velocity (of the sun and the moon) and divided by the difference of the daily velocities

(of both). (The longitudes at sunrise of) the sun and the moon, when decreased or increased by (that) result, (occupy) the same minute. (The longitude of) the node is (computed) for that time.]
(BSS IV, 4, as translated by Pingree and Montelle)

The verse suggests the following procedure. Determine the time (Δt) from sunrise until or after the end of the tithi, the longitudes of the sun and the moon at sunrise (λ_s and λ_m), and their velocities (v_s and v_m). The following two distances are calculated:

$$\Delta\lambda_m = \Delta t \times \frac{v_m}{(v_m - v_s)} \text{ and } \Delta\lambda_s = \Delta t \times \frac{v_s}{(v_m - v_s)},$$

and added to or subtracted from the longitudes at sunrise to get the longitudes at syzygy [$\lambda(s)$]:

$$\lambda_m(s) = \lambda_m \pm \Delta\lambda_m \text{ and } \lambda_s(s) = \lambda_s \pm \Delta\lambda_s.$$

One is instructed to consider *only* the minute components of the amounts. In effect, the degrees can be ignored because they will be either the same (in the case of a solar eclipse), or 180° different (in the case of a lunar eclipse). Therefore, if

$$\text{minutes of } \lambda_m(s) = \text{minutes of } \lambda_s(s),$$

the longitudes are *samalipta* (literally "the same minute"). There is no indication for what to do if they are not equal. Presumably, some sort of iteration procedure was to be employed until equality was reached.

The next rule concerns the latitude of the moon, giving a formula for how to calculate the latitude and a way to determine its direction with respect to the ecliptic:

khatrighana 270 guṇā vyāsārdhabhājitā candrapātayogajyā |
vikṣepakalāḥ saumyāḥ ṣaḍrāśyūne 'dhike yāmyāḥ ||
[The sine of the sum (of the longitudes) of the moon and (its) node is multiplied by 270 and divided by the radius. (The result is) the minutes of the (lunar) latitude. When (the sum of the longitudes of the moon and the ascending node) is less than six zodiacal signs [i.e., 180°], (the minutes of the moon's latitude) are to the north (of the ecliptic), when greater (than six signs), to the south.]
(BSS IV, 5, as translated by Pingree and Montelle)

The rule calculates the latitude as follows:

$$\beta = \frac{270 \times \text{Sin}(\lambda_m - \lambda_n)}{R}$$

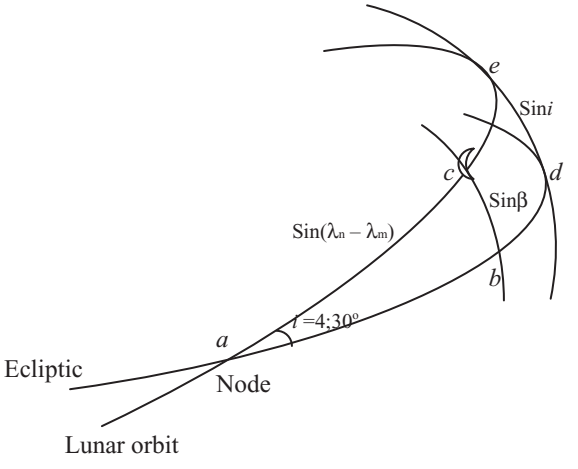


Figure 5.10 Brahmagupta's computation of the lunar latitude.

where the maximum latitude is taken to be $270' = 4;30°$.[85] This particular rule invokes the properties of similar triangles on a sphere (fig. 5.10):

$$\frac{\mathrm{Sin}bc}{\mathrm{Sin}ac} = \frac{\mathrm{Sin}de}{\mathrm{Sin}ae},$$

$$\mathrm{Sin}bc = \frac{\mathrm{Sin}ac \times \mathrm{Sin}de}{\mathrm{Sin}ae}$$

$$= \frac{\mathrm{Sin}(\lambda_m - \lambda_n) \times 270}{R}.$$

In this formulation, $\mathrm{Sin}\beta$ has been approximated by β. This is a reasonable approximation as, at the time of an eclipse, the latitude β is very small, and thus its sine is very close to its angular size.

Brahmagupta next (IV, 6) sets out three formulas to calculate the apparent angular diameters of the sun, the moon, and the earth, respectively.

> raviśaśibhuktī bhavadaśaguṇe nakhaiḥ svarajinairhṛte māne |
> tattvāṣṭaguṇitabhuktyor vivaraṃ ṣaṣṭyā hṛtaṃ tamasaḥ ||
> [The daily velocities of the sun and the moon are multiplied by 11 and by 10 (respectively) and divided by 20 and by 247 (respectively. The results are) the measures (of the apparent diameters of the sun and the moon). The difference of the two daily velocities, (having been) multiplied by 8 and by 25 (respectively), is divided by 60. (The result is the apparent diameter) of the shadow.]
> (BSS IV, 6, as translated by Pingree and Montelle)

Here, Brahmagupta uses the relation that the velocity of a planet is inversely proportional to its geocentric distance. Accordingly, the following three

formulas give the diameters in minutes:

$$d_s = \frac{11 \cdot v_s}{20},$$

$$d_m = \frac{10 \cdot v_m}{247},$$

$$d_u = \frac{(8v_m - 25v_s)}{60}.$$

From Brahmagupta's mean daily velocities (using $v_s = 0;59,8$ and $v_m = 13;10,34$), the following mean angular diameters can be calculated:

$$d_s = 0;32,31^\circ,$$

$$d_m = 0;32^\circ,$$

$$d_u = 1;20,46,12^\circ.$$

The above values almost exactly conform to the Greek 2½:1 ratio of the diameter of the shadow to the diameter of the moon[86] which is more or less in agreement with what is seen.

Next, Brahmagupta presents rules for determining the amount of obscuration and defines a total and a partial eclipse in terms of the amount of obscuration:

> chādyacchādakamānaikyārdhaṃ vikṣepahīnitaṃ channam |
> sarvagrahaṇaṃ grāhyād adhike khaṇḍagrahaṇam ūne ||
> [Half the sum of the measures (of the apparent angular diameters) of the eclipsed and eclipsing bodies decreased by the (lunar) latitude is the obscured portion (at mid-eclipse). When (this amount) is greater than the eclipsed body, there is a total eclipse, and when less, a partial eclipse.]
> (BSS IV, 7, as translated by Pingree and Montelle)

This rule is very simply verified (fig. 5.11):

$$\begin{aligned} \text{Obscured portion} &= ac \\ &= ab + bc \\ &= R - \beta + r \\ &= R + r - \beta \\ &= \frac{1}{2}(D + d) - \beta, \end{aligned}$$

where $R = \frac{1}{2}D$ and $r = \frac{1}{2}d$ are the radii of the eclipsing and the eclipsed body, respectively. This formula holds for both a lunar and a solar eclipse.

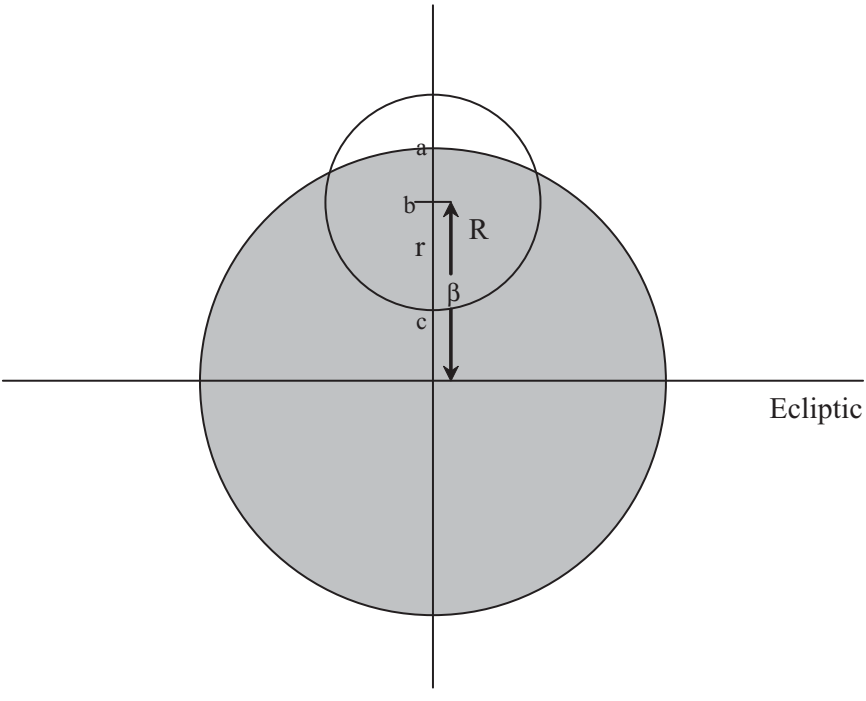
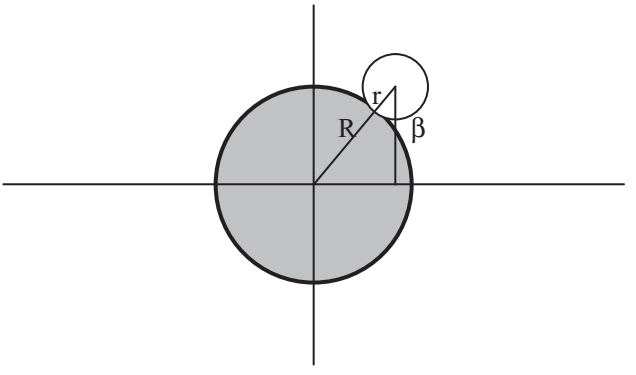


Figure 5.11 Amount of obscuration.

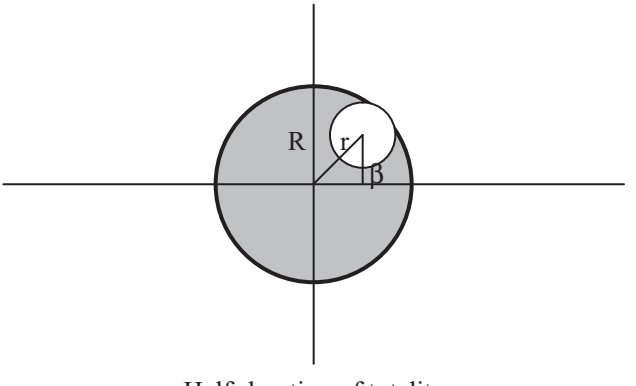
From Brahmagupta's formula it is clear that he must intend to compute the obscured portion at mid-eclipse, that is, when the two radii are along the same vertical as the latitude. The obscured portion is simply a linear amount measured along the diameter, rather than an actual area of the disk. This is in a similar spirit to the work Ptolemy, of who divides the diameter up into twelve equal parts and gives a measure of obscuration with respect to these units.

Following this, Brahmagupta gives rules (IV, 8) for the half-duration (*sthityardha*) and half-duration of totality (*vimarda*) measured in *ghaṭikās* (sixtieths of a day) (fig. 5.12).

> chādyena yutonasya chādakamānasya taddalakṛtibhyām |
> vikṣepakṛtiṃ prohya pade tithivat sthitivimardārdhe ||
> [Subtracting the square of the (lunar) latitude from the two squares of half of that, (namely) of the measure (of the apparent diameter) of the eclipsing body increased and decreased by (the apparent diameter of) the eclipsed body, the square-roots (of the two remainders converted into time) like a tithi, are the half-duration and the half-duration of totality.]
> (BSS IV, 8, as translated by Pingree and Montelle)



Half-duration



Half-duration of totality

Figure 5.12 Half-duration and half-duration of totality.

For the half-duration,

$$\frac{\Delta t}{2} = \sqrt{(R+r)^2 - \beta^2}.$$

For the half-duration of totality,

$$\frac{\Delta t_{\text{tot}}}{2} = \sqrt{(R-r)^2 - \beta^2}.$$

Again Brahmagupta has made the assumption that the latitude is kept constant throughout, and thus he can employ the Pythagorean theorem, treating β as a constant linear distance. To convert these time intervals to minutes of arc, one multiplies by 60 and divides the product by the difference of the true daily motions in arc minutes per day.

The following rule finds the changes in longitude during the half-duration and/or half-duration of totality:

ṣaṣṭyā vibhājitā sthitivimardadalanāḍikā guṇā svagatiḥ |
ādau ravīndupāteṣv ṛṇam asakṛt teṣu dhanam ante ||
[The daily velocity of each is multiplied by the ghaṭikās of the half-duration or of the half-duration of totality and divided by 60. (The result is applied) negatively to (the syzygy longitudes of) the sun, the moon, and the node at the beginning (of the eclipse or the beginning of its totality, and applied) positively to these at the end, repeatedly.]
(BSS IV, 9, as translated by Pingree and Montelle)

This gives the following equation for the longitudinal increment $\Delta\lambda$:

$$\Delta\lambda = \frac{v \cdot \Delta t}{60},$$

where one can substitute for Δt the half-duration or the half-duration of totality, and for v the lunar or solar velocity, depending on what is required. This rule too is valid only when the lunar orbit is parallel to the ecliptic. One is instructed to carry out this rule "repeatedly," that is, to iterate the procedure. This may be an attempt to take into account the changing velocities of the luminaries.

Verse 10 gives the relationship between the half-duration and the half-duration of totality: the difference between the former and the latter gives the time of immersion after first contact or the time from the end of totality until release. This rule maintains the symmetry of the half-durations.

sparśān nimīlanaṃ sthitidale vimardārdhahīnite paścāt |
mokṣād arvāg unmīlanaṃ vimardas tadaikyārdhaḥ ||
[When the half-duration is decreased by the half-duration of totality, (the result is) the time of totality after (first) contact and the time of reappearance before release. The (half-)duration of totality is half the sum of these (two).]
(BSS IV, 10, as translated by Pingree and Montelle)

However, it seems that Brahmagupta does not continuously hold the assumption that the lunar latitude is constant throughout the duration of the eclipse, as the next quartet of verses shows:

bhuktyantaram iṣṭonasthitidalaghaṭikā guṇaṃ hṛtaṃ ṣaṣṭyā |
bāhuḥ prāgvat tatphalahīnayutaiḥ sūryaśaśipātaiḥ ||
tātkālikavikṣepaḥ koṭis tadvargayutipadaṃ karṇaḥ |
mānaikyārdhāt karṇaṃ viśodhya tātkāliko grāsaḥ ||
[The difference between the (two) daily velocities is multiplied by the ghaṭikās of the half-duration decreased by a given (time) and divided by 60. The bāhu [i.e., base of the right-angled triangle] as before (is obtained) by (the syzygy

longitudes) of the sun, the moon, and the node decreased or increased by that result. The (lunar) latitude at that time is the koṭi [i.e., the upright of that right-angled triangle]. The square-root of the sum of the squares of these (two amounts) is the hypotenuse. Subtracting the hypotenuse from half the sum of the measures (of the apparent angular diameters, the result) is the amount of obscuration at that time.]
(BSS IV 11–12, as translated by Pingree and Montelle)

If the lunar orbit was assumed to be parallel to the ecliptic, so that lunar latitude remained constant throughout the eclipse, it would be straight forward to compute the amount of obscuration at any time. However, here, Brahmagupta has not adopted this simplifying assumption. His rule gives the obscured portion at a particular time after contact or before separation, and, in contrast to previous verses, takes into account the changing lunar latitude. To determine the magnitude of an eclipse for a given time t^*, he first computes the elongation of the moon from the shadow center ($\Delta\lambda^*$) at that time:

$$\Delta\lambda^* = \Delta t^* \frac{v_m - v_s}{60},$$

where Δt^* is the difference between the half-duration ($\frac{\Delta t}{2}$) and the time between the beginning of the eclipse and the given time.

This amount is then added to (or subtracted from) the longitudes of the appropriate luminaries at syzygy, which gives (fig. 5.13) the base (*bāhu*) of

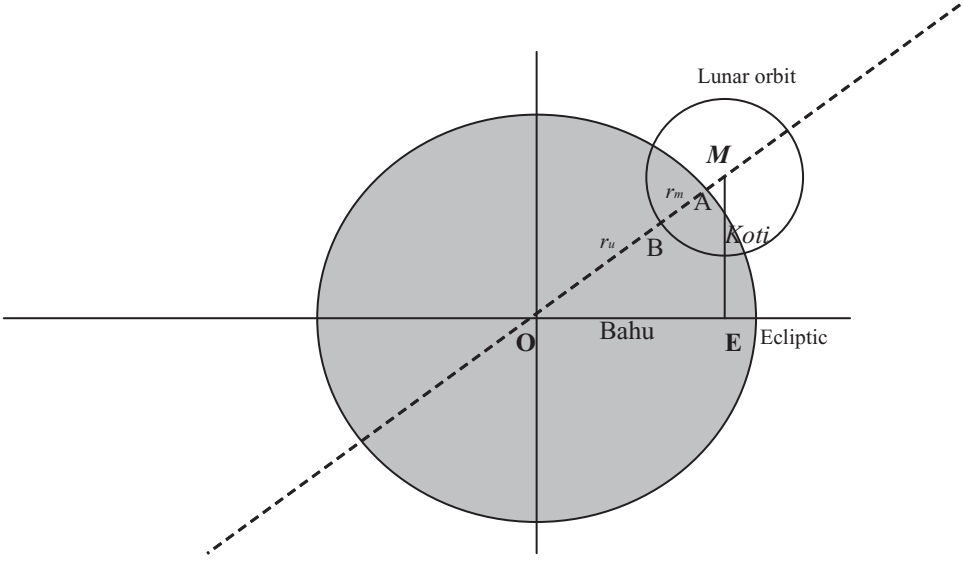


Figure 5.13 Brahmagupta's duration.

a right-angled triangle (OE). The upright of that triangle (*koṭi*) is the lunar latitude at t^* (ME). The hypotenuse of this triangle (OM) is found as:

$$OM = \sqrt{(OE^2 + ME^2)}.$$

Then μ, the amount obscured at that time, is (as in the demonstration for verse 8)

$$\mu = AB = r_u + r_m - OM.$$

Again, the amount of obscuration is a distance along the diameter of the moon's disk, and not an area. Although Brahmagupta considers the lunar orbit as inclined to the ecliptic, he has made a simplifying assumption of a different sort. Underlying his rule is the assumption that mid-eclipse and syzygy occur at the same time, which is not always the case.

In contrast to the last two verses, the next two verses give the converse rule, deriving the time at which the given obscured portion occurs. The second verse in this pair states rules to compute the time since the beginning of the eclipse and the time until release from the time at which a given obscuration is determined.

asakṛd grāsakalonapramāṇayutidalakṛter viśodhya kṛtim |
tātkālikavikṣepasya śeṣamūlaṃ kṛtaṃ tithivat ||
pragrahaṇasthityardhāt prohya pragrahaṇato bhavet kālaḥ|
maukṣaṃ viśodhya mokṣasthityardhāt prāg bhaven mokṣāt ||
[Subtracting the square of the (lunar) latitude at that time iteratively from the square of half the sum of the measures (of their apparent angular diameters) decreased by the minutes of the amount of obscuration. The square root of the remainder is computed like a tithi. Subtracting (this time) from the half-duration at the beginning of the eclipse, (the remainder) is the time since the beginning of the eclipse. Subtracting (the time computed) for the release from the half-duration at the release, (the remainder is the time) before the release.]
(BSS IV, 13–14, as translated by Pingree and Montelle)

Again, the lunar latitude is not fixed for the whole eclipse, but rather is to be found for the time in question. At the time of the desired obscuration, the latitude of the moon is unknown and thus must be computed by iteration. Presumably this iterative procedure implies that one starts with a rough guess, such as the latitude at mid-eclipse. From this latitude, a corresponding time is computed. Then the longitudes of the moon and the node are calculated for this time, a new latitude is found corresponding to those longitudes, and the procedure is iterated until the desired result is achieved.

Brahmagupta describes the various phases of the eclipse next:

sphuṭatithyante madhyaṃ pragrahaṇaṃ sthitidalonake 'bhyadhike |
mokṣo nimīlanonmīlane vimardārdhahīnayute ||
[Mid(-eclipse) is at the accurate end of the (syzygy) tithi. When it is decreased by the half-duration, (the result is) the beginning of the eclipse. When increased, the release. When it is decreased and increased (respectively) by the half-duration of totality, it is the time of total immersion and the time of reappearance after total immersion.]
(BSS IV, 15, as translated by Pingree and Montelle)

An important insight can be gained from considering all of Brahmagupta's verses which concern obscuration-time computations. Here, Brahmagupta presents two distinct methods to find the latitude set out side by side. The first is very simple: it is based on the assumption that the latitude remains constant throughout the eclipse. The second drops this assumption, and instructs one to compute the latitude for the time in question by the use of an iterative method. This technique, in which a simple approximative rule is followed by a more accurate rule, is frequent within Brahmagupta's work. It does not indicate a contradiction, nor a confusion on the part of the author, but rather gives the reader options. The first is traditional, and in most cases is based on material seen in the *Paitāmahasiddhānta*; the second is Brahmagupta's improvement and is generally more sophisticated as it attempts to take into account more detail. There are some exceptions to this. For example, the next verse, IV, 15, reasserts the symmetry of the half-durations, despite the previous rule that treats it as changing.

Next, Brahmagupta presents three verses for the determination of the *valana* or deflection:

prāk paścān nataviṣuva jyayor vadhāt trijyayāptacāpāṃśaiḥ |
uttarayāmye pūrvā viṣuvadvṛttāt tribhe grāhyā ||
samamaṇḍalād viṣuvato grāhyāt trigṛhādhikād udagyāmyaiḥ |
krāntyaṃśair apamaṇḍalapūrvāsyāś candravikṣepaḥ ||
ekānyadiśor yutiviyuter jyā pragrahaṇamadhyamokṣeṣu |
valanaṃ nimīlanonmīlaneṣṭakāleṣv ato 'nyadiśām ||
[Multiply the sine of the depression (in ghaṭikās from the meridian) to the east or west by the sine of the terrestrial latitude and divide (the result) by the radius. By the degrees of arc (corresponding to the quotient) the eastern (direction) is to be taken in a quadrant from the equatorial circle to the north or south from the prime meridian. Increase (the longitude of) the body to be eclipsed by three signs and (find) the degrees of declination (of that point) to the north or south of the equator. By (those) degrees (is determined) the eastern (direction) of the ecliptic. The latitude of the moon is (measured)

from this (eastern direction). The sine of the sum or the difference (of these two inclinations) as they are in the same or opposite directions is the (total) inclination, at the beginning of the eclipse, the middle, and the release, or at the beginning of totality, the reappearance after total immersion, or at given times.]
(BSS IV 16–18, as translated by Pingree and Montelle)

The concept of the *valana*, as discussed in relation to the *Pauliśasiddhānta*, is one that has no consequences mathematically or astronomically, but gives an additional "quality" to the occurrence of an eclipse that is loosely based on the angle of impact, which was deemed significant for astrological purposes. An observer could quite easily construct the north-south and east-west grid on the disks or ground, relative to the horizon. With respect to these coordinates, one can also observe the direction of impact of the eclipsing body and roughly set up a vector which runs through this point and the center of the eclipsed body. The *valana* measures the angle between these two (fig. 5.14).

Brahmagupta's treatment of this topic is the same as that presented in the *Sūryasiddhānta* as set out in the *Pañcasiddhāntikā* (XI, 1–5); they perhaps go

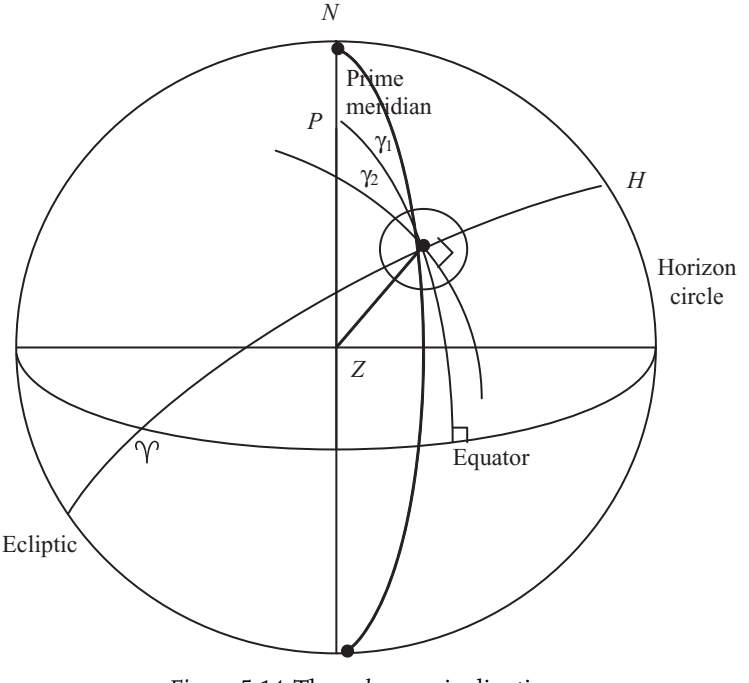


Figure 5.14 The *valana* or inclination.

back to a lost section of the *Paitāmahasiddhānta*. The first component, the *akṣavalana* (here γ_1), is given as

$$\gamma_1 = \frac{\text{Sin}(\Delta\tau) \times \text{Sin}\phi}{R},$$

where $\Delta\tau$ is the hour-angle (*nata*) measured in *ghaṭikā*s, and the *ayanavalana* (γ_2) is

$$\gamma_2 = \text{Sin}\delta(\lambda + 90^\circ).$$

The *akṣavalana* is an angle that is dependent on terrestrial latitude (ϕ). This angle is that made between the great circle going through the north point, the center of the eclipsing body, and the south point, and the great circle going through the center of the body, perpendicular to the equator. Thus, when the bodies are on the meridian, the *akṣavalana* is at its minimum value of zero. When the bodies are on the prime vertical, the angle is at its maximum, which is ϕ.

The *ayanavalana* (fig. 5.15) is an angle which is independent of location, and, as the formula indicates, is a function of declination, which in turn is a function of longitude translated by 90°. This angle is between the great circle

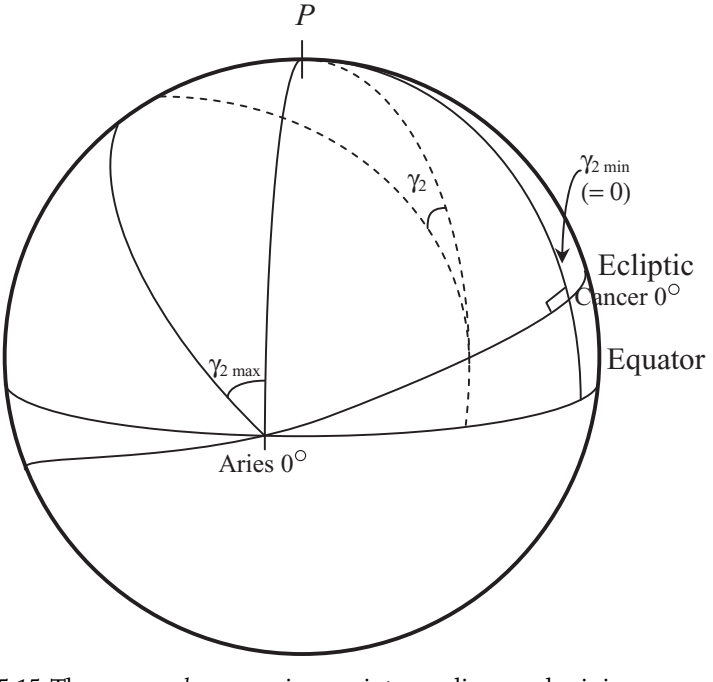


Figure 5.15 The *ayanavalana*: maximum, intermediary, and minimum positions.

perpendicular to the ecliptic which passes through the body, and the great circle passing through the body which is perpendicular to the equator.

When the eclipse occurs at a solstice, so that the two great circles coincide on the same great circle, the *ayanavalana* is zero, its minimum. When the eclipse occurs at the equinox, the two great circles differ by an amount equal to the maximum obliquity of the ecliptic. The angle varies sinusoidally between these two extrema. Thus, from Aries 0°, the *ayanavalana* is decreasing until it reaches the minimum at the solstice (Cancer 0°). Then the *ayanavalana* increases until it reaches the opposite equinox. Therefore the *ayanavalana* angle is simply the declination with the longitude displaced 90°, so that it increases and decreases in the right order. The total *valana* is just the sum or difference of these two components.

Although the underlying principle of the inclination or deflection is the same in Greek (*prosneusis*) and Indian sources, and both use two components whose sum or difference produces the total angle, the Indian *valana* differs from the Greek with respect to the two components themselves. In the Indian tradition, the angle at which one body hits the other is not a factor in the calculation. In the Greek tradition, one is to join up the centers of the sun and the moon, but in the Indian tradition, for ease of computation, one simply assumes that the moon is on the ecliptic. This is a good example of a concept that has been transformed substantially from its original formulation.

The fact that the two accounts of the *valana* are different may suggest that the Indian sources received this concept directly from Mesopotamian sources and not after a Greek passage. However, that there is a substantial difference in the reckoning of the two points at something much more important—the inclination is of little consequence astronomically, and therefore there were no restrictions as to how it should be reckoned. Scholars could define it as they liked. There were no mathematical or astronomical consequences beyond the expression itself. This may be another strong reason as to why its actual formulation changed from culture to culture.

Beyond the differences between cultures for the expression of the *valana*, Indian scholars themselves disagreed as to the precise formulation of the *valana*. There is a significant difference between Āryabhaṭa's expression of the *valana* and Brahmagupta's. The former uses the versine, the latter the sine. This difference was not limited to Āryabhaṭa and Brahmagupta, many prominent astronomers insisted upon the correctness of one or the other. The selection of either function in the formula initiated some fairly intense disagreement between generations of Indian astronomers. It was a feature that was irrespective of their loyalty to a particular *pakṣa*.

Versine		Sine	
Āryabhaṭa	(A 45)	Brahmagupta	(BSS)
Bhāskara I	(MB v 42–45)		(KKD)
	(LB iv 15–17)	Bhāskara II	(SŚi I 20–23)
Lalla	(ŚDV v 23)		(II ix 16–22)
Pṛthūdaka	(BSS iv 16–17)	Jñānarāja	(SS V, 26–27)
	(KSG 99–100)		
Bhaṭṭotpala	(KKD vol II p. 116–19)		
Śripati	(SŚe v 19–20)		
Āmarāja	(KKD)		
Parameśvara	(GM, GND)		

This table presents the particular preferences of various authors. Such a disagreement is puzzling. In fact the sine and the versine are in phase with each other from 0 to 90° (fig. 5.16). Thus, whether sine or versine is used, the difference in the overall result will be minimal: the extrema will be the same, but the points in the middle may vary a little. Therefore, with respect to this contention, it seems as though as early as Āryabhaṭa's time, the original

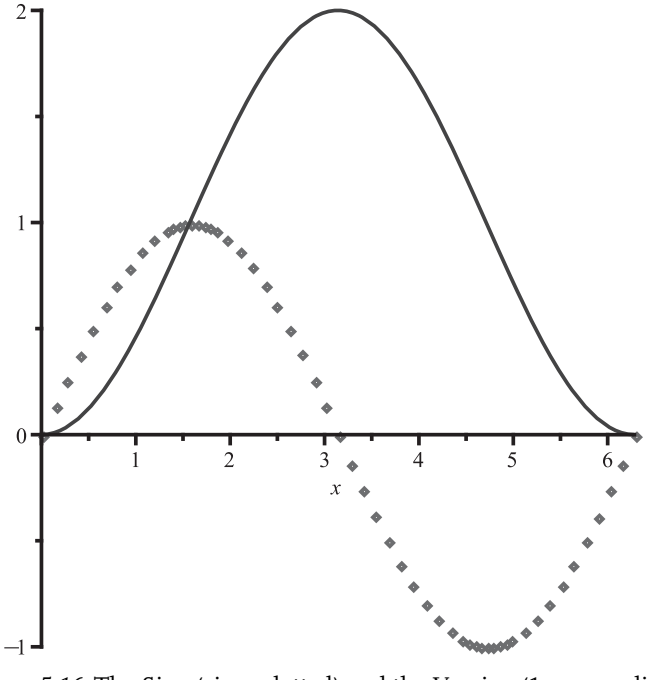


Figure 5.16 The Sine ($\sin x$; dotted) and the Versine ($1 - \cos x$; line) function; $R = 1$.

statement of the formula, from some very early text, had already been corrupted and the precise meaning had been lost.

Brahmagupta gives the colors for an eclipse as follows:

ādyantayoḥ sa dhūmraḥ kṛṣṇaḥ khaṇḍagrahe 'rdhato 'bhyādhike |
grāse sa kṛṣṇatāmraḥ sarvagrahaṇe kapilavarṇaḥ ||
[At the beginning and at the end (of the eclipse, the color of the moon is) smoky; in a partial eclipse, black; when the amount of obscuration is greater than half, a coppery-black; and in a total eclipse, reddish-brown.]
(BSS IV, 19, as translated by Pingree and Montelle)

Unlike the *Pauliśasiddhānta*, but following Āryabhaṭa, Brahmagupta assigns colors depending on the phase and/or the amount of obscuration of the eclipse, an identification that was to become standard. However, there was some variation of opinion over which colors to ascribe to the different cases. Here, Brahmagupta has repeated the colors of Āryabhaṭa (*Gola* 46).

Color	Translation	Situation
dhūmra	Smoky	Beginning or end
kṛṣṇa	Black	Partial
kṛṣṇatāmra	Coppery-black	Obscuration more than half
kapila	Reddish-brown (literally monkey colored)	Total

Brahmagupta finishes the section on lunar eclipses with the following summary verse:

mānavimardasthitidalavalaneṣṭagrāsasamakalādyeṣu |
candragrahaṇādhyāyo viṃśatirāryāścaturtho 'yam ||
[The fourth chapter, the eclipse of the moon, with respect to the measures, the half-durations of totality and half-durations, the inclination, given magnitudes, equal times, and so on (consists of) twenty āryas.]
(BSS IV, 20, as translated by Pingree and Montelle)

Chapter V of the *Brāhmasphuṭasiddhānta* is devoted to solar eclipse theory. Most of the material in this chapter covers the computation of parallax and how to apply it. The rules set out in chapter IV to find durations, obscuration, and so on, for solar eclipses are to be employed with the appropriate parameters. Brahmagupta at the outset of this chapter criticizes Āryabhaṭa, who uses the "five sines" referred to here (*Gola* 33–34):

dṛggaṇitaikyaṃ na bhavati yasmāt pañcajyayā ravigrahaṇe |
tasmād yathā tadaikyaṃ tathā pravakṣyāmi tithyante ||

[Because there is no unity of observation and computation in a solar eclipse by means of the five sines, therefore I will say how the unity of these at the end of the (syzygy) tithi (is computed).]
(BSS V, 1, as translated by Pingree and Montelle)

These are Sin$\alpha(S)$ (the altitude of the sun), Sinη (the rising amplitude), Sin$z(M)$ (the sine of the zenith distance of the meridian ecliptic point), SinVS (the sine of the longitudinal difference of the sun and the nonagesimal), and Sin$z(V)$ (the sine of the zenith distance of the nonagesimal). Brahmagupta in his rules for parallax often gives two options; the first is based on the use of the above measures for which he criticizes Āryabhaṭa, and the second replaces these values with alternative amounts.

Brahmagupta clearly distinguishes the two components of parallax, longitudinal (*lambana*) and latitudinal (*avanati*), by setting out the conditions under which each one has no effect.

vitribhalagnasame 'rke na lambanaṃ tadadhikonake bhavati |
tasya krāntijyodak yadākṣajīvāsamā na tadā ||
avanatir ato 'nyathā bhavati sambhave tad udayair vilagnasamam |
kṛtvā tad uditaghaṭikās tacchaṅkus taccaraprāṇaiḥ ||
[When (the longitude of) the sun is equal to (the longitude of) the nonagesimal, there is no longitudinal parallax; when it is greater or less than that, there is. When the sine of the declination is to the north (by an amount) equal to the sine of terrestrial latitude, then there is no latitudinal parallax. When it is other than this, there is. At the occurrence (of an eclipse) computing what is equal to the ascendant by means of the rising times (of that terrestrial latitude, calculate) the ghaṭikās of its (i.e., of the nonagesimal) ascension. The sine of the altitude (of the nonagesimal is calculated) by means of the time-minutes of its ascensional difference.]
(BSS V, 2–3, as translated by Pingree and Montelle)

He first states that there is an absence of longitudinal parallax when the longitude of the sun is equal to that of the nonagesimal. The total parallax then, if there is any, is entirely composed of a latitudinal correction. This follows from the formula for longitudinal parallax which is given by Brahmagupta in the next verse, a rule that has as a factor the amount SinVS, the sine of the longitudinal distance between the sun and the nonagesimal. If they are in the same place, it will be zero; thus the longitudinal component of parallax will be zero. By this statement Brahmagupta also exposes one of his fundamental assumptions, based on the procedures of his Indian predecessors: that is, that the parallax is dependent on the distance of the sun from the nonagesimal, not the zenith. This interesting premise, made for computational facility, has

some important consequences which affect the accuracy of the outcome, as will be discussed below.

He states moreover that when $\mathrm{Sin}\phi = \mathrm{Sin}\delta$ there will be no latitudinal parallax. This again is a direct consequence of his rule for latitudinal parallax, which has in its formulation the sine of the zenith distance of the nonagesimal, approximately $\mathrm{Sin}(\phi - \delta)$. When this element is zero, the latitudinal parallax is zero, so that the total parallax is entirely composed of longitudinal parallax. This implies that latitudinal parallax is based upon where the ecliptic is with respect to the horizon, and not the moon itself.

At the end of these two verses, Brahmagupta explains how to calculate the altitude of the nonagesimal. He states that one should first determine the ascendant using information about the rising times for a particular locality. From this the position of the nonagesimal is obtained by adding 90° to the longitude of the ascendant.

If the interpretation of the fragmentary material remaining in the *Paitāmahasiddhānta* is correct, then it seems that Brahmagupta bases his formula for longitudinal parallax on the one given there. He gives two rules:

> trijyākṛteścaturguṇaśaṅkuhṛtāyāḥ phalena bhaktāyāḥ |
> tātkālikārkarāśitrayonalagnāntarajyāyāḥ ||
> lambanaghaṭikā labdhaṃ lagnāt tātkālikāt trirāśyūnāt |
> ṛṇam adhike 'rke hīne dhanam asakṛt pañcadaśyante ||
> [Multiply the square of the radius by the sine of the latitude multiplied by four. By (that) result, divide the sine of the difference between (the longitudes of) the sun at that time and the ascendant diminished by three signs. The quotient is the ghaṭikās of longitudinal parallax. It is negative when (the longitude of) the sun is greater than (the longitude of) the ascendant at that time decreased by three signs, and positive when it is less. (This is to be calculated) iteratively at the end of the fifteenth (tithi).]
> (BSS V, 4–5, as translated by Pingree and Montelle)

Also

> karṇaguṇād vyāsārdhād vasuvedavibhājitāt phalavibhaktā |
> lambanānāḍyo bhāskaravitribhalagnāntarajyā vā ||
> [Or, multiply the radius by the noon hypotenuse and divide by 48. By (that) result divide the sine of the difference between (the longitudes of) the sun and the nonagesimal. (The result is) the naḍīs of longitudinal parallax.]
> (BSS V, 6, as translated by Pingree and Montelle)

These correspond to

$$\pi_\lambda = \frac{4 \cdot \mathrm{Sin}\alpha(V)\mathrm{Sin}(VS)}{R^2}$$

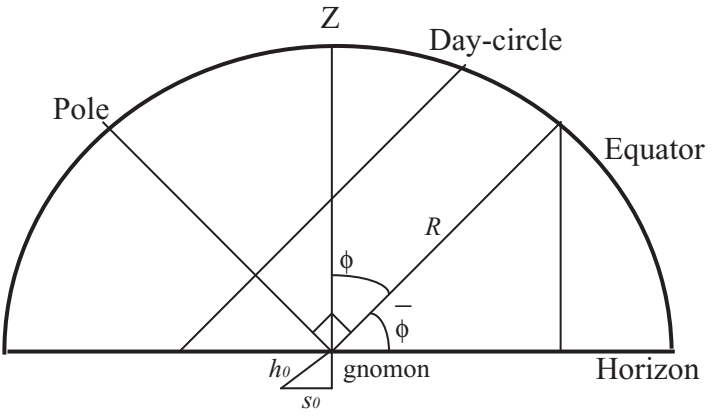


Figure 5.17 Similar triangles for alternative parallax formula.

and

$$\pi_\lambda = \frac{48 \cdot \mathrm{Sin}(VS)}{R.h_n},$$

in which 48 is four (maximum parallax in *ghaṭikās*) times twelve (the standard height of a gnomon in digits) and h_n is the "hypotenuse of the shadow." The second rule is simply a substitution of a rule of three ratio from similar spherical triangles (fig. 5.17):

$$\frac{h_n}{12} = \frac{R}{\mathrm{Sin}\alpha_n}.$$

Brahmagupta then gives a rule for the application of longitudinal parallax to the longitudes:

> raviśaśipātagatikalā lambanaghaṭikāguṇā hṛtāḥ ṣaṣṭyā |
> yadi lambanam ṛṇam ūnā dhanam adhikāḥ svaphalaliptābhiḥ ||
> [The minutes of the daily velocities of the sun, the moon, and the node are multiplied by the ghaṭikās of longitudinal parallax and divided by sixty. If the longitudinal parallax is negative, (their respective longitudes) are decreased by the minutes of their own results and (if) positive, increased.]
> (BSS V, 7, as translated by Pingree and Montelle)

Correspondingly, he gives two rules for the latitudinal parallax, but these are preceded by rules that describe the application (i.e., positively or negatively) of latitudinal parallax to the positions of the luminaries:

> akṣajyāyā vitribhalagnāt svakrāntir uttarā 'rkasya |
> indorvā yadyadhikā 'vanatiḥ saumyā 'nyathā yāmyā ||
> vitribhalagnād uttaradakṣiṇavikṣepahīnasaṃyuktam |
> śaṅkudhanur uttarāyām adhikonaṃ dakṣiṇāvanatau ||

tajjyenduśaṅkur ādyaḥ savitur dṛkkṣepamaṇḍale yukte |
apamaṇḍalena bhānoścandrasya vimaṇḍalena yute ||
[If (the sine of) the declination of the sun or the moon, to the north of the nonagesimal, is greater than the sine of the terrestrial latitude, the latitudinal parallax is to the north; otherwise it is to the south. When the latitudinal parallax is to the north, the arc of the sine of the altitude (of the nonagesimal) is decreased or increased by the (lunar) latitude as it is to the north or the south of the nonagesimal. (When it is) to the south, increased or decreased. The sine of the altitude of the moon is its sine; similarly, in the case of the sun; (the altitude) of the sun is where the circle of zenith distance joins the ecliptic, and of the moon, where it joins the lunar orbit.]
(BSS V, 8–10, as translated by Pingree and Montelle)

The two rules for latitudinal parallax are:

trijyāvargāvūnau svaśaṅkuvargeṇa tatpade dṛgjye |
raviśaśimadhyagatiguṇe tithiguṇitavyāsadalabhakte ||
tacchāyāguṇite vā madhyagatī tithiguṇasvakarṇahṛte |
phalayor diksāmye 'ntaram avanatir aikyaṃ diganyatve ||
[The two squares of the radius are decreased by the square of the sine of their altitudes [i.e., of the sun and the moon]. The two square roots from that are the two sines of the zenith distances. (These) are multiplied by the mean daily velocities of the sun and the moon (respectively) and divided by the radius multiplied by fifteen. Or, the two mean daily velocities are multiplied by their (noon) shadows and divided by their (noon) hypotenuses multiplied by fifteen. When their directions are the same, the latitudinal parallax is the difference of the two results. When the directions are different, the sum.]
(BSS V, 11–12, as translated by Pingree and Montelle)

The first rule is

$$\pi_\beta = \frac{\mathrm{Sin}z(V) \cdot \overline{\nu}}{15R}.$$

The latitudinal parallax of the sun uses the mean daily velocity of the sun, and the lunar latitudinal parallax uses that of the moon. As with the longitudinal parallax, the second rule has replaced the component with the nonagesimal in it:

$$\pi_\beta = \frac{\overline{\nu} \cdot s_n}{15 \cdot h_n}.$$

Here $\mathrm{Sin}z(V)$ has been replaced according to the following ratio:

$$\frac{h_n}{s_n} = \frac{R}{\mathrm{Sin}z(V)},$$

where s_n is the length of the shadow of a 12-digit gnomon at noon. The latitudinal parallax is applied to the lunar latitude as follows:

saṃyogāntaram avanatiśaśāṅkavikṣepayoḥ samānyadiśoḥ |
sphuṭavikṣepaḥ śaśivat sthityardhavimardadalanāḍyaḥ ||
[The sum or difference of the latitudinal parallax and the latitude of the moon (as they are) in the same or the opposite direction is the corrected lunar latitude. Like (an eclipse of) the moon, the naḍīs of half-duration or half-duration of totality (are computed).]
(BSS V, 13, as translated by Pingree and Montelle)

There are certain assumptions that Brahmagupta makes by invoking these parallax rules:

1. The zenith distance of the nonagesimal $z(V)$ is sufficiently close to the zenith distance of a body on the meridian so that they can be considered the same:

$$\mathrm{Sin} z(V) \approx \mathrm{Sin} \overline{\alpha}_n.$$

This is calculated from $\mathrm{Sin}(\phi \pm \delta)$. Similarly, the zenith distance of the moon is equal to $\mathrm{Sin}(\phi \pm \delta \pm \beta_m)$ (i.e., the position of the ecliptic when it crosses the meridian increased by the latitude of the moon).

2. The mean daily motions are sufficiently close to the true ones.

Moreover, the widely varying geocentric distances according to this model are not taken into account. There is no accounting for parallax in terms of the eclipse limits. There is no accounting for the variability of latitude of place in the maximum parallax.

By computation of the appropriate parallax for each phase of the eclipse, the true times, durations, and magnitudes can be obtained. Brahmagupta describes the effects of parallax on the half-duration:

prāgval lambanam asakṛt tithyantāt sthitidalena hīnayutāt |
adhikonaṃ tanmadhyād ṛṇayor ūnādhikaṃ dhanayoḥ ||
yadyadhikaṃ sthityardhaṃ tadāntareṇānyathonam ṛṇam ekam |
anyad dhanaṃ tadaikyenādhikam evaṃ vimardārdhe ||
[First, the longitudinal parallax (is to be computed) iteratively from the end of the (syzygy) tithi decreased or increased by the half-duration. When the two (times of longitudinal parallax) are negative they are added to and subtracted from the middle of that (eclipse). When positive, subtracted from and added to. The half-duration is increased by their difference, or else decreased. If one is negative and the other positive, (the half-duration) is decreased or increased by their sum. In the same way, the half-duration of totality (is computed).]
(BSS V, 14–15, as translated by Pingree and Montelle)

Further details about the application of parallax are described:

sphuṭatithyantāl lambanam asakṛt sthityardhahīnayuktādvā |
tatsphuṭavikṣepakṛtasthityardhonayutatithyantāt ||

tatspaṣṭatithicchedāntare sphuṭe sthitidale vihīnayutāt |
svavimardārdhenāsakṛd evaṃ spaṣṭe vimardārdhe ||
śaśivad bāhuḥ sphuṭavikṣepakṛtasthitidalena saṅguṇitaḥ |
spaṣṭasthityardhahṛto bhavati bhujaḥ pūrvavaccheṣam ||
grāsāt kālaḥ śaśivat spaṣṭasthitidalaguṇo 'sakṛd bhaktaḥ |
sphuṭavikṣepakṛtasthitidalena śodhyaḥ sthitidalāt syāt ||
[The longitudinal parallax (is computed) iteratively from the accurate end of the (syzygy) tithi or (from that) decreased or increased by the half-duration, (or) from the end of the (syzygy) tithi decreased or increased by the half-duration computed by means of the corrected (lunar) latitude. The two half-durations, (using) the differences from that corrected division between the tithis, are (made) true. (From the true end of the syzygy tithi) decreased or increased by its half-duration of totality (the longitudinal parallax) is (computed) iteratively. In the same way, the half-durations of totality (are made) true. Like (an eclipse) of the moon, the bāhu is multiplied by the half-duration computed by means of the corrected (lunar) latitude and divided by the half-duration corrected (by longitudinal parallax. The result is) the true bhuja. The rest is as previously. The time (of the given phase before or after the time of maximum) obscuration like (an eclipse of) the moon, is multiplied by the half-duration corrected (by the longitudinal parallax) iteratively and divided by the half-duration computed with the corrected (lunar) latitude. Subtract (the result) from the half-duration.]
(BSS V, 16–19, as translated by Pingree and Montelle)

Brahmagupta states the following as limits for a solar eclipse:

valanādi śaśivad anyad grahaṇaṃ taikṣṇyād raver anādeśyam |
dvādaśabhāgād ūnaṃ svacchatvāt ṣoḍaśād indoḥ ||
[The remaining (calculations, that is of) the inclination and so on, (are computed) as in the case (of an eclipse) of the moon. An eclipse of the sun is not to be predicted if less than a twelfth (of its disk is to be obscured) because of its brilliance, (and an eclipse) of the moon (is not to be predicted) if less than a sixteenth (of its disk is obscured) because of its clearness.]
(BSS V, 20, as translated by Pingree and Montelle)

Following his disapproval at the outset of this chapter, Brahmagupta continues to be openly critical of Āryabhaṭa and his methods, setting aside a whole verse plus a portion of another to raise his objections (V, 21 and 25). His criticism seems focused on the lack of longitudinal corrections to the sun, the moon, and the node, as well as the use of the sine of the altitude of the nonagesimal in parallax computations:

na sphuṭam āryabhaṭādiṣv arkagrahaṇaṃ yatas tataḥ spaṣṭam |
śaṅkujyayā kṛtaṃ laghu laghutaram evaṃ raver grahaṇam ||

[Since (the computations for) an eclipse of the sun in (the works of) Āryabhaṭa and so on are not accurate, therefore the easy eclipse of the sun, corrected by means of the sine of the altitude, is made even easier.]
(BSS V, 21, as translated by Pingree and Montelle)

pūrvavad anyat spaṣṭaṃ brahmoktaspaṣṭasūryaśaśipātaiḥ |
nāryabhaṭādibhir uktā[87] yato 'sphuṭās te tato 'spaṣṭam ||
[The rest is correct as previously by means of the corrected (longitudes of) the sun, the moon, and the node pronounced by Brahma. Since they are not mentioned by Āryabhaṭa and so on, therefore their (longitudes) are not accurate. (His computations) are incorrect.]
(BSS V, 25, as translated by Pingree and Montelle)

Additional rules for the computation of parallax are stated, and as promised by Brahmagupta, use the sine of the altitude:

lagnāt trirāśihīnādapakramākṣāṃśayutiviśeṣonāt |
bhatritayājjyā chedastrijyārdhakṛteḥ phalena hṛtā ||
vitribhalagnārkāntarajīvā ghaṭikādi lambanaṃ labdham |
vitribhalagnāpakramavikṣepākṣāṃśayutiviyuteḥ ||
jīvā śaśāṅkabhāskaramadhyamabhuktyantareṇa saṅguṇitā |
pañcadaśabhirguṇitayā vibhājitā trijyayā 'vanatiḥ ||
[The sine of 90° decreased by the sum or difference of the terrestrial latitude and the declination of the ascendant decreased by three signs [i.e., the nonagesimal] is the divisor of the square of the radius. [The result is the sine of the altitude of the nonagesimal.] By the result divide the sine of the difference between (the longitude of) the nonagesimal and the sun. The quotient is the longitudinal parallax (measured in) *ghaṭikās*, etc. The sine of the sums and differences of the declination of the nonagesimal, the lunar latitude and the terrestrial latitude is multiplied by the difference between the mean daily velocities of the moon and the sun and divided by the radius multiplied by fifteen. (The result is) the latitudinal parallax.]
(BSS V, 22–24, as translated by Pingree and Montelle)

These are the same as those given in the *Paitāmahasiddhānta*. The longitudinal parallax is

$$\pi_\lambda = \frac{\mathrm{Sin}[\lambda(V) - \lambda(S)] \times \mathrm{Sin}[\alpha(V)]}{R^2}$$

and the latitudinal parallax:

$$\pi_\beta = \frac{\mathrm{Sin}[\delta(V) + \beta + \phi] \times (\nu_m - \nu_s)}{15R}.$$

Last, Brahmagupta describes the colors associated with a solar eclipse:

aṅgulamātre virate raktaḥ śaśimaṇḍale bhaved varṇaḥ |
bhānos tu punaḥ kṛṣṇo varṇaḥ sarvatra nirdiṣṭaḥ ||

[When the disk of the moon has stopped at a digit (of obscuration), the color of the sun is red. But everywhere (else) the color is shown to be black.]
(BSS V, 26, as translated by Pingree and Montelle)

In most early astronomical traditions, it was less common to give the colors of a solar eclipse. This is puzzling as, despite the fact that solar eclipses occur much less frequently, they are far more visually spectacular than lunar eclipses. One would imagine that an annular eclipse and the light effect that this created would be mentioned in the above verse.

He closes his chapter on solar eclipses, with the following summary verse:

iṣṭagrāsavimardasthityardhāvanatilambanādyeṣu |
āryāṣaḍviṃśatyā 'rkagrahaṇaṃ pañcamo 'dhyāyaḥ ||
[The fifth chapter, the eclipse of the sun, with respect to the amount of obscuration at a given (time), the duration of totality, the half-duration, and latitudinal parallax, the longitudinal parallax and so on (consists of) twenty-six āryas.]
(BSS V, 27, as translated by Pingree and Montelle)

Thus ends Brahmagupta's initial treatment of lunar and solar eclipses. Brahmagupta does revisit this area later in his work, but that material is supplementary and represents Brahmagupta's improvements over these first chapters.

Chapter XVI covers graphical projections. Among the topics covered in this chapter, Brahmagupta gives the rules for predicting eclipses (BSS 16, 29–33). Here he uses a concept called the *parvan*, an interval of six months or 177 civil days. For each parvan, Brahmagupta gives corresponding values, which he calls *cālana* (longitudinal increments). These are the increments in mean motions for 177 days of the relevant phenomena: those of the sun, the moon, the moon's *manda* epicycle, and the moon's node. These are to be added to the mean longitudes of the bodies of the previous eclipse possibility.

Brahmagupta explains how to utilize these *cālana* values:

pātārkayutir bhārdhāt cakrād vonādhikā kalā bhaktā |
tadgatiyutyāptadinair āsanno 'rkasya māsānte ||
parvendoḥ pakṣānte prāg adhikonā bhavati paścāt |
tanmadhyena grahaṇaṃ yadi bhānoḥ pañcajinabharasāḥ |
5 | 24 | 27 | 6 || 30 ||
indor viṣayā dviyamā divākarās triviṣayās taduccasya |
5 | 12 | 12 | 53 |
vyomātidhṛtir dviyugāni rasaśarāś candrapātasya |
0 | 19 | 42 | 56 || 31 ||
khaṃ nandā dviyamāḥ svābdhayo
0 | 9 | 22 | 40 |

gṛhādyās tatheṣṭaparvaguṇāḥ |
kṣepyāḥ parvaṇy eṣye śodhyā madhye tv atikrānte ||
grahaṇe yathā ravīndvoḥ spaṣṭīkaraṇādyam uktavaj jñātvā |
evaṃ parvajñānaṃ grahaṇajñānaṃ sphuṭaṃ gaṇitāt ||
[The minutes in the sum of the (longitudes of) the node and the sun, which are less than or greater than a semi-circle or a circle, are divided by the sum of their motions; by the resulting days is calculated the parvan of the sun at the end of the month, that of the moon at the end of the pakṣa, before, if (the sum) is greater, after, if less. If there is an eclipse, to their mean (longitudes) 5, 24, 27, 6 of the sun; 5, 22, 12, 53 of the moon; 0, 19, 42, 56 of its apogee; and 0, 9, 22, 40 of its node in signs and so on, multiplied by the given parvan, are to be added to the mean (longitude of each) if the parvan is in the future, subtracted if it is passed.]
(BSS XVI, 29–32, as translated by Pingree and Montelle)

These *cālana*s given by Brahmagupta are

Sun	$5^s\ 24;27{,}06°$
Moon	$5^s\ 22;12{,}53°$
Manda	$19;42{,}56°$
Node	$9;22{,}40°$

These numbers are generated by simply multiplying the mean daily motions by 177:

$$5\ 24;27{,}6 \div 177 = 0;59{,}8{,}10{,}10, \ldots,$$

$$5\ 22;12{,}53 \div 177 = 13;10{,}34{,}52{,}32{,}32, \ldots,$$

$$0\ 19;42{,}56 \div 177 = 0;6{,}40{,}59{,}39{,}39, \ldots,$$

$$0\ 9;22{,}40 \div 177 = 0;3{,}10{,}44{,}4{,}4, \ldots.$$

These values are approximately those that Brahmagupta gives in I, 44–49.

In this chapter, Brahmagupta also discusses a method for finding the longitudinal difference between the calculated and the observed times of eclipses (XVI, 27–28), which is taken from the *Paitāmahasiddhānta* (III, 27).

In chapter XXI, Brahmagupta repeats the formulas given in the *Paitāmahasiddhānta* for the calculation of the circumferences of the orbits of the planets in *yojanas* (XXI, 11–14), including the crude estimate for the value of $\pi = \sqrt{10}$. He then gives a slightly different value for the diameters of the sun,[88] the moon, and the earth (XXI, 32):

Sun	6522 *yojanas*
Moon	480 *yojanas*
Earth	1581 *yojanas*

Although the diameters look different from those that were presented in the *Āryapakṣa* section, they use the same base values, simply assuming different amounts for the number of *yojanas* equal to one minute of arc at the moon's distance. Brahmagupta uses 15, Āryabhaṭa 10. This may imply that Brahmagupta considers the moon to be farther away than Āryabhaṭa does. Thus the (mean) angular diameter for the moon is

$$\frac{480}{15.60} = 0;32^\circ,$$

which is identical to that calculated above (IV, 6).

From the above value for the diameter of the earth, one can indirectly derive the lunar horizontal parallax:

$$\frac{1581}{2.15.60} = 0;52,42^\circ.$$

This provides a suitable check of the soundness of Brahmagupta's approximation of the maximum parallax as 1/15 of the daily lunar velocity:

$$\pi_{\max} = \frac{13;10,34\ldots}{15} = 0;52,42,\ldots^\circ.$$

Here he also gives an additional rule for finding the diameter of the earth's shadow at the moon's distance (XXI, 33):

$$d_u = d_e - (d_s - d_e)] \cdot \frac{k_m}{k_s},$$

which is based on similar triangles (see fig. 5.7).

5.6.5 *The* Ārdharātrikapakṣa: *Brahmagupta's* Khaṇḍakhādyaka

The *Khaṇḍakhādyaka* of Brahmagupta, with the epoch of 23 March 665,[89] was written some forty years after the *Brāhmasphuṭasiddhānta*. The title of this work reveals its author to have a rich and playful sense of humor—*Khaṇḍakhādyaka* literally means "Consuming Candies"; a whet to the appetite, it seems, as Brahmagupta seeks to tempt his potential audience with the promise of sweet intellectual satisfaction. It seems strange that Brahmagupta who is so openly critical of Āryabhaṭa, the originator of the *Ārdharātrikapakṣa*, decided to write an astronomical work in that tradition originally conceived of by his rival. However, it may be that Brahmagupta believed the best way to criticize and trump his intellectual rival would be to outshine him in his own system. Indeed, Brahmagupta expresses his justification for the *Khaṇḍakhādyaka* as providing an alternative far superior to Āryabhaṭa's "midnight system." He is direct about this motivation:

vakṣyāmi khaṇḍakhādyakam ācāryāryabhaṭatulyaphalam ||
prāyeṇāryabhaṭena vyavahāraḥ pratidinaṃ yato 'śakyaḥ |
udvāhajātakādiṣu tatsamaphalalaghutaroktir ataḥ ||

> [I now write the *Khaṇḍakhādyaka* which gives the same results as those obtained from Āryabhaṭa's formulas. His rules are lengthy and hence impractical for daily purposes, such as marriage, birth, and the like. Mine, on the other hand, are brief, yet yield similar results.
> (KKD I, 1–2, as translated by Chatterjee [1970])

In a fashion similar to the *Brāhmasphuṭasiddhānta*, Brahmagupta organizes his work into two sections. In the first section Brahmagupta (claims to) present the astronomical constants given by Āryabhaṭa. The second section, which he calls the *uttara* (literally "latter") section, is where Brahmagupta adds his own elaborations and corrections to the material presented in the first section. He reminds the reader again in this second section of his intention to improve upon the methods of his rival:

> na sphuṭam āryabhaṭoktaṃ spaṣṭīkaraṇaṃ yatas tato vakṣye |
> [I write (the second part) because the rules of Āryabhaṭa do not give correct results.]
> (KKD (*uttara*) I, 1, first half, as translated by Chatterjee [1970])

Even though the *Khaṇḍakhādyaka* has an epoch different from that of the *Brāhmasphuṭasiddhānta*, to a large extent the eclipse calculations remain unaffected by this change. One immediately obvious difference is that in the *Khaṇḍakhādyaka* Brahmagupta uses $R = 150$ for the value of the trigonometric radius, and, as a result, many of the rules contain different multipliers.

Various commentaries on the *Khaṇḍakhādyaka* exist, but one of the more notable is that by the twelfth-century commentator Āmarāja. One of the significant features of Āmarāja as a commentator is that he tackles and attempts to explain many of the tricky procedures and concepts that other commentators gloss over (see section 5.1 for an excerpt from his commentary). In his lengthy explanations and derivations, one can gain insight into how these rules were received and understood by students and practitioners. His prose is punctuated with hypothetical questions, some trivial, some complex, which he then goes on to answer. Moreover, Āmarāja's commentary is important because is reveals the ways in which Brahmagupta's text was understood and justified in the tradition of later scholars. The *Khaṇḍakhādyaka* presents rules for eclipses that are similar to those presented in the *Brāhmasphuṭasiddhānta*, but it does contain unique features which will be highlighted here.

The first verse gives a rule for determining the celestial latitude of the moon. Āmarāja expands upon this rule. He gives the maximum latitude of the moon as 270 minutes, or 4;30°. Āmarāja then quotes a verse[90] that he ascribes to the *Brāhmasphuṭasiddhānta* that gives the values of the diameters of the sun, moon and shadow as 6522, 480, and 1581, respectively.[91] He shows how to

derive the rules seen in the *Brāhmasphuṭasiddhānta* for the calculation of the apparent diameters by means of their velocities. He uses the assumption that all planets move with the same linear velocity on their own orbits.

Āmarāja gives the following quotients[92] and shows that by extracting a common factor (592 and 48, respectively), they give the multipliers that are in Brahmagupta's formulas:

$$\frac{6522}{11858;45} \approx \frac{11}{20} \text{ and } \frac{480}{11858;45} \approx \frac{10}{247}$$

The *Khaṇḍakhādyaka* gives rules for eclipse limits, colors, and *cālana*s, and these are identical to those given in the *Brāhmasphuṭasiddhānta*.

Āmarāja attempts to clear up the disagreement over the exact formulation of the rule for the *valana* in texts predating this commentary. Āmarāja finds himself caught between in the *Khaṇḍakhādyaka* what Brahmagupta says and what his predecessors say. Brahmagupta repeats the rule he presented in the *Brāhmasphuṭasiddhānta*, which uses the sine in the formula for γ_1. Unusually, Āmarāja disagrees openly with Brahamgupta's, criticizing his use of the sine rather than the versine. In a delightful and thoroughly confusing passage, Āmarāja seeks to resolve the contention. Significantly, his explanation focuses on the geometry of the model, rather than the arithmetical patterns:

> svadeśakhāgranyaste bhagole samamaṇḍalād viṣuvanmaṇḍalaṃ svākṣabhāgairdakṣiṇena tiryag bhavati | yadā viṣuvanmaṇḍalagatyā prāk grāhyabimbaṃ nīyate tadā bimbavat pūrvā tayaiva gatyā valaty ataḥ samamaṇḍalānusāreṇa grāhyabimbe yā pūrvā tasyā uttareṇa pāramārthikī pūrvā varttate | tena pūrvayā ākṣā valanāṃśā uttarā bhavanti | te ca kiyanta iti tadarthaṃ trairāśikam | yadi trijyātulyanatotkramajyayā 'kṣajyā valanajyā bhavati tadiṣṭanatotkramajyayā kiyatīti phalaṃ valanajyā | yato yāmyottaramaṇḍalasthe grāhye valanajyābhāvo viṣuvanmaṇḍalasamamaṇḍalasampāte tv akṣajyātulyaṃ paramaṃ valanaṃ tac ca yāmyottaramaṇḍalāt prabhṛty utkrameṇopacīyate | ata utkramajyayā valanānayanam upapannam |
>
> [When the orb of the nakṣatras is set up with the tips of the latitude of its place [being flat], the equator is oblique to the south of the six o'clock circle by the degrees of its latitude. Whenever the body to be eclipsed is brought into the eastern [hemisphere] by the motion of the equator, then, like the body, the "purva" is deflected by just that motion [of the equator]. Whatever is the purva of the disk of the body to be eclipsed with respect to the *samamaṇḍala,* the real purva is to the north of that. The degrees of the *akṣavalana* are north of the purva by that amount. There is a rule of three to answer the question how much are these: If the sine of the latitude is equal to the versine of the *nata* when it is equal to the Radius, how much is the sine of the *valana* measured by the versine of a given *nata*? The result is the

> sine of the *nata*, since when the body to be obscured is placed on the meridian, there is no sine of the *valana*, but when it is placed on the intersection of the *samamaṇḍala* and the equator the *valana*, being maximum, is equal to the sine of the latitude, and this increases as you go down, beginning from the meridian. It is [thus] shown that the calculation of the *valana* is to be produced by the versine [and not by the sine as Brahmagupta says!].]
> (KKD p. 135 line 16ff, as translated by Montelle)

Āmarāja rightly points out (without explanation) that the *akṣavalana* is zero when it is on the meridian. This is because the line going through the body to and from the north and south points is the same as the great circle going through the pole of the equator. He then continues that the *akṣavalana* is at a maximum, equal to $\mathrm{Sin}\phi$, when the body is at the intersection of the prime vertical and the equator (i.e., at the horizon). He then infers that this angle increases as one progresses toward this point of intersection. He concludes that because of this one must use the versine and not the sine.

However, this conclusion is by no means correct. The values of both the sine and the versine ($= R - \cos\theta$) increase as the angle increases (although with different increments). Therefore, this is not a valid reason for using the versine.

In order to provide further information about the method of *parvan*s given by Brahmagupta, Āmarāja cites a rule from an earlier commentary by the tenth-century commentator Bhaṭṭopala on the *Khaṇḍakhādyaka*:

> dinavṛndāt khaśara 50 ghnāt ṣaṇnavaravi 1296 bhir vibhājitād āptam |
> rāmarttukhendu 1063 sahitaṃ kṣiped dyuvṛnde bhajet khadhṛti 180 bhistat ||
> labdhaḥ kamalajapūrvaḥ parvagaṇaḥ saptabhājite śeṣaḥ |
> gatagamye tithyū 15 ne khaguṇo 30 ne candrasūryaparva syāt ||
> [One should add the quotient, (produced) from the number of days multiplied by 50 and divided by 1296, increased by 1063, to the number of days. One should divide this by 180. The result is the quantity of the *parvan* beginning with Brahma. When what has passed or what is to come is divided by 7, then the remainder if it is less than 15 or less than 30, is the *parvan* of the Moon or Sun (respectively).]
> (KKD p. 145 ll. 17–20. See also Chatterjee [1970] vol. II, p. 104, lines 3–6)

As Āmarāja later explains there are seven *parvan*s. He cites a verse from Varāhamihira which lists these seven *parvan*s:

> ṣaṇmāsottaravṛddhyā parveśāḥ sapta devatāḥ kramaśaḥ |
> brahmaśaśīndukuverā varuṇāgniyamāś ca vijñeyāḥ ||
> [The seven deities, in order, Brahma, Śaśi, Indra, Kubera, Varuṇa, Agni, and Yama are to be known as the lords of the Parvans by the increase of the difference of six months.]

(KKD p. 146 ll. 6–7. See also *Bṛhatsamhitā* of Varāhamihira V, 19, as translated by Bhat [1996])[93]

There are seven lords of the *parvan*s, perhaps because of the fact that eclipses occur six months apart for seven eclipse possibilities (or sometimes eight—see section 3.4.1), and then a 5-month eclipse interval occurs. Varāhamihira's original text goes on to explain that each *parvan* was presided over by one of these deities and has a different effect for the people and their lands (such as prosperity of cattle, good crops, drought famine, destruction of crops, trouble for the king).

Given the number of days since the epoch (a), the *parvan* is computed as follows:

$$\text{The current } parvan + \text{a remainder} = \frac{50a + 1063}{1296}.$$

The latter part of this verse gives a relation the meaning of which is unclear. Supposedly, if

$$\frac{\text{remainder}}{7} \text{ or } \frac{1 - \text{remainder}}{7} < 15,$$

the *parvan* is for the moon, and if the above inequality is less than 30, the *parvan* is for the sun. It is not clear what is intended here.

Āmarāja gives a detailed digression on solar eclipse limits which shows how one applies the components of parallax to get a more accurate limit, as well as how they are derived.[94] He gives the sum of the radii of the sun and the moon to be 0;34,16, and the maximum parallax in latitude to be 0;48,44 (shown below). Taking the sum, (83,0), he states that this is the limit in latitude at which the moon does not eclipse the sun. Brahmagupta's rule for lunar latitude is

$$\beta = \frac{9}{5} \cdot \text{Sin}(\lambda_m - \lambda_n).$$

He calculates:

$$\begin{aligned} 83{,}0 &= \frac{9}{5} \cdot \text{Sin}(\lambda_m - \lambda_n) \\ \Rightarrow \text{Sin}(\lambda_m - \lambda_n) &= 46{,}7 \\ \Rightarrow (\lambda_m - \lambda_n) &= 17{,}52. \end{aligned}$$

To this limit he adds the maximum equation of the sun, which he gives as 2;10,49°. He gives the sum as 20,2,55 (in fact, it is 20,2,49, but such oversights by Āmarāja are common—they may suggest that he knows the value he wants to end up with and will adjust intermediary values accordingly). This amount

is converted into minutes by multiplying by 60 and dividing by the difference in the daily velocities of the sun and the node:

$$\frac{20{,}2{,}55 \cdot 60}{62{,}19} = 19{,}18 \text{ minutes.}$$

He goes through a similar procedure to derive the lunar eclipse limits, except that the method (correctly) does not require the application of parallax.

The sum of the radii of the moon and the shadow he gives as 0;56,23.[95]

$$\begin{aligned} 40, 38 &= \frac{9}{5} \cdot \mathrm{Sin}(\lambda_m - \lambda_n) \\ \Rightarrow \mathrm{Sin}(\lambda_m - \lambda_n) &= 31{,}19 \\ \Rightarrow (\lambda_m - \lambda_n) &= 12;2{,}52^\circ. \end{aligned}$$

Adding on the maximum equation of the sun, he gets

$$12{,}2{,}52 + 2{,}10{,}49 = 14{,}13{,}41$$

and finally

$$\frac{14{,}13{,}41 \times 60}{62{,}19} = 13{,}45 \text{ minutes.}$$

(In fact the multiplication should result in 13,42 minutes.)

Āmarāja, however, warns:

> etāni sthūlāni | trijyāyā laghutvāt | mahatyā ca trijyayā sūkṣmāṇi caturvedoktāni jāyante |
> [These (limits) are crude (*sthūlāni*) because of the smallness of the Radius, but accurate (limits) spoken by Caturveda are born (because of) the large Radius.]
> (KKD p. 151 ll. 4–5)

Here, Āmarāja's reference to "Caturveda" must be to the commentator Pṛthūdakasvāmin. Āmarāja must be referring to the fact that the base radius for the trigonometric functions in the *Khaṇḍakhādyaka* is 150, versus that of the *Brāhmasphuṭasiddhānta* which is 3270. A larger radius gives a more accurate integer output than a smaller radius.

Āmarāja also gives a demonstration of the rule for the maximum parallax.[96] He sets out the following values for the orbital radii of the luminaries and true diameter of the earth:

$$\begin{aligned} k_s &= 684{,}870 \textit{ yojanas}, \\ k_m &= 51{,}229 \textit{ yojanas}, \\ r_e &= 790;30 \text{ (from } d_e = 1581) \textit{ yojanas}. \end{aligned}$$

He calculates the longitudinal parallax by the following rule of three:

$$\frac{r_e}{k_s} = \frac{\pi_\lambda}{k_s - k_m}$$

so that

$$\pi_\lambda = \frac{790;30 \times 633641}{684870}$$

$$\approx 731 \textit{ yojanas}.$$

He uses another rule of three to convert π_λ into minutes, which is based on Brahmagupta's relation that fifteen *yojanas* equals one minute of the orbit of the moon at its mean distance.

$$\frac{3415;30 \textit{ arcminutes}}{51229 \textit{ yojanas}} = \frac{\pi_\lambda(\textit{arcminutes})}{731 \textit{ yojanas}}$$

$$\pi_\lambda(\textit{arcminutes}) = \frac{3415;30 \times 731}{51229}$$

$$= 48;44, 11, \ldots \textit{ arcminutes}.$$

He then proceeds to convert the parallax into *ghaṭikā*s:

> yadi ravicandrabhuktyantareṇa ṣaṣṭir ghaṭikā bhavanti tallambanaliptābhiḥ kiyatya iti phalaṃ catasro lambanaghaṭikāḥ paramāḥ |
> If 60 *ghaṭikā*s are measured by the difference of the daily velocities of the sun and the moon, how many are measured by the minutes of its longitudinal parallax? The result is the maximum longitudinal parallax (measured in) *ghaṭikā*s (which is) four.
> (KKD p. 155, lines 14–15)

Therefore, if 48;44 is divided by 12—the approximate daily difference between the velocities of the sun and the moon being $13 - 1$—the result is $4;03,40 \approx 4$ *ghaṭikā*s. Thus, in a moment rare in Indian astronomy, Āmarāja has carefully and systematically shown why the Indian astronomers use the approximation of horizontal parallax $= 4$ *ghaṭikā*s.

The rest of the material in the *Khaṇḍakhādyaka* is similar if not identical to the rules and parameters given in the *Brāhmasphuṭasiddhānta*.

5.6.6 *The* Āryapakṣa*: Bhāskara I*

Bhāskara I (fl. 629) was an important seventh-century astronomer who followed the *Āryapakṣa* school and was one of the main proponent's of Āryabhaṭa's astronomical system. He composed three works, the *Mahābhāskarīya*, the *Laghubhāskarīya*, and the *Āryabhaṭīyabhāṣya*.

Bhāskara I's most infamous addition to eclipse theory is his claim[97] that parallax must be applied in the case of a lunar as well as a solar eclipse. This statement is repeated in several places, so that, despite the best efforts of the fourteenth-century astronomer Parameśvara to save him, it is clear that this was indeed his intention.

In Bhāskara's *Mahābhaskarīya*, chapter V is dedicated to eclipses. The diameters of the bodies are the same as Āryabhaṭa's, but the orbital radii have been altered (V, 2):

$$k_s = 459{,}585 \text{ } yojanas,$$

$$k_m = 34{,}377 \text{ } yojanas.$$

The true diameters are to be calculated as follows:

$$d_s = \left(\frac{5}{9}\min + \frac{5}{36}\sec\right)\nu_s,$$

$$d_m = \left(\frac{1}{25}\min - \frac{1}{100}\sec\right)\nu_m,$$

$$d_u = \left(\frac{1}{10}\min - \frac{1}{16}\sec\right)\nu_m,$$

so the mean diameters are

$$d_s = 0;32{,}54,\ldots,$$

$$d_m = 0;31{,}44,\ldots,$$

$$d_u = 1;19{,}49,\ldots.$$

Notably, the diameter of the shadow is a function only of the motion of the moon, and not of the difference between lunar and solar motion.[98] Maximum lunar latitude is 4;30° (V, 14).

Chapter V, 8–18 contain a discussion and rules for the computation of the meridian ecliptic point (*madhyalagna*), the sine of the amplitude of the rising point of the ecliptic (*udayajyā*), and the sine of the zenith distance of the meridian ecliptic point (*madhyajyā*). Bhāskara I, in an attempt at greater accuracy, notes that strictly the sines of the amplitudes *udayajyā* for the sun and for the moon will be distinct, because their orbits are inclined by different amounts. He also notes this for the respective zenith distances. He deems it necessary to calculate the two separately, a step that is hardly required given the fact that at the time of the eclipse the sun and the moon are almost in the same place.

The rest of his computation of parallaxes is the same as Āryabhaṭa's and his rules for finding the magnitudes and durations of eclipses are extensions of

those found in the *Āryabhaṭīya* (V, 33–41). His computation of the *akṣavalana* and the *ayanavalana* is standard, and like Āryabhaṭa, he uses the versine (*utkramajyā*) for the *akṣavalana*. Detailed instructions are given (V, 48–67) for the graphical representation of the eclipse and as is common within the *Āryapakṣa*, lunar eclipse theory (V, 68–70) follows the material on solar eclipses. Here, techniques that were presented in the solar eclipse section are to be applied (with the orientations reversed where appropriate), even parallax!

5.6.7 *The* Āryapakṣa: *Lalla's* Śiṣyadhīvṛddhidatantra

Lalla was one of the leading Indian astronomers of the eighth century[99] and followed the two traditions set out by Āryabhaṭa. His work, the *Śiṣyadhīvṛddhidatantra*, is the most comprehensive work in the *Āryapakṣa* school, but uses many techniques developed by his predecessors who were outside the *Āryapakṣa*. He covers the material related to eclipses in chapter V (lunar eclipses), chapter VI (solar eclipses), chapter VII (possibility of an eclipse), and chapter XX, a section on refutations of traditional beliefs, including the notion that Rāhu is the cause of eclipses. He begins the lunar eclipse section by instructing the reader to ascertain the longitudes of the sun and the moon and the ascending node on the 15th *tithi* (lunar day) at sunset.

Lalla's value (V, 2) for the circumference of the sky and thus the circumference of the lunar orbit[100] is

$$\begin{aligned} C &= 21{,}600 \times 10 \times \text{ revolutions of the moon in a } \textit{yuga} \\ &= 21{,}600 \times 10 \times 57{,}753{,}336 \textit{ yojanas} \\ &= 12{,}474{,}720{,}576{,}000 \textit{ yojanas}. \end{aligned}$$

This value is then used to determine the corresponding radius by the relation[101] $C = 2\pi r$, so that the mean distances of the sun and the moon (V, 4) are

$$k_s = 459{,}585 \textit{ yojanas},$$

$$k_m = 34{,}377 \textit{ yojanas}.$$

These are identical to the values of Bhāskara I. These mean distances are converted into the true ones (V, 5) by multiplying by their *mandasphuṭa* hypotenuses and dividing by R (cf. *Paitāmahasiddhānta* [V, 2]; see section 5.6.1). In addition, Lalla gives an alternative rule: one can multiply the orbital radii by the quotient of mean motion to true motion (cf. *Romaka* VIII, 15; see section 5.5.2).

The solar and lunar diameters are given (V, 6) as 4410 and 315 (cf. BSS XXI, 32). The height of the shadow cone is the sun's distance from the earth

multiplied by 5⁄16 (cf. MB V 72):

$$k_u = 459{,}585 \times \frac{5}{16} = 143{,}620;18{,}45 \textit{ yojanas}.$$

The diameter of the earth's shadow is given as (V, 7)

$$d_u = \frac{1050(k_u - k_m)}{k_u},$$

so that at mean distance the following amount can be derived:

$$d_u \approx 2 \times 799 \textit{ yojanas}$$

$$= 1598 \textit{ yojanas}.$$

Conversion from linear into angular diameters is given next (V, 8). The conversion is based on the proportionality of the diameter to its distance from the earth:[102]

$$d_{\text{minutes}} = \frac{d_{yojanas} \times R}{k}.$$

Lalla gives an alternative conversion using the true and mean motions of the planet in question. This rule is copied from Brahmagupta (IV, 6).

Lalla gives a glimpse into his conception of the mechanics of the eclipse in V, 10. He notes that the shadow is the thing that causes the moon to be eclipsed and the moon is what causes the sun to be eclipsed. He mentions that both total and partial eclipses can occur, and states that whether it is a solar or lunar eclipse depends on which body is being obscured (as opposed to which body is obscuring).

The formula for the latitude of the moon (V, 11) is based on Brahmagupta's. Lalla's formulas for the magnitudes and durations (V, 12–22) are standard. His calculation of the *valana*s uses the versine (*utkramajyā*). Lalla has a section on the graphical representation of eclipses that gives detailed instructions for depicting the various phases of an eclipse in a diagram.[103]

Lalla has three different expressions for longitudinal parallax (VI, 6–9). The first is

$$\pi_\lambda^{\text{moon}} = \frac{525.\sqrt{\text{Sin}^2 z(S) - \text{Sin}^2 z(V)}}{k_m},$$

$$\pi_\lambda^{\text{sun}} = \frac{525.\sqrt{\text{Sin}^2 z(S) - \text{Sin}^2 z(V)}}{k_s}.$$

$z(V)$ is the zenith distance of the nonagesimal and $z(S)$ is the zenith distance of the sun. The square root of the difference of the squares of these amounts

is the sine of the distance between the nonagesimal and the sun (or the moon) and can vary from 0° to 90°. This rule is essentially

$$\frac{\pi_\lambda = 525 \cdot \mathrm{Sin}(VS)}{k_m}.$$

The rule Lalla gives is based on taking the extremes of parallax and varying it sinusoidally between these two extremes along this 90° arc, so that the remaining elements in the rule are explained as follows:

$$\text{Maximum parallax} = \frac{\text{Radius of earth} \times R}{\text{Sun's distance}}$$
$$= \frac{525 \times R}{k_s}$$

The total longitudinal parallax is

$$\pi_\lambda = \frac{525 \cdot \sqrt{\mathrm{Sin}^2 z(V) - \mathrm{Sin}^2 s(V)}}{k_m - k_s} \text{ minutes}$$
$$= \frac{525 \cdot \sqrt{\mathrm{Sin}^2 z(V) - \mathrm{Sin}^2 s(V)}}{k_m - k_s} \times \frac{60}{v_m - v_s} \; ghaṭikās$$

This is to be added to or subtracted from the conjunction to find the corrected time. Thus, where Bhāskara I has taken the zenith distances of the sun and moon separately, Lalla keeps them the same. This formula is based on the one in the *Sūryasiddhānta* (see section 5.6.2) in the *Pañcasiddhāntikā* (IX, 19–23).

The second formula Lalla (VI, 8) gives is

$$\pi_\lambda = \frac{\mathrm{Sin}\Delta\tau \times \mathrm{Sin}\alpha(M)}{2{,}954{,}961},$$

where $\Delta\tau$ is the hour-angle, $\alpha(M)$ is the altitude of the meridian ecliptic point, and 2,954,961 is $\frac{R^2}{4}$ ($R = 3438$), 4 being the maximum parallax. This rule is based on the rule-of-three proportion given in the *Pauliśasiddhānta* and is similar to the one given by Brahmagupta in the *Brāhmasphuṭasiddhānta* (V, 4–5 and 22–23):

$$\pi_\lambda = \frac{4 \cdot \mathrm{Sin}(VS) \times \mathrm{Sin}\alpha(V)}{R^2}$$

where $R = 3438$, so that $2{,}954{,}961 = 4/3438^2$ and VS has been approximated by the hour-angle.

Lalla's third method (VI, 9) is the following, given in *ghaṭikās*:

$$\pi_\lambda = \frac{\sqrt{\mathrm{Sin}^2 z(S) - \mathrm{Sin}^2 z(V)}}{860}$$

where 860 is $\frac{3438}{4}$.

Next, Lalla states explicitly that "(parallax) should be added. . .if the sun is in the western hemisphere and subtracted if in the eastern hemisphere." This prescription suggests that the sign of the parallax correction depends on whether the sun is west or east of the meridian. In VI, 10, he admits that the parallax "should be added provided the moon is to the south of the sun, and if the sun is to the south of the moon the parallax should be subtracted."

Lalla gives two formulas for the calculation of latitudinal parallax (VI, 11):

$$\pi_\beta = \frac{525 \cdot \mathrm{Sin}z(V)}{k}$$

and

$$\pi_\beta = \frac{(\nu_m - \nu_s) \cdot \mathrm{Sin}z(V)}{51{,}570}.$$

The second formula is essentially the same as the first, with the substitution of the maximum parallax being 1/15 of the daily motion: $\pi_{\max} = {}^1\!/_{15}\nu_s$ or ν_m, and $51{,}570 = R \cdot 15 = 3438 \cdot 15$. This rule is similar to that given in the *Sūryasiddhānta*, but the numbers differ due to the different base parameters. What is notable about the presentation of parallax is that there is more than one formula given for each component, with different underlying assumptions. This is not contradictory, but rather intended to give the reader options.

Solar eclipse limits differ from the 1/8 of the sun's diameter commonly attested in the *Āryapakṣa*. Instead, Lalla repeats the 1/12 seen in *Brāhmapakṣa* texts. Interestingly enough, he argues that a lunar eclipse can be viewed down to 1/16 of its diameter, because of its clarity (VI, 17). This is clearly not an empirical derivation, as in practice the edge of the shadow is rather indistinct.

Lalla has a whole chapter devoted to the computation of the possibility of an eclipse, the *parvan* (VII, 1–11). In this chapter, further rules are given for the computation of the angular diameters of the sun, the moon and its shadow, parallaxes, and celestial latitude. He adds some new material on the probability of an eclipse at any time in which the *cālanas* or longitudinal increments for *parvans* are given.

The limit for a lunar eclipse is given as (VII, 2)

$$|\lambda_m - \lambda_n| < 12^\circ.$$

A rather cryptic commentary on this verse by a later commentator, Mallikārjuna,[104] may indicate a recognition of the effect of terrestrial latitude on the bounds of the solar eclipse limit. He states:

> atra viśeṣam āha—gataṃ gamyaṃ vā pūrvoktam amāvāsyāyāṃ dvādaśaiva caikāṅgulaviṣuvacchāyādeśe 'rkagrahaṇaṃ na syāt | yatra pañcāṅgula-viṣuvacchāyā tatra gataṃ gamyaṃ vāṃśāḥ pañcadaśa cedarkagrahaṇaṃ na

syāt | yatra navāṅgulā viṣuvacchāyā tatra gataṃ gamyaṃ vāṣṭādaśāṃśāś ced arkagrahaṇaṃ na syāt |
somagrahaṇe tu gataṃ gamyaṃ vā dvādaśāṃśāś cen madhyabhuktitaḥ sphuṭabhuktir alpā tulyā vā syāt tadendugrahaṇaṃ na syāt | sphuṭabhuktir adhikā ced grahaṇam asti | yadā gataṃ gamyaṃ vāṣṭādaśāṃśāś cet tadā somagrahaṇaṃ na syāt |
[Here he says the difference: At new moon, what is elapsed or is to elapse, which has been mentioned previously is 12°. In the place where the equinoctial shadow is one digit (*aṅgula*), there is no solar eclipse. Wherever the equinoctial shadow is 5 digits, there if the degrees are 15, there will be no solar eclipse. Wherever the equinoctial shadow is 9 digits there if the degrees are 18, there will be no solar eclipse. But in the case of a lunar eclipse, if the difference is 12° and if the true motion of the moon is either equal to or less than its mean motion, there is no possibility of a lunar eclipse. If the motion is greater than the true there is a possibility. If the difference is 18°, there is no possibility of a lunar eclipse.]
(ŚDV p. 120 lines 7–13)

This seems to suggest that Mallikārjuna is trying to account for the fact that the solar eclipse limit changes with respect to terrestrial latitude because of parallax. In this case, parallax widens the solar eclipse limit by up to 6°. The equinoctial shadow gauges the changing latitude. An equinoctial shadow of one digit would refer to a place such as Śrī Laṅka; five digits to Ujjain; nine digits to a location yet further north. The amounts "the elapsed" and "the is to be elapsed" probably refer to the distance between the eclipsed body and the eclipser, or, in other words, the elongation of the sun and the moon.

According to Mallikārjuna's description, the scheme below results, extending the solar eclipse limits due to the parallax correction as the latitude varies. The equinoctial shadow (a function of the terrestrial latitude) is related to the elongation in the following way. Mallikārjuna gives the maximum elongation for which an eclipse is not possible, but presumably for all values less than this amount an eclipse is possible:

Equinoctial shadow	Elongation
1	12
5	15
9	18

For lunar eclipses, the situation is different, as there is no parallax correction. Instead the lunar eclipse limits can be extended when the moon is traveling faster than its mean velocity. But the maximum lunar eclipse limit seems to be 18°.

Condition	Elongation	Occurrence
$\nu_m \leq \overline{\nu_m}$	12	No possibility
$\nu_m > \overline{\nu_m}$	12	A possibility
—	18	No possibility

Two rules are given for the calculation of the angular diameters. These are simply those of Brahmagupta multiplied by ⅞. This scaling factor is based on Lalla's proportion of minutes to digits, 2⅓:1 (VII, 3) (compare the 2½:1 given by Āmarāja in section 5.1):

$$d_m = \frac{4}{17}\nu_m,$$

$$d_s = \frac{10}{577}\nu_s,$$

$$d_u = \frac{5}{28}\nu_s - \frac{2}{25}\nu_m.$$

The rules for parallax set out in the previous chapter are repeated, and a "more accurate parallax in longitude" (VII, 5–7) is given:

$$\pi_\lambda = \frac{4 \cdot \mathrm{Sin}z(M) \cdot \mathrm{Sin}\alpha(M)}{R^2}.$$

This seems to be an imperfect copying of the formula given in the *Brāhmasphuṭasiddhānta* (V, 4), which in turn was based on the *Paitāmahasiddhānta*. Lalla has substituted the meridian ecliptic point where Brahmagupta has used the nonagesimal. This highlights the tendency of the astronomers to manipulate equations at will, without any consideration of the practical consequences. Substitutions such as these are commonly made by Indian composers, but rarely justified. Neither were the motivations given for these modifications. It seems most likely that these substitutions were made for computational facility. Moreover, it may be that the theoretical geometrical consequences of these substitutions were in fact considered, but found to be negligible, which validated these changes.

Parallax in latitude in digits is given by the following (VII, 8):

$$\pi_\beta = \frac{21 \cdot \mathrm{Sin}z(M)}{R} \cdot \frac{(\nu_m - \nu_s)}{\overline{\nu_m} - \overline{\nu_s}},$$

in which 21 fingers = 49 minutes (using the 2⅓:1 ratio), which is the horizontal parallax. This too seems to be an elaboration on Brahmagupta's rule (V, 11–12), again using the zenith distance of the meridian ecliptic point and not the nonagesimal which Lalla gave earlier (VI, 11). Then Lalla claims that its direction is the same as that of the zenith distance. This seems to be an error. The direction of *total* parallax is in the direction of the zenith distance circle,

but parallax in latitude will be added onto the latitude, which is perpendicular to the ecliptic (and thus not usually parallel to the zenith distance circle).

The chapter closes with the *cālanas* (longitudinal increments). As well as listing these amounts for 177-day intervals (VII, 9–10), Lalla also adds those for 15 civil days (VII, 11):[105]

	177 days	15 days
Sun	5^s 24;27,6°	0^s 14;47,2°
Moon	5^s 22;12,53°	6^s 17;38,42°
Manda	0^s 19;42,53°	0^s 1;40,13°
Node	0^s 9;22,51°	0^s 0;47,45°

5.6.8 Āryapakṣa: *Vaṭeśvara's* Vaṭeśvarasiddhānta

Vaṭeśvara (b. 880) has been identified as a figure representative of the maturation of astronomy between Brahmagupta (toward whom he is harshly critical) and Bhāskara II (b. 1115).[106] Vaṭeśvara states that he resided in Ānandapura, which has a latitude of 24°—probably a locality in northern Gujarat. He composed the *Vaṭeśvarasiddhānta* when he was only 24. It was notably large in size (although it has not survived in its complete form) and collected and described standard techniques and methods of astronomers in his time. It is, as far as is known, the last significant text that represents the *Āryapakṣa* in the northern reaches of India. Of particular interest are his rules for the computation of parallax[107] which describe an iterative technique to establish the correct value of the parallax as a time interval between true and apparent conjunction. Vaṭeśvara states:

> The sine of the longitudinal component of the zenith-distance multiplied by four and divided by the Radius [is] the longitudinal parallax in terms of *ghaṭikās*. [When that is] subtracted from the time of conjunction or added to it in the manner stated before, repeatedly, [it is] here stated [to be] the true longitudinal parallax.
> (*Vaṭeśvarasiddhānta* V, 16, as translated by Shukla [1986] I, p. 244 and cited by Plofker [2002] p. 178)

The first part of this verse gives a rule for the computation of the longitudinal component of parallax. Let the time corresponding to the parallax in longitude be $\pi_\lambda^{(g)}$ *ghaṭikās* and the longitudinal component of the zenith distance be $z(\lambda)$. Vaṭeśvara sets out the following ratio:

$$\frac{\pi_\lambda^{(g)}}{4} = \frac{\mathrm{Sin}z(\lambda)}{R},$$

implying that the longitudinal component of parallax in time is given by

$$\pi_{\lambda}^{(g)} = 4\mathrm{Sin}z(\lambda)\ \textit{ghaṭikās}.$$

The ratio invoked has simply taken the extreme values of parallax, namely 4 and 0, and varied the intermediary values sinusoidally. However, Vaṭeśvara then chooses to add (subtract) it to (from) the time of conjunction "repeatedly," which suggests an iterative procedure. The procedure seems to be used because the precise moment of conjunction needs to be established so that parallax can be applied appropriately. Vaṭeśvara seemed to realize that the known positions of the sun and the moon were the ones after the correction had been applied (i.e., the apparent positions), as opposed to the ones to which the correction was to be applied. The iterative procedure, then, seems to be as follows: select an initial value that approximates the one sought after, compute the correction for this circumstance, apply with the opposite sign, and then repeat the process until the value sought for becomes fixed.[108]

Accordingly, supposing the time of conjunction is T and that of apparent conjunction T' with the corresponding longitudinal parallax $\pi_{\lambda}(T')$ *ghaṭikās*, Vaṭeśvara states:

$$T = T' \pm \pi_{\lambda}(T')$$

However, mathematical computations can only produce T, and not T'. Thus, taking as a first approximation $T'_0 = T$, Vaṭeśavara requires:

$$T'_{k+1} = T \mp \pi_{\lambda}(T'_k)\ \textit{ghaṭikās}$$

to be iterated until T' becomes fixed. Hence the amount of the parallax in time between true and apparent conjunction can be determined. Similar techniques seem to have been used by Islamic scholars, and the techniques set out here may help clarify the details behind the proposals of these Arabic scholars.

5.7 Parameśvara and the Astronomical School in Kerala

While the progress of mathematical astronomy was constantly disrupted by political and social unrest in the north of India after the 1200s, pockets of activity elsewhere ensured that the astronomical tradition continued.[109] One of these havens was in the south-western region of Kerala. Astronomical activity was fathered by astronomers such as Vararuci I (fourth century CE), traditionally considered the founder of the *kaṭapayādi* numeral system and the moon sentences (*cadravākyas*), and Haridatta (ca. 650–700). However, the use of Malayalam language in which the majority of these texts have been composed or commented upon and the difficulty of the medieval Malayalam script have prevented or deterred many scholars (particularly western scholars) from properly studying these texts.

A notable element characterizing the groups of scholars working in this region in the fourteenth century and beyond was a new, distinct spirit of inquiry. More than ever before, a desire for conformity of computation with observation provided great impetus to correct existing theoretical models and parameters or even to devise new systems that would produce a better correspondence with reality.

Eclipses played a prominent role in this spirit of observation, as eclipses themselves can conveniently be used as means with which to observe the correctness of parameters, or indeed to derive them. Parameśvara[110] (b. ca. 1380; d. ca. 1460) left documentation of thirteen eclipse observations he gathered which span a period of thirty-four years, and other authors attest to his having made observations for fifty-five years. The detailed and numerous references to actual observation suggest a scientific discipline that placed more emphasis than ever before on the importance of recording and checking repeated and sustained observations. Furthermore, unlike the majority of his predecessors, Parameśvara, in his pursuit of accuracy, acknowledged that his system was imperfect for a variety of reasons, including some that he could not anticipate. He openly admitted this in his works on several occasions, as in the following passage:

> kālo 'nena ca siddhaḥ kadācid api bhidyate svalpam |
> [The times (of the phases of eclipses) as obtained by this (computation, i.e., the one set out in his work) may at times differ slightly.]
> (*Grahanamaṇḍana* 96, as translated by Sarma [1965])

This statement suggests that by the time of Parameśvara the very perception of scientific practice was changing among Indian astronomers. Now it was no longer adequate to simply uphold ancient authority at the expense of the facts. Rather, empirical facts themselves, as observed in the heavens, were identified as the basis from which scientific calculation and progress were to be made. There was also an understanding that an astronomer may not be in a position to have identified all the causes and factors that influenced the motions of the heavens, but that, little by little, these could be fully accounted for.

Parameśvara wrote several works devoted exclusively to eclipse theory. This itself was unprecedented—this topic had always been included as part of a broader astronomical treatise by previous astronomers and never treated separately. Parameśvara's works on eclipse theory include the *Grahaṇanyāyadīpikā*, the *Grahaṇamaṇḍana*, the *Grahaṇāṣṭaka*, and the *Siddhāntadīpikā*. It is difficult to establish with certainty the order of these works from the existing cross-references in them. However, it is certain that the *Grahaṇanyāyadīpikā* must come last as it mentions all three other titles.[111]

Parameśvara was influenced by some impressive individuals. His teachers were Nārāyaṇa Paṇḍita (fl. 1356) and Mādhava (lived between ca. 1340 and ca. 1425), two prominent scholars from the same region, who were responsible for introducing significant results into the mathematical scene. Mādhava in particular is credited with being the founder of the Mādhava School, a lineage of scholars who produced many important mathematical discoveries. Indeed, Mādhava is well known for his exposition of the power series for the computation of certain trigonometric functions.[112] There was much innovation and activity in the field of astronomy as well. Two new astronomical traditions were introduced and provided inspiration to scholars to further revise existing models. The first was the *parahita* method produced by Haridatta, which was set out in his *Grahacāranibandhana* and his *Mahāmārganibandhana*. The *parahita* system was a conscious revision of Āryabhaṭa's systems. The second method, the *dṛggaṇita* system was set out by Parameśvara and was intended as a response to the first *parahita* tradition. In his work *Dṛggaṇita* in 1431, Parameśvara claimed that he was correcting those parameters set out in the *parahita* system, as a result of his observations.

5.7.1 The Grahaṇamaṇḍana

The *Grahaṇamaṇḍana* (literally "Ornament of Eclipses") is a *karaṇa* consisting of one hundred verses. The epoch is July 15 1410, or 1,648,157 days after the beginning of the kaliyuga. Almost a third of the work is devoted to computing the mean positions of the appropriate phenomena: the sun, the moon, and the nodes. From these mean positions, the syzygies are identified, and then in turn those for which there is an eclipse possibility are computed. The rules that address eclipses, both lunar and solar, occupy the main section of the work (verses 37–72). Graphical methods associated with eclipse phenomena are described next (verses 73–87) as well as instructions for drawing an eclipse diagram for an instant during the course of an eclipse (verses 88–94).

Rules are given for the calculation of the mean positions in terms of *ahargaṇa* (civil) days after the epoch (verses 5–8). These are as follows (where a is the *ahargaṇa* days since the beginning of the kaliyuga minus 1,648,157):

$$\begin{aligned} &\text{Sun} && a \times \frac{58}{21185} \\ &\text{Moon} && a \times \frac{143}{3907} \\ &\text{Apogee} && a \times \frac{4}{12931} \\ &\text{Node} && a \times \frac{4}{20378} \end{aligned}$$

Parameśvara then gives corrections (the *dhruvas* or additive constants) to add to the mean positions of the sun, the moon, the apogee, and the node (verses 9–12) as calculated from the zero date.

	Additive constant
Sun	3^s 18;15,7°
Moon	2^s 0;17,1°
Apogee	2^s 5;23,18°
Node	11^s 1;41,4°

Once the details concerning the mean positions are established, the conditions for syzygy occurrences are discussed (verses 26-30). In verse 31, eclipse limits are given:

> rāhūnārkabhujāṃśā viśvonāścenniśāpateścintyam ||
> grahaṇaṃ te sapalāṅgularudronāścet tayā savituḥ |
> [If the degrees of the bhuja (*bhujāṃśa*) of (the longitude of) the sun decreased by (the longitude of) the node are less than 13°, an eclipse of the moon is to be expected. If (the *bhujāṃśa*) are less than 11 plus the degrees of the equinoctial shadow, (an eclipse) of the sun.]
> (Grahaṇamaṇḍana 30; second half—31, as translated by Sarma [1965])

The derivation of the solar eclipse limits by adding information about the equinoctial shadow to the *bhujāṃśa* seems to take into account the fact that parallax affects the limits at which a solar eclipse may occur (see section 5.6.7 and the related procedure set out by commentator Mallikārjuna).

The measures of the diameters are given (verses 31–34) as follows:

$$d_s = \nu_s \times \frac{5}{9} + 8 \text{ seconds},$$

$$d_m = \frac{\nu_m}{25} - 8 \text{ seconds},$$

$$d_u = \frac{\nu_m}{10} \times \overline{\nu_s} \div \left(\frac{(\overline{\nu_s} + \nu_s)}{2} + 50 \right) \text{ seconds}.$$

These give the mean values (using 13;10,34 and 0;59,8)

$$\begin{aligned} d_s &\approx 0;32,52 + 8 \text{ seconds} \\ &= 0;33, \\ d_m &\approx 0;31,38 - 8 \text{ seconds} \\ &= 0;31,30, \\ d_u &= 1;17,57,28,58, \ldots . \end{aligned}$$

The intention of adding or subtracting the 8 may be related to the desire to round the figures up or down. After this, Parameśvara gives a charming analogy to demonstrate the characteristics of the obscuration:

> kumbhāntareṇa kumbho yathā tathā chādyate raviḥ śaśinā | vāripraveśavat syāt candrasya chādana tamasā ||
> [The sun is covered by the moon (in a solar eclipse) just like a water-pot by another water-pot. The covering of the moon by the (earth's) shadow (in a lunar eclipse) is like an entrance into water.]
> (*Grahaṇamaṇḍana* 36, as translated by Sarma [1965])

The computation for the latitude of the moon is presented by means of two rules, the second rule purportedly "more correct" (*sphuṭataraṃ*) than the first (verses 37–38):

$$\beta_m = \frac{4}{51}\mathrm{Sin}(\lambda_m - \lambda_n)$$

$$\beta_m = \frac{4}{51}\mathrm{Sin}(\lambda_m - \lambda_n) \times \frac{\nu_s}{\overline{\nu_s}}.$$

The next section, which includes calculations "especially for solar eclipses," deals with parallax: the longitudinal component of parallax (verses 39–55), and then the latitudinal component (verses 56–58). This formula for parallax is very similar to that offered by Brahmagupta; however, Parameśvara calculates everything in terms of *nāḍikā*s (identical to *ghaṭikā*s—1 *nāḍikā* = 6°). The general procedure rests on the computation of each of the following: the ascendant (*lagna*), the nonagesimal (*dṛkkṣepalagna*), the "elongation" between the nonagesimal and the sun (*lambananata*), and the zenith distance of the nonagesimal (*dṛkkṣepanata*). These elements form the foundation for his rule for longitudinal parallax.

Parameśvara first instructs the reader to find the ascendant, and from this the nonagesimal (*dṛkkṣepalagna*) (verse 40b) (the nonagesimal by definition is 90° west of the ascendant measured along the ecliptic). Having established this (and from the position of the sun, presumably computed previously) he introduces the following quotient (41), the result of which he calls the *lambananata-nāḍikās*, literally the *nāḍikā*s of the depression for longitudinal parallax:

$$nata = VS^{\mathrm{nad}} = \frac{\lambda(V) - \lambda(S)}{6}.$$

This is simply the elongation of the sun from the nonagesimal converted from degrees to *nāḍikā*s. The result, if greater than 15, is to be subtracted from 30, to get a final result:

$$1 \leq VS \leq 15.$$

The ecliptic always has 180° above the horizon, so that the divisor of 6 divides the ecliptic into 30 sections, fifteen on either side of the nonagesimal. These represent the equivalent of one *nāḍikā*. The maximum distance in longitude between the nonagesimal and the sun is 90° (i.e., at the instant of sunrise or sunset). Parameśvara tabulates the corresponding sines for each of these 15 values, using the value $R = 239$. It seems that the reason he has selected 239 as his radius here is because he wants to express the arc of the ecliptic in terms of the equator and

$$239 = 23;54 \times 10 = 10 \cdot \epsilon,$$

where $\epsilon = 23;54$ is the obliquity of the ecliptic and the factor of 10 allows these amounts to be expressed with greater integer precision. The sines, $\mathrm{Sin}(VS)$, are given by him as follows (verse 43–44 first half) :

$(VS)^{\text{degrees}}$	$(VS)^{\text{nad}}$	$\mathrm{Sin}(VS)$	$(VS)^{\text{degrees}}$	VS^{nad}	$\mathrm{Sin}(VS)$
6	1	25	54	9	193
12	2	50	60	10	207
18	3	74	66	11	219
24	4	97	72	12	227
30	5	120	78	13	234
36	6	140	84	14	238
42	7	160	90	15	239
48	8	177			

The next measure Parameśvara requires is the zenith distance of the nonagesimal. Like his predecessors, Parameśvara computes this by taking the sum or difference of the terrestrial latitude and the declination. He articulates a detailed procedure for the calculation of one's terrestrial latitude (verse 45 second half), in *nāḍikā*s (1 *nāḍikā* = 6°). It is a very practical method which is based on the measurement of the noon equinoctial shadow (s_0), but, more importantly, for certain values of s_0 Parameśvara uses a quadratic correction term, presumably intended to improve the accuracy. For $s_0 \leq 3$ digits:

$$\phi = \frac{s_0 \times 5}{7} nāḍikās.$$

If, however, $s_0 > 3$ *fingers*, then Parameśvara instructs one (46) to compute the amount

$$\phi = \frac{s_0 \times 5}{7} - \frac{(s_0 - 3)^2}{45} nāḍikās.$$

The above computation of latitude is based on approximations to the sine function by a quadratic equation.[113]

Next, Parameśvara describes how one finds the declination of the nonagesimal–this is to be added to or subtracted from the terrestrial latitude to find the zenith distance of the nonagesimal. In order to find this declination, Parameśvara does not simply take the complement of the altitude of the sun at noon and then consider this approximately equal to the declination of the nonagesimal as his predecessors did, but uses a more involved procedure. To find the declination of the nonagesimal itself, its longitudinal separation from the intersection of the equator and the ecliptic has to be measured. First one has to take into account that, because the Indian system is sidereal, Aries 0° changes its position with respect to this intersection. Thus a correction is necessary. With the nonagesimal longitude as corrected for precession (*bhujāṁśa*), λ_V^*, one is to find the corresponding sine. Parameśvara lists the sine for each half sign as follows:

λ_V^*	$Sin(\lambda_V^*)$
15	20
30	39
45	56
60	69
75	77
90	80

The declination in *nāḍikā*s is this sine divided by 20. A maximum longitude of 90° corresponds to 80 in the table or 4 *nāḍikā*s, which is $\frac{4}{60} = \frac{1}{15}$ of a day; $\frac{1}{15} \cdot 360° = 24°$ which is the maximum declination. That he does not simply give the values of $\mathrm{Sin}(\lambda_V^*)$ divided by 20 in the table may be because he is avoiding fractional parts. The zenith distance of the nonagesimal $z(V)$ is the sum or the difference of the latitude and the declination thus computed, also given in *nāḍikā*s.

With these measures established, Parameśvara states the parallax in longitude as

$$\pi_\lambda = \frac{\mathrm{Sin}(VS) \cdot \mathrm{Sin}\alpha(V)}{239} nāḍikās$$

where $\alpha(V) = 15 - z(V)$ *nāḍikā*s. This is similar to the formula given by Brahmagupta (V, 4–5), with the $\frac{4}{R}$ term left out because Parameśvara is working in *nāḍikā*s. Calculating the whole thing in *nāḍikā*s, although cumbersome, allows Parameśvara to add (subtract) the value directly to (from) the *nāḍikā*s elapsed from sunrise to the time of conjunction. With this new time of conjunction, the positions are to be recomputed. Then the parallax in longitude is recomputed again, and the above procedure is repeated until the longitudinal

parallax is fixed. With this amount of parallax, the original conjunction time can be corrected.

Parameśvara's parallax in latitude (56–58) is standard and is based on the sine of the zenith distance of the nonagesimal,

$$\pi_\beta = \frac{\mathrm{Sin}[z(V)] \cdot (\nu_m - \nu_s)}{60},$$

in which the sine of the zenith distance is calculated via the above table of Sin(VS) (i.e., the maximum is 239). Thus, the mean relative horizontal parallax (taking the mean velocity of the moon to be 13;10,34 and that of the sun 0;59,8) occurs when the nonagesimal is on the horizon and the sine of the zenith distance of the nonagesimal equals the radius:

$$\begin{aligned}\pi_\beta &= \frac{239 \cdot (13;10,34 - 0;59,8)}{60} \\ &= \frac{239 \cdot 12;11,26}{60} \\ &= 48;33,32\ldots \text{minutes}.\end{aligned}$$

With parallax established, Parameśvara describes the computation of all other aspects of solar eclipses with the parallax correction taken into account. The half-duration Δt for a solar eclipse is given next (verses 59–68), found by the rule

$$\frac{\Delta t}{2} = \sqrt{(r_s + r_m)^2 - \beta^2}.\frac{60}{(\nu_m - \nu_s)}.$$

This formula assumes a constant latitude throughout, that is, the latitude which has been corrected for parallax at mid-eclipse. Parameśvara explicitly observes that this must be further corrected due to the modifications of parallax at the times of first contact or release. The latitudinal and longitudinal parallaxes for either of these times are computed. With a new latitude for either of these times, adjusted for parallax (β_a), the new half-duration is to be calculated as follows:

$$\Delta t = \sqrt{(\beta \pm \beta_a)^2 + (r_s + r_m)^2 - {\beta_a}^2}.\frac{60}{(\nu_m - \nu_s)}.$$

The computation of the half-duration of a lunar eclipse is then given, but Parameśvara rightly asserts it is to be done without the application of parallax.

Eclipse limits are reasserted in 71, second half–72. Here a lunar eclipse is said to pass by if the lunar latitude is greater than half the sum of the diameters; again implicit in this statement is the assumption that the lunar orbit is parallel to the ecliptic. The minimum latitudes required for a total and partial eclipse, expressed with respect to the relations between the diameters, are also given.

The formulas for the *valana*s (73–76a) commence with the *akṣavalana*:

$$\gamma_1 = \frac{s_0}{12} \times \Delta\tau \text{ minutes}$$

in which $\Delta\tau$ is the hour-angle.

The computation of the *ayanavalana* is different from that done by Parameśvara's predecessors. Here, Parameśvara gives three possible values for the *ayanavalana*, namely, the sines of the *ayanavalana* are 1, 3, or 6, in minutes. The derivation or intent of this component of the *valana* is unclear.

He then uniquely adds in a third component to be computed and applied to the total *valana*. This he calls the *vikṣepavalana*, that is, the deviation due to celestial latitude. Its amount is calculated from

$$\gamma_{3,m} = \frac{\beta_m \times 2}{7}$$

for the moon (76, second half–77, first half), and

$$\gamma_{3,s} = \frac{\beta_m}{2}$$

for the sun (77, second half–78, first half). The origin and intent of these formulas still remains a puzzle.

The total *valana* is then computed (78, second half–80) as follows:

$$\gamma_{\text{total}} = \frac{D*}{32}(\gamma_1 \pm \gamma_2 \pm \gamma_3),$$

where $D*$ is the true diameter of the disk of the body to be eclipsed, and 32 may be an approximation to its mean diameter. Parameśvara notes that the sums are to be taken if the deviations are "in the same direction" and differences if they are "in different directions." This detail of eclipse computations remains elusive.

The section on the *parilekha* or eclipse diagram follows (80b–94). Instructions are given for the representation of the *valana* in terms of the eclipsed and eclipsing bodies, although they shed no light on the calculation of this amount. Following are instructions on the construction of a general diagram which represents an eclipse at any stage (and not just at impact or release). The limits beyond which an eclipse will not be visible are stated (95) to be for the sun less than ⅛ and for the moon less than 1/16 part of the disk.

As Parameśvara concludes his work, he makes reference to a verse of Varāhamihira from his *Bṛhatsaṃhitā* (V, 26):

ūnātiriktakāle phalam uktaṃ pūrvaśāstradṛṣṭatvāt
[Predictions of the effects (of eclipses) occurring (a little) earlier or later than

the times due are given (by me) on the authority of ancient texts on the subject.]
(*Grahaṇamaṇḍana*, 97, as translated by Sarma [1965])

As discussed earlier (see section 5.3) while there is a possibility that this may connect to an element from the tradition of *EAE* in which eclipses that do not occur "at the allotted time" are part of the protasis material, Varāhamihira interprets it to mean there was an additional factor not recognized by these early texts, or an incorrect method that is responsible for the discrepancy. Parameśvara agrees with this; he takes it to mean (98) that "a correction not stated in old texts must exist."

Parameśvara also reveals his scientific outlook by noting (99) that such corrections must be postulated by astronomers after observing a large number of eclipses, with attention to the principles of spherics (*yuktigola*):

dṛṣṭvā bahūparāgān sañcantya ca golayuktim iha gaṇakaiḥ |
kalpyaḥ sa tu saṃskāras tasmād athavā gurūpadeśena ||
[Such a correction has to be postulated by astronomers after observing a large number of eclipses and with due consideration to the principles of spherics, or in the light of instructions of masters.]
(*Grahaṇamaṇḍana*, 99, as translated by Sarma [1965])

This emphasizes the importance that Parameśvara ascribes to observations over a long period of time. However, he is no more specific than this; even though we have records of his eclipse observations, he does not reveal the ways in which his own observations influenced his theory and parameters.

His admission that corrections may exist that he has not the ability to comprehend reveals an attitude to the discipline that is quite different from that of his predecessors. Elsewhere, Parameśvara openly admits that his system has limitations and encourages future astronomers to build upon the results he has achieved:

evaṃ dṛggaṇitaṃ śāke trīṣuviśvamite kṛtam |
paramādīśvareṇaitat prāyo bhavati dṛksamam ||
[In this way in Śaka 1353, the *drggaṇita* was written by Parameśvara. This is more or less (*prāyo*) in accord with observation.]
(*Dṛggaṇita* II, 46, as translated by Sarma [1963])

Another interesting remark (100) that shows his concern with the practicalities of observing eclipses is his admission that the observation of a solar eclipse is not easy due to the brilliance of the rays that dazzle and overwhelm the eyes. He thus instructs one to view the solar eclipse by looking at the shadow it casts on the ground. He also admits (101) that it is difficult to measure the

magnitude of an eclipse of the moon accurately by the naked eye, despite the fact that he asserts you can see, at minimum, 1/16 part of its diameter being eclipsed. All of these statements indeed convey his awareness of the challenges of observing!

5.7.2 *The* Grahaṇāṣṭaka

The *Grahaṇāṣṭaka* (literally "octet [of verses] of Eclipses," that is, eclipse computations given in eight verses—compare the one hundred verses of the *Grahaṇamaṇḍana*!), is, as its name implies, a heavily abbreviated manual for the computation of eclipse phenomena. Because it is so concise, it is intended not as an introduction, but rather for readers familiar with Parameśvara's other work(s) on eclipses (possibly the *Grahaṇamaṇḍana*), to memorize and retain.

Despite this, Parameśvara claims that the audience for this work are the *mandabuddhihita* (literally "the dull-witted")! However, although many elements of eclipse theory in this work have been simplified, or precomputed, the computations are not given in a progressive order and the text is very dense. Topics are presented in the following order:

Verse	Topic
1	Correction to the longitudes of the sun, moon, and nodes
1–2	Angular diameters
2	Nonagesimal defined
2	Six sines of longitudinal difference between nonagesimal and sun
3	Declination of nonagesimal
4	Terrestrial latitude
5	Zenith distance of the nonagesimal
5	Longitudinal component of parallax
5	Application of longitudinal parallax to the conjunction
5	Latitudinal component of parallax
5	Lunar latitude
6	Lunar latitude corrected by latitudinal parallax
7	Half-duration of solar eclipse
7	Beginning and end of solar eclipse
7	Lunar eclipse
8	Duration of a lunar eclipse
8	Magnitude of solar and lunar eclipse
8	Direction of first and last contact for both solar and lunar eclipses

Parameśvara explicitly states that his exposition gives but a "rough calculation of eclipses" (*sthūloparāgagaṇitaṃ*) and, on comparison with his other works devoted to eclipses, this proves true. There are parameters in this work that have been rounded to "nicer" numbers. Aspects such as the *valana* or

inclination have been greatly simplified. The inclination, given as a number of degrees in other works, is now reduced to the identification of one of two cardinal directions. Amounts that are given in other works in days are recast in less precise but smaller numbers of years. This can be seen in the very first verse, which gives corrections to the computed longitudes. Parameśvara gives these not in days, but rather the years (K) since the beginning of the kaliyuga, and those which are small he reckons to be zero:

	Correction
Sun	None
Moon	$\frac{K-3623}{46}$ minutes
Apogee	None
Node	$\frac{K-3623}{46}\frac{11}{7}$ minutes

The diameters are given next, and although they resemble in format those presented in the *Grahaṇamaṇḍana*, they are distinctly different.

$$d_s = \frac{v_s \times 5}{9},$$

$$d_m = \frac{v_m}{25},$$

$$d_u = \frac{6(v_m - 0;0,5)}{v_s}.$$

The nonagesimal is then defined as Parameśvara introduces the material on parallax. As in the *Grahaṇamaṇḍana*, the longitudinal difference between the nonagesimal and the sun is divided into six equal intervals, and then sines tabulated for each of these successive differences of 15°. Rather than the more precise $R = 239$, this work uses $R = 240$.

VS	Sin(VS)
15°	62
30°	120
45°	169
60°	207
75°	231
90°	240

These values coincide with those given in the *Grahaṇamaṇḍana* except for the last. The basis of Parameśvara's rule for longitudinal parallax is Brahmagupta's formula (BSS V, 4 and V, 6). In the case when the longitudinal difference is not a tabulated value, Parameśvara does not specify whether one

should use the value for the nearest 15° interval or compute intermediate values by linear interpolation.

The sine of the maximum distance between the sun and the nonagesimal (90°) is 240 (or 239 in the *Grahaṇamaṇḍana*). This is the obliquity of the ecliptic multiplied by 10, presumably to maintain precision but avoid fractions. In the next verse Parameśvara stipulates that his tabulations of Sin(VS) should be divided by 10.

Unstated is the assumption that the zenith distance of the nonagesimal is the sum or difference of the declination of the nonagesimal and the terrestrial latitude. Rather than give a general rule, Parameśvara gives three precomputed values for the declination for each 30° of arc of the ecliptic. Presumably, interpolation between the tabulated values supplies a closer amount for an arbitrary arc:

Degrees between the nonagesimal and the equinox	Sin$\delta(V)$
30	12
60	21
90	24

Next an approximate rule for the calculation of the terrestrial latitude is given:

$$\mathrm{Sin}\phi = \frac{s_0(100 - 2s_0)}{20}.$$

This, as in the *Grahaṇamaṇḍana* (45, second half–46), is an approximation of the sine function by a quadratic equation, but less involved.

With the declination of the nonagesimal and the terrestrial latitude, the zenith distance of the nonagesimal can be computed. This calculation is based on the approximation that was frequently invoked by Parameśvara's predecessors: the assumption that the latitude is measured on a triangle that has the meridian ecliptic point as one of its corners. The nonagesimal will not necessarily be at this point, but sufficiently close to it:

$$z(V) \approx \phi \pm \delta(V).$$

One adds or subtracts depending on whether the ecliptic is to the north or the south of the equator. Having found the two major components Sin(VS) and $z(V)$ [and from then $\alpha(V) = 90 - z(V)$], the parallax is computed as follows:

$$\pi_\lambda = \frac{\mathrm{Sin}\alpha(V) \cdot \mathrm{Sin}(VS)}{24}$$

which is the same as the formula given in the *Grahaṇamaṇḍana*. Parameśvara then proposes a correction to this parallax, which he notes is based on the longitudinal differences between the sun and the nonagesimal:

1	20
2	38
3	51
4	56
5	56
6	47
7	28
8	6

With each number of *vināḍīs* (1 *nāḍī* = 60 *vināḍīs*), one is to multiply by $\mathrm{Sin}\alpha(V)$ and divide by 24. This correction is then added to the parallax to get the true parallax. This is essentially a correction to $\mathrm{Sin}(VS)$ so that parallax becomes

$$\pi_\lambda = \frac{\mathrm{Sin}[\alpha(V)][\mathrm{Sin}(VS) + \text{correction}]}{24}.$$

This is to be added to the time of conjunction which Parameśvara states is the corrected instant of mid-eclipse.

In order to compute the parallax in latitude, the zenith distance of the nonagesimal is calculated, the sine is taken, and the following rule applies:

$$\pi_\beta = \frac{\mathrm{Sin}[z(V)] \cdot (\nu_m - \nu_s)}{60}.$$

This gives a maximum parallax (taking the mean velocity of the moon to be 13;10,34 and that of the sun 0;59,8) of

$$\pi_\beta = \frac{240.(13;10,34 - 0;59,8)}{60}$$

$$= \frac{240.12;11,26}{60}$$

$$= 48;45,44 \text{ minutes}.$$

Next, Parameśvara instructs one to find the lunar latitude for the corrected conjunction as follows:

$$\beta = \frac{\mathrm{Sin}(\lambda_m - \lambda_n).\nu_m}{149},$$

which produces a maximum lunar latitude a little under 5;15°, slightly too high. To or from this result in arc minutes one adds or subtracts the parallax in latitude, π_β. The result is the lunar latitude to be used in a solar eclipse.

Parameśvara gives an ingenious method to determine the half-duration of solar eclipses. He assumes a simple relation between the magnitude of the eclipse and its duration; a crude approximation, but useful as a rule-of-thumb method. One is to determine the magnitude μ by means of the following formula:

$$\mu = \frac{1}{2}(d_m + d_s) - \beta_a,$$

where β_a is the parallax-corrected latitude at mid-eclipse. Parameśvara gives a list of values of μ or five "remainders" with their corresponding half-durations in sixtieths of a *nāḍikā*, or *vināḍi*s.

μ (min)	Vināḍis of half-duration
2	55
6	93
12	125
20	148
32	160

Here it has been assumed that $d_m = d_s = 0;32$ and that the maximum length of a solar eclipse is $2 \times 2;40$ *nāḍī*s $= 2 \times 64$ minutes, a little over two hours.

An identical procedure, but with different parameters, can be applied to a lunar eclipse. Here the main problem is that lunar eclipses have unequal half-durations of totality, and thus a total lunar eclipse can have varying durations, a fact which this rule of thumb cannot encompass.

As in the case of a solar eclipse, one is to find half the sum of the angular diameters of the eclipsing and eclipsed bodies, in this case the shadow and the moon, and deduct the latitude. The remainder indicates the whole duration in *nāḍī*s tabulated by Parameśvara as follows:

Remainder (min)	nāḍīs of whole duration
1	2
5	4
13	6
28	8
56	9

Thus, the longest eclipse is nine *nāḍika*s, that is, 9×24 minutes $= 224$ minutes ≈ 4 hours.

The beginning and end of a solar eclipse (7) are given by subtracting (and adding), respectively, the half-duration from (to) the time of the middle of the eclipse, so that the eclipse is symmetrical. The magnitude for both solar and lunar eclipses is given by the following formula (8):

$$\mu = \frac{1}{2}(d_{\text{eclipser}} + d_{\text{eclipsed}}) - \beta.$$

Parameśvara gives the limits of a solar and lunar eclipse as 1/8 and 1/16 of the disk, respectively. The last section states that the obscuration for a solar

and lunar eclipse begins in the west and east, respectively, and clears in the opposite direction. This may be a crude statement of the *valana* or inclination which is otherwise unmentioned.

Parameśvara closes his work with the familiar phrase *prāyo bhavati dṛksamam*, "this is in accordance more or less with observation." But this time he has explicitly stated that his rules and procedures are "crude," which is responsible for the approximate nature of the results.

5.7.3 *The* Grahaṇanyāyadīpikā

The *Grahaṇanyāyadīpikā* (literally "the lamp of the rationale for eclipses") is a self-contained work setting out and explaining Parameśvara's astronomical constants and methods related to eclipses. It follows his two other works dedicated to eclipses, namely, the *Grahaṇamaṇḍana* and the *Grahaṇāṣṭaka*. Unlike its two predecessors, this work is distinctly characterized by Parameśvara's attempts to give the reasons behind the techniques that he sets out. In contrast with the *Grahaṇāṣṭakā*, this work is declared to be for the *gaṇakottamaiḥ* (literally "expert calculators").

Parameśvara's standard distances and diameters are given (2–6). These are

$$d_s = 4410,$$
$$d_m = 315,$$
$$d_e = 1050.$$

Parameśvara states that these can be converted into minutes of arc by dividing these amounts by their true distances and multiplying by the radius, where $R = 3438$.

The size of the shadow at the moon's distance is then calculated (7) and explained by the following analogy:

> bhānor vyāsadalaṃ dīponnatiḥ śaṅkūnnatir bhuvaḥ |
> śaṅkudīpāntarāle bhūḥ syād raver yojanaśrutiḥ ||
> [The radius of the sun is (equal) to the height of a lamp, that of the earth the height of a gnomon, and the distance of the sun in yojanas the horizontal distance between the gnomon and the lamp.]
> (*Grahaṇanyāyadīpikā*, 7, as translated by Sarma [1966])

Humorously, Parameśvara describes the convergence of the angle of the shadow cone as tapering "like a cow's tail" (9) (*gopuccha*).

As he promised from the outset, Parameśvara offers an explanation for the phenomena he gives rules for. The fundamental reasons for the durations are given:

ekadṛksūtragau yāvac candrārkau grahaṇaṃ raveḥ |
tāvan niśākṛto yāvat tāvat syāt tamasi sthitiḥ ||
[The sun's eclipse lasts as long as the moon and the sun continue to be on the same line of vision. Likewise, in the case of the moon, (the eclipse lasts) as long as it remains in the shadow.]
(*Grahaṇanyāyadīpikā*, 13, as translated by Sarma [1966])

Mid-eclipse is described as being at opposition, and that parallax may cause a difference between conjunction and new moon is recognized:

sthitir indostamo madhye parvānte syāt śaśīnayoḥ |
yutiḥ parvāntataḥ prāgvā paścād vā lambanād bhavet ||
[At full moon, the moon will be at the middle (of its passage through) the shadow. The conjunction of the moon and the sun may occur (a little) before or after the time of new moon, on account of parallax.]
(*Grahaṇanyāyadīpikā*, 14, as translated by Sarma [1966])

The theory of parallax is dealt with next (15–41). True to his word, rather than simply state the rules, Parameśvara carefully unpacks each of the elements that make up the rules and their interpretation. He derives the underlying mathematical proportions, illustrates the resulting geometrical configurations, and explains the applications and the various changes in units. It is perhaps the most detailed treatment of the subject of parallax in any astronomical *mūla* (noncommentary) text.

Parameśvara introduces this topic by defining the meridian ecliptic point and the nonagesimal. The sine of the zenith distance of the meridian ecliptic point is to be computed from the following:

$$\mathrm{Sin}z(M) = \mathrm{Sin}[\delta(M) \pm \phi].$$

Parameśvara then notes how the altitude of the meridian ecliptic point is related to zenith distance. Next, he gives a rule of three for the determination of the altitude of the nonagesimal (see fig. 5.1):

$$\mathrm{Sin}\alpha(V) = \frac{\mathrm{Sin}\alpha(M) \cdot R}{\mathrm{Sin}(MH)},$$

where MH is the arc along the ecliptic from the ascendant to the meridian ecliptic point. This rule is based on the fact that these arcs are part of two similar spherical triangles and the spherical triangle MVZ is right.

Parameśvara introduces the sine of the longitudinal distance between the sun and the nonagesimal (*dṛggati*). With all elements defined he then states the rule for the computation of the longitudinal displacement between the sun

and the nonagesimal (verse 20):

$$\mathrm{Sin}(VS) = \sqrt{[\mathrm{Sin}z(S)]^2 - [\mathrm{Sin}z(V)]^2}.$$

Parameśvara evokes the geometrical configuration responsible for the discrepancy between planetary positions as observed on the surface of the earth and those registered from the center (verses 21–24). He highlights when this difference will be maximized and minimized and notes that this discrepancy is proportionate for intermediary positions.

> dṛgbhedajo yayaḥ kṣepo dṛkkṣepajyeti socyate |
> chāyāvaśāddhi dṛgbhedo bhaved dṛgjyā tato 'tra sā ||
> śūnyaṃ khamadhye harije bhūvyāsārdhamito bhavet |
> dṛgbhedo nijakakṣyāyām antarāle 'nupātataḥ ||
> [That from which (the portion of) the latitude caused by the *dṛgbheda* [i.e., the difference of the view from the surface and the center of the earth] is obtained is called the sine for the *dṛkkṣepa*. The difference of view is in accordance with the sine of the zenith distance. Hence, here the sine of the *dṛkkṣepa* is the same as the sine of the zenith distance of the nonagesimal. The *dṛgbheda* is zero at the zenith. It is equal to the radius of the earth at the horizon, in its own orbit. In between, it is proportionate (to the sine of the zenith distance).]
> (*Grahaṇanyāyadīpikā* 21–22, as translated by Sarma [1966])

Having defined these key elements and effects, Parameśvara introduces the concept of parallax for the very first time in the work. Most significantly he talks about a *single* effect of parallax that shifts what is observed "downward" (verses 23–24), and from this he distinguishes the two components of parallax: the parallax in longitude which he states is derived from the longitudinal displacement between the sun and the nonagesimal [$\mathrm{Sin}(VS)$] and is measured along the ecliptic, and the parallax in latitude which is dependent on the zenith distance of the nonagesimal ($\mathrm{Sin}[z(V)]$) and is measured along an arc perpendicular to the ecliptic.

Parameśvara then describes the derivation of the rules for the two components of parallax. First, he gives the proportions for parallax in *yojanas* as measured along the planet's own orbit:

$$\pi_\beta = \frac{\mathrm{Sin}[z(V)] \cdot r_e}{R}, \quad \pi_\lambda = \frac{\mathrm{Sin}[VS)] \cdot r_e}{R}.$$

These are converted to measures along the great circle by dividing by their true distances from the earth and multiplying by R. Parameśvara then describes how, in the case of an eclipse, the combined parallaxes of the sun and the moon should be taken and the lunar parallax is computed using the sun's position, because solar eclipses occur when the luminaries are at (or close to)

conjunction. He then describes the rule for the longitudinal component of parallax given in *nāḍikās*:

$$\pi_\lambda = \frac{\mathrm{Sin}(VS) \cdot r_e}{k_m} \frac{60}{\nu_m}.$$

Multiplication by the radius of the earth over the mean distance of the moon converts the measure from *yojanas* to minutes of arc, and multiplication by 60 over the mean velocity of the moon converts it to *nāḍikā*s (compare this to the procedure described by Āmarāja, section 5.6.5). Parameśvara immediately gives an alternative:

$$\pi_\lambda = \frac{\mathrm{Sin}(VS)}{863}$$

where $863 = \frac{4}{3438}$. The latitudinal component of parallax is given in a parallel manner:

$$\pi_\beta = \frac{\mathrm{Sin}[z(V)]}{863} \frac{\nu_m - \nu_s}{60}.$$

Parameśvara then carefully describes how the components of parallax are to be applied when the sun is to the east or the west of the nonagesimal. He also instructs the reader to iterate the longitudinal component of parallax until there is no difference remaining.

In the next verses (41–64) Parameśvara describes general features relating to eclipses and the various significant elements, such as the five phases, lunar latitude, half-durations, and half-durations of totality. The rules for the computation of the *valana* or inclination are then set out. A different formulation is given for the *akṣavalana* from the one given in the *Grahaṇamaṇḍana*:

$$\gamma_1 = \frac{\mathrm{Sin}\phi.\mathrm{Versin}(\Delta\tau)}{R}$$

Parameśvara is also torn between using the sine or the versine. He insists that the versine is to be used (67):

> ādhikyād harije tiryaggater utkramasaṃgrahaḥ
> [The versine is to be used because at the horizon the change of deviation is a maximum.]
> (Parameśvara *Grahaṇanyāyadīpikā*, 67,as translated by Sarma [1966])
> dyuvṛtta[stha]sya bimbasya hy udaye triguṇaḥ śrutiḥ
> tiryaggatiḥ palajyā ca na syāt sāto 'nupātataḥ
> [The body is on its diurnal circle, and at rising the sine of three zodiacal signs is the hypotenuse, and the sine of the latitude gives the deviation. Therefore the deviation cannot be proportionate (to the sine of the hour angle).]
> (Parameśvara *Grahaṇanyāyadīpikā*, 68, as translated by Sarma [1966])

As noted previously, the sine and versine functions have similar extrema and points of inflection. Thus Parameśvara's reasoning is incorrect.

The *ayanavalana* is given (69–71), where K_m is the *koṭi* of the moon:

$$\gamma_2 = \frac{\mathrm{Sin}\epsilon \cdot \mathrm{Versin}(K_m)}{R}.$$

Again he defends the use of the versine:

> 'trādhikyād utkramagrahaḥ ||
> tiryaggatir hi bimbasya golānte paramāpamaḥ |
> trijyā karṇas tv ayanānte na tv ataḥ sānupātataḥ ||
> [The versine is used since the maximum variation is at these points (i.e., spring and autumn equinoxes). At the ends of the *golas*, (the deviation in the direction of the orb is equal to) the maximum declination, but the hypotenuse is equal to the radius at the ends of the *ayanas* (i.e., at the beginning of Cancer and of Capricorn). Therefore the deviation cannot be proportionate (to the sine).]
> (*Grahaṇanyāyadīpikā*, 70b–71, as translated by Sarma [1966] with modifications)

The total deflection is given by (72)

$$\gamma_{\mathrm{total}} = \frac{\mathrm{Diameter}}{R}(\gamma_1 \pm \gamma_2)$$

where "diameter" is the diameter of the sun or the moon.

Eclipse limits (80) are repeated from the previous two works. Then Parameśvara gives several verses dedicated to the correction of the predicted time for reduction to equatorial time.

He finishes the work by commenting:

> upāyāntaram apy asti parvalambādibhir vinā |
> siddhāntadīpikāyāṃ tal likhitaṃ golavitsmṛtam ||
> [There is another method (for computing the solar eclipse) without finding the parallax at new moon etc. ... This has been explained in the *Siddhāntadīpikā*, as given by "the *golavid*" (literally "expert in spherics").
> (*Grahaṇanyāyadīpikā*, 84, as translated by Sarma [1966])

This "expert in spherics" is probably Mādhava, Parameśvara's teacher. What method can replace parallax in computing a solar eclipse still remains unclear.

5.7.4 *The* Siddhāntadīpikā

As has been seen, Parameśvara refers to the need to use observations over long periods of time in order to perfect astronomical parameters and mechanisms. Such references are not without substance, as Parameśvara has

recorded observations made by himself. Parameśvara had a reputation in his time among his contemporaries and pupils as somebody who made observations. For example, his son's pupil, Nīlakaṇṭha Somayājin (b. 1444), states:

> parameśvarācāryeṇa punaḥ grahaṇagrahayogādikam yantraiḥ pañcapañcāśadvarṣakālaṃ samyak parīkṣitam
> [Eclipses and conjunctions of the planets etc. . . . were observed over an entire time interval of fifty-five years by the teacher Parameśvara by means of instruments.
> (*Āryabhaṭīya bhāṣya, Gola* 48)

This claim is largely confirmed by a work written by Parameśvara. The *Siddhāntadīpikā* (literally "The Light of the *Siddhānta*") includes a record of eclipses which Parameśvara maintains he observed, spanning an interval of some thirty-four years. These observations are set out at the end of the *Siddhāntadīpikā*, a work that is a commentary on Govindasvāmin's commentary on the *Mahābhāskarīya* composed by Bhāskara I (see section 5.6.6). However, it is unclear to what extent Parameśvara's eclipse observations were actually used by him (if at all) to modify his parameters.

Parameśvara begins the *Siddhāntadīpikā* by pointing out the need to reconcile theories with observations:

> iti vistārataḥ proktaṃ grahaṇaṃ sūryacandrayoḥ |
> dṛśyate tatra dṛgbhedaḥ kāle bimbe kadācana ||
> tithiviśvasame śāke prārabhya grahaṇaṃ mayā |
> anekam īkṣitaṃ teṣu bhinnaḥ kālo dṛśā sadā ||
> pratyakṣakālas teṣu prāggaṇitānītakālataḥ |
> ataḥ kāryo 'tra saṃskāro yaḥ kaścid gaṇakottamaiḥ||
> . . . viśeṣo 'yam dṛṣṭisāmyasiddhaye kriyate 'dhunā |
> [Thus, a solar and lunar eclipse are explained in full detail. A difference in observation there is seen. At times, beginning in the Śaka year 1315, many an eclipse was seen by me, among them, there was always a different time with respect to observation. The time observed (was different) from the time obtained by previous calculations. Therefore, some correction is to be made here by the most eminent astronomers . . . This difference now is computed for the attainment of equality with observation.]
> (*Siddhāntadīpikā* 1–3, 6, first half)

Parameśvara was working in the village of Ālattūr, on the banks of the river Nilā. He states his coordinates of that place:

> āsmin grāme palajyā syāt saptavedarasair mitā |
> samarekhā tv ataḥ prācyāṃ yojane 'ṣṭādaśe kila ||

[In this village, the Sine of the latitude has the measure 647. The prime meridian is to the east of that at 18 *yojanas*.]
(*Siddhāntadīpikā* 8)

Using the Sine table given in the *Grahaṇamaṇḍana* (in which $R = 3438$), the latitude can be calculated to be 10;51° (N). For the longitude, Parameśvara states he is situated 18 *yojana*s east of the prime meridian. Supposing he is referring to the line through the city Ujjain (75;46E in longitude), the longitude of Ālattūr would be placed at about 76;46E as 18 *yojana*s is about. However, the prime meridian was not accurately nor consistently determined by Indian astronomers, and the information derived from references to it must be used only as a tentative guess.

In the *Siddhāntadīpikā* (verses 69ff.), Parameśvara gives 13 *actual* observations of eclipses, seven solar followed by five lunar eclipses. The observations are not arranged chronologically, nor by any other obvious theme. For each report, in addition to the date (given in *ahargaṇa* or accumulated days after the epoch), the following information may also be included: the type (solar or lunar), the geographical place, the magnitude (partial, total), the duration (half-duration, half-duration of totality, with the amount, if given, measured in *nāḍī*s), the color, and for solar eclipses and one lunar eclipse, the *padabhā* (discussed below), given for first contact (*sparśa*) and/or release (*mokṣa*).

69. When the ahargaṇa measured 1,653,387, an eclipse of the sun was seen in Gokarṇa. (It was) not (seen) here on the banks of the Nilā river.
70. When the *ahargaṇa* was equal to 1,655,130, an eclipse of the sun was seen in Gokarṇa. (It was) not (seen) here at the confluence of the sea and the Nilā river.
71. Also on the day in question, a difference of color on the side of the disk was imagined by some youths even here on the banks of the Nilā river.
72. When the *ahargaṇa* measured 1,652,000, a very bright eclipse of the Sun was seen in Nāvākṣetra.
73. At the instant of first contact, the padabhā measured 40. But some say that it measured 35, because of individual observation.
74. When the *ahargaṇa* measured 1,655,662, the sun was seen eclipsed slightly on (the banks of) the Nilā river by keen observers.
75. At the time of first contact, the *padabhā* measured 15. At the time of release, 9;30.
76. On that day, the two eyes of the men observing the disk of the sun were not especially injured; therefore at this time a dullness of the rays is to be assumed.
77. When the *ahargaṇa* was 1,643,524, (a solar eclipse occurred). At the beginning, the *padabhā* was 11. The release was seen in the afternoon.

78. Here, the time of the duration (of the eclipse) was greater than nine *nāḍikā*s. Therefore, the *aviśeṣa* etc. for the sun is to be computed by an intelligent person.
79. When the *ahargaṇa* measured 1,647,156, when the sun was released (i.e., when the eclipse ended), the setting was seen, and there was a difference of a quarter of a *nādī* (between release and setting).
80. When the *ahargaṇa* measured 1,648,722, at the first contact, the *padabhā* of the sun measured 24.
81. When the *ahargaṇa* measured 1,655,484, the *padabhā* of the sun measured 5;30 at the time of release.
82. When (the *ahargaṇa*) measured 1,655,647, (there was an eclipse) of the moon. At the time of first contact the padabhā was 15 and at release 2.
83. When the *ahargaṇa* measured 1,655,293, an (expected) eclipse of the moon was not seen in the sky.
84. When the *ahargaṇa* measured 1,652,694, the moon was seen by men to be eclipsed slightly in the sky.
85. When the *ahargaṇa* measured 1,654,614, (an eclipse) of the moon was seen. (There was) a half-duration of totality. Like that, (there was also a lunar eclipse when the ahargaṇa was equal to) 1,653,403.
86. Eclipses of the sun other than those mentioned were seen by me and also many (eclipses) of the moon were seen, but these are not related here. (*Siddhāntadīpikā* 69–86)

The eclipses and their particulars are tabulated as follows:

Verse	Type	Date	Date	Place	*Padabha*	Comments
69	S	1653387	10 November 1425	Gokarṇa		
70	S	1655130	19 August 1430	Gokarṇa		Very
72	S	1652000	22 January 1422	Nāvākṣetra	$F = 40$ or 35	bright
74	S	1655662	1 February 1432	Nilā	$F = 15$, $R = 9;30$	Partial
77	S	1643524	9 November 1398		$F = 11$	> 9 *nādis*
79	S	1647156	19 October 1408			Set after rel.
80	S	1648722	31 January 1413		$F = 24$	
81	S	1655484	8 August 1431		$R = 5;30$	
82	L	1655647	17 January 1432		$F = 15$, $R = 2$	
83	L	1655293	28 January 1431			Not seen
84	L	1652694	17 December 1423			Partial
85	L	1654614	20 March 1429			Total
85	L	1653403	25 November 1425			Total

In order to assess the quality of these observations, several assumptions need to be made. As Parameśvara does not always give the location from which he is observing, the terrestrial coordinates 11;17° N and 76;15° E have

been assumed, that is, it is assumed he was observing from his home (Nilā). However, for four of his solar eclipses he states specific locations: the first is Gokarṇa, for which the modern equivalent could be either modern Gokarn (14;33 N and 74;19 E, about 400 kilometers away from Nilā) or Goa (15;30 N and 73;55 E). From the details that Parameśvara gives, as in verse 70 where he mentions an eclipse that was visible in Gokarṇa but not in Nilā, one needs to take the latter locality to satisfy this requirement. Nāvākṣetra could likely be identified as modern Kochi or Cochin (9;58 N and 76;13 E).

The following are modern recomputations of the solar eclipses listed by Parameśvara given the above coordinates (times are stated in hours and decimal fractions of an hour):[114]

Date	First contact	Mid-eclipse	Fourth contact	Magnitude
9 November 1398	8.52	10.18	12.21	0.88
19 October 1408	15.70	16.84	17.84	0.55
31 January 1413	7.38	8.53	9.82	0.67
22 January 1422	6.94	7.43	7.95	0.11
10 November 1425	15.48	16.14	16.75	0.11
19 August 1430	6.61	7.21	7.87	0.10
8 August 1431	6.59	7.86	9.36	0.66
1 February 1432	7.74	8.19	8.66	0.07

Parameśvara's duration for the 1398 eclipse is notably good. He is correct in maintaining (verses 69 and 70) that the eclipses in 1425 and 1430 were not visible in Nilā. However, the eclipse that he states occurred in 1422 at Nāvākṣetra was also visible in Nilā, and, in fact, was slightly greater there than at Nāvākṣetra.[115] Parameśvara makes no mention as to whether it was seen in Nilā or not. This could well be due to other factors, such as bad weather, for example. His remark that the eclipse of 1408 ended and then the sun set is also correct; the local sunset for this day is 17:53, with the eclipse finishing immediately before this at 17:50.

The lunar eclipses are recomputed as follows:

Date	Beginning	Ending	Magnitude
17 Dec 1423	19.58	20.96	0.04
25 Nov 1425	0.86	4.41	Total (1.20)
20 Mar 1429	2.16	5.68	Total (1.08)
28 Jan 1431	—	—	—
17 Jan 1432	19.39	23.18	Total (1.24)

As Parameśvara has rightly observed, there was no eclipse of 1431—it was most probably a penumbral darkening. Thus all of Parameśvara's records seem

to be based on real observations. As noted above, they are not arranged chronologically in Parameśvara's text. The order in which they appear in the text may roughly reflect increasing magnitude. The exceptions are the eclipse of 1422 (observed in Nāvākṣetra) and the eclipse of 1398.

The smallest eclipse (0.07) is in fact too small to observe according to Parameśvara's own eclipse limits. The *Grahaṇāṣṭaka* (8)[116] gives the solar eclipse limit as ⅛ of the sun's diameter, which is equivalent to 0.125 in magnitude; three of his solar eclipses would fall into this category (1425, 1430, and 1432). Similarly, with his lunar eclipses Parameśvara's limit of 1⁄16 of the moon's own diameter is again larger than one of the eclipses recorded by him. The minimum value 1⁄16 is equivalent to 0.0625, but the lunar eclipse of 1423 had a magnitude of 0.04.

Parameśvara gives his measurements of the eclipses with respect to a *padabhā*. This term is unique to Parameśvara and translates literally as "foot-shadow." It most probably refers to the measurement of the shadow made using the gnomon on the back of an astrolabe. On an Arabic astrolabe such components for measuring the shadow are called "shadow squares." This Sanskrit term may be derived from the fact that the Arabic astronomers used the term *al-aqdām* (literally "feet") to measure this shadow.[117]

The following is a table comparing Parameśvara's *padabhā* measurements with those of the modern recomputations of the altitude of the eclipsed body at impact and clearing:

Date	*Padabhā* at impact	*Padabhā* at clearing	Altitude at impact	Altitude at clearing
9 Nov 1398	11	—	30.36	59.58
31 Jan 1413	24	—	16.77	48.76
22 Jan 1422	40 or 35	—	10.39	24.32
8 Aug 1431	—	5;30	11.07	51.25
17 Jan 1432	15	2	23.10	74.78
1 Feb 1432	15	9;30	21.01	33.87

As one would expect, under the assumption that the *padabhā* is a shadow measurement, the *padabhā* measurement is inversely proportional to the altitude. All the measurements show the correct relationship. This further confirms that Parameśvara was indeed using actual observations.

The circumstances in which Indian astronomy was practiced, its format and content, the nature of its origin, and the attitudes of its practitioners ensured that the trajectory of its development was significantly different from that of its counterparts in the other astronomical cultures. Its relation to these other

traditions is therefore both direct and nuanced. As a result, there is no question as to its vital historical significance. Characterized as a receiver and remodeler of foreign astronomical ideas, Indian astronomy was the product of a complex history of cross-cultural scientific exchange and development. Astronomical knowledge was delimited and channeled into five distinct schools or *pakṣa*s, which ironically found their derivation from at most two distinct external sources stemming from Greek planetary tables. Indeed, most elements that are the product of eclipse reckonings can be related back to Greek sources. However, these ideas, originating in a foreign culture, were suitably modified, and in many cases improved with superior mathematical techniques, so that they could be regarded as distinctly Indian.

That its contents were identified as being the product of divine revelation entailed a marked conservatism on the part of the practitioners of Indian astronomy and therefore instilled a strong reluctance in them to tamper with the fundamental models and parameters. This attitude brought with it a distinct approach to many elements of astronomical inquiry; there was always a complex and delicate relationship between observation and computation—the upholding of divine origins entailed a relaxation of accountability for the strict correspondence between the two. Indian astronomers were largely sheltered from insights and considerations in natural science, physics, philosophy, and other subjects which may have challenged their views. The necessities of oral culture posed additional challenges to the practitioners, encouraging the priority of memorability, conciseness, and recall in their compositions; the exclusivity of *śāstra* promoted esotericism but encouraged scholars to protect the arcanal qualities of their efforts. Astrological requirements and religious application continually ensured the pursuit of astronomy was a practical endeavor. As a result, Indian astronomers made little improvement in their abilities to predict eclipses and account for their features over time.

However, Indian mathematical astronomy significantly influenced the early stages of this discipline in the Islamic empire. Some of the first works written in Arabic in this field were closely related adaptations of works originally composed in Sanskrit. Along with mathematical conventions, standard parameters, and superior computational techniques, the whole Indian astronomical model was incorporated and investigated by Islamic astronomers. It provided much inspiration and support for their own astronomical pursuits.

CHAPTER SIX

A Science of Configuration
Arabic Eclipse Reckoning

It was clear to the sight that the world was lit up, the stars had disappeared, the sun was about to rise, and the moon was about to set behind the mountains which screened it. A small portion of the eclipse (still) remained in its body and I was unable to observe it (i.e., the time of clearance) exactly.

—al-Bīrūnī in *Kitāb Taḥdīd Nihāyāt al-Amākin li-Taṣḥīḥ Masāfāt al-Masākin*, as translated by Said and Stephenson (1997) 47.

To find the truth is tough . . . But God has not protected scientists from error . . . The seeker after the truth is, therefore, not he who studies the writings of the ancients and, following his natural disposition, puts his trust in them, but rather the one who suspects his faith in them and questions what he gathers from them, the one who submits to argument and demonstration and not to the arguments of a human being whose nature is fraught with all kinds of imperfection and deficiency.

—Ibn al-Haytham *Optics*, as translated by A. I. Sabra (1998).

6.1 Overview

When Caliph Abū Ja'far al-Manṣūr founded Baghdad in 762 CE, he did so with the intention of establishing the heart of the Islamic nation; the nascent dynasty of 'Abbāsid rulers had far-reaching ambitions for this state. Indeed, not only did Baghdad become the established and enduring center for the nation of Islam, but it formed the cultural, political, and intellectual locus for what was at its peak to be a vast empire stretching as far as India, Spain, and Central Asia. Although intellectual activity languished in some parts of the world during this period, in these quarters it flourished. Initially, scholarly activity was energized by the solicitation of material from sources distant in time and place. The ensuing transmission of foreign scholarship invigorated the intellectual

environment of this incipient empire, and the industry with which this foreign scholarship was incorporated and processed makes it a significant period in the history of science. Not only is the activity of Islamic scholars critical to the history of the classical tradition, but also the resulting Islamic intellectual legacy had a significant impact on the European Renaissance in the thirteenth century and beyond.

There were many factors during this period, roughly the eighth to the fifteenth century, which encouraged astronomical activity. Islamic religious practice was a significant motivation for research in astronomy.[1] Astronomical details such as the visibility of the new crescent, the layout of the months (lunar), and daily time-keeping (solar), as well as related requirements, such as the determination of the sacred direction (the *Qibla*) of prayer, were all fundamental to the Islamic religious tradition. In addition, despite the fact that they found no precedent in the dominant religious tradition, astrology and related arts were fundamental to the political and social needs of those living in the Islamic empire. Even more essential to the enterprise of science were the general conditions under which scholarship thrived. The Islamic nation at this time has been characterized as multicultural,[2] and for the most part open-minded, persistently curious, deeply inquisitive, and conservative and revisionist as well as progressive and innovative.

Furthermore, the focus on the overriding supremacy of one religious tradition, namely Islam, over many meant that its primary language, Arabic, united this multicultural empire. This was critical for science; the Arabic language connected scientific practitioners by means of a shared medium of communication. In the diversity of influences from other cultures and the constant flow of information from strikingly different intellectual traditions, Arabic provided a common forum. This was an ingredient essential to the flourishing of science; the careful scrutiny of foreign ideas as a result of translation into the Arabic language and the philological exactitude required therein encouraged scholars to be deeply mindful about the content they worked on and motivated them to develop and improve it. Accordingly, a single language for science fostered collaboration and facilitated the smooth flow of ideas among Islamic scholars, and its passing on to later generations.

The practice of astronomy was generally captured by the Arabic term *'ilm al-hay'a*,[3] which literally translates as "knowledge of the structure on configuration (of the universe)." Also commonly used were *'ilm al-falak*, the "knowledge of the celestial sphere" and *'ilm al-nujūm*, the "knowledge of the stars"; the latter term can be applied to astrology as well as astronomy. The majority of astronomical works that survive consist of tables with accompanying explanations on their use, called by their authors *zījes*, a word that is Persian in origin.

There are estimated to be about ten thousand Islamic manuscripts extant,[4] and it is reckoned that these represent only a small portion of the original corpus. As further evidence of the astronomical activity of this period around one thousand astronomical instruments survive. Thus the sources on which historians can base their recovery of these efforts are not excessive. The many complexities involved in editing and penetrating these huge works have meant that only three out of some two hundred *zījes* have been properly published in modern times. Despite this, even the small remnant available is informative and gives a rich impression of the industry of scholars of this period.

Almost immediately after the establishment of the new capital at Baghdad came the initiation of a massive program of translating works from Indian, Iranian, and, most comprehensively, Greek originals into Arabic, a project that continued for well over two centuries.[5] By the late tenth century, the majority of all Greek philosophical and scientific treatises that were accessible to Islamic scholars were translated into Arabic, sometimes more than once. What is significant about this movement is that it was based upon the importation of foreign texts gathered not by peripheral or fortuitous exchanges of traders and explorers, but by professional scholars as a dedicated deliberate endeavor. Carried out with thoroughness and attention to detail, the translation movement was supported by influential social groups and gained momentum from its own progress.

The impact of the translation movement on astronomy and related disciplines was profound. The promise of entirely new ways of tackling existing problems must have been very alluring. Astronomical texts and theories were sought from India, Greece, and Iran, and thus formerly unconnected techniques and theories were intermingled and synthesized. The very activity of translating and its exacting standards for penetrating a text resulted in new forms of scholarly scrutiny. Translation efforts demanded the creation of new vocabulary, the articulation of novel ways of thinking, and the incorporation of new-fangled symbolic representation of numbers, geometrical mechanisms, and diagrams, and other mathematical concepts and techniques into an emergent field. Translating scholars were compelled to analyze texts in a way they never otherwise would have. Such thorough scrutiny and meticulous consideration of content highlighted the areas in which astronomical theories from different sources were in disagreement. The existence of differing models naturally turned the attention of the Islamic astronomers toward issues of consistency. How were they to judge the superiority of one system over another? Such circumstances not only forced scholars to seek resolution from internal considerations, but also accentuated the absolute necessity of obtaining empirical evidence of their own to enable them to apply consistent criteria for

evaluating theories. The eventual outcome was the reliance on one, fundamentally Ptolemaic, model but with new mathematical techniques and better parameters derived from updated observations. But the critical activity that had brought about its dominance also meant that this model itself became subject to critique and intensive examination, until it too was eventually overhauled. The refuge that mathematics provided to astronomy was slowly dawning on scholars and was revealing itself as increasingly unsatisfactory. The Andalusian philosopher Ibn Rushd (b. 1126), later known in the Latin world as Averroes, appears to express this sentiment—namely of the limitations of mathematical astronomy:

> In my youth, I hoped to make a complete study of this, but now that I have grown old, I have given up this idea because of the obstacles I found in my way before. But this explanation will perhaps induce somebody to study these things later. In our time, astronomy is not about something real; the [model of the] sphere existing in our time is a model conforming to calculation, not to reality.
> (Ibn Rushd *Tafsīr Mā ba'd al-ṭabī'a* XII c. 45, as translated by Gerhard Endress, in Hogendijk and Sabra (2003) p. 122)

Indeed, foreign ideas were studied, critiqued, and developed with such industry by Arabic-literate scholars that, by the end of the first millennium, the astronomy practiced in the Islamic Near East could be firmly and distinctly called "Arabic." The breadth of the Islamic empire meant that the impact of this astronomy was far-reaching and substantial well beyond the Near East itself.

6.1.1 A Brief Survey of the Development of the Astronomical Tradition

Before the foundation of Islam, the peoples who inhabited the Arabian peninsula maintained simple practical astronomical knowledge—folk astronomy—such as the rising and setting of stars, phases of the moon, basic time-reckoning techniques, and the behavior of the sun and its relation to agriculture. From these simple schemes arose more mathematically based reckonings of phenomena with increasing attention to detail, partly as a result of increasing confidence in tracking the phenomena and partly as a result of external inspiration. The first technical astronomical treatises were the result of an Indian-Iranian tradition, which found its roots in pre-Ptolemaic Greek astronomy. These initial influences were from the Iranians of the Sasanian period. Epitomizing this tradition is a late version of a work called the *Zīk-i Shahriyārān* prepared during the reign of Yazdigird III (d. 651), the parameters of which are clearly derived from the Indian *ārdharātrikapakṣa* (see sections 5.1.1 and 5.6.5). This work was

composed in Pahlavī and formed a reference for early development. All these early works were characterized by their preference for simplified mathematical techniques and pragmatic methods rather than a concentration on theoretical models and geometrical structures of the celestial realm.[6]

By the eighth century, the earliest astronomical works written in Arabic appeared. Their content marks a new phase in astronomical activity, as they have clearly been influenced by works originating in India and Afghanistan. Furthermore, these works were written in a format that was to prove foundational and dictated the ways in which Islamic scholars were to present their results for centuries to come. This format was the *zīj*, a work consisting of tables and explanatory text; it derives from a Persian word (*zīk*) meaning "thread." It has been suggested that this appellation refers to the actual layout of these works; the interlacing of the ruled tables which housed the astronomical parameters resembled the warp and weft in woven cloth. The first, in 735, was the *Zīj al-Arkand*, which uses the Sanskrit work the *Khaṇḍakhādyaka* written by Brahmagupta in 665 (see section 5.6.5) for inspiration, but was also influenced by the *Zīk-i Shahriyārān*. A blending of Sanskrit and Pahlavī texts can be detected in other works of this time, such as the *Zīj al-Harqan* (which has an epoch of 742).

Another major influence was from Syrian sources. Syrian astronomy itself was influenced by the Indian tradition via Sasanian Iran, but ultimately the Ptolemaic tradition dominated Syrian scholarship. Syrian scholars knew both Greek and Arabic equally well. In particular, the seventh-century monk Severus Sebokht of Nisibis had knowledge of the *Handy Tables* as well as the wider tradition of Greek astronomy, and was acquainted with Indian numerals. Theophilus of Edessa served as the chief astrologer of Caliph al-Mahdī from the 750s until his death in 785. Theophilus knew Greek, Syriac, and Arabic, and had familiarity with Indian material.

Early ʻAbbāsid court politics and the demands of structuring a new empire, largely due to Caliph al-Manṣūr (754–775) and Hārūn al-Rashīd (775–785), saw a rise in institutional support for astronomical among other intellectual endeavors. At this time, Greek sources and the very first translations of Ptolemy's works appeared on the scene, but there was also a further influx of Indian material. This additional inspiration from Indian astronomy reached the Islamic tradition through diplomatic visits around 772,[7] at which time a work based on an intermediary of Brahmagupta's *Brahmasphuṭasiddhānta* (628) (see section 5.6.4), supposedly called the *Mahāsiddhānta*, was translated into Arabic by al-Fazārī[8] (fl. second half of eighth century) under orders from al-Manṣūr. It was entitled the *Zīj al-Sindhind al-kabīr*, and proved influential for many works to come. A contemporary and collaborator of al-Fazārī was Yaʻqūb ibn Ṭāriq. He wrote a *zīj* that contains elements traceable to the *Mahāsiddhānta* (literally

"the Great Siddhānta"), the *Zīj al-Shāh*, and (probably) the *Zīj al-Arkand*,[9] as well as Ptolemaic elements.

The first translations of Ptolemy's *Almagest* and *Handy Tables* into Arabic appeared during the reign of Hārūn al-Rashīd.[10] Additional translations, such as that completed by Arabic scholar al-Ḥajjāj ibn al-Maṭar[11] in 829 or 830, were commissioned, with the result that there were at least five translations of Ptolemy's *Almagest* circulating in the late eighth and ninth centuries. The definitive translation of this important work most frequently consulted by scholars was that prepared by Naṣir al-Dīn al-Ṭūsī in the late ninth century. Ultimately, Ptolemy's *Almagest* formed the basis for the cosmological picture of Arabic scholars, whether they accepted it or used it as a platform for adjustments. After the initial translation efforts, Ptolemy's work was commented on, corrected, and critiqued. Programs of observation gained momentum, and in the ninth century the tradition of the observatory was institutionalized.

By the ninth century, astronomy in the Islamic Near East was a complex combination of indigenous traditions, mixed in with elements that had been transmitted both directly and indirectly from various cultures, including Indian, Greek, and Persian. In response to this, new directions for astronomy came from on high. Under the reign of Caliph al-Ma'mūn (813–833), among other considerable initiatives in the intellectual environment, there was a marked increase in activity of observatories, particularly in Baghdād and Damascus, largely as a response to the need to reconcile the contradictory parameters and incompatible models that existed. Astronomy matured during this time according to two distinct routes.[12] One strand adopted the Indian school and updated it by means of the Iranian tradition as well as the material from the *Handy Tables*. The other strand adopted the Ptolemaic models but made modifications to them, based both on their own observations and on other sources taken from the Indian or Iranian traditions, particularly when these aspects were deemed simpler than their Ptolemaic equivalents. Indeed, such activity can be characterized by eclectic rearrangement of disparate sources, their incorporation and adjustment as a result of contemporary observations, the adoption of the latest mathematical techniques and insights, and minor tinkering with the underlying geometrical models. Naturally, these practices only further accentuated the internal discrepancies which were not to be overcome without some major changes.

These practices are well documented in the works produced by scholars at the time. Ḥabash al-Ḥāsib (d. 864/874) of Marw was appointed the head of the Damascus observatory by al-Ma'mūn and engaged in observational activity from 829 to 864. As well as writing prolifically, he produced work that became fundamental for later scholars. From two extant astronomical manuscripts,

one can conclude that, although his basic models are Ptolemaic, he adopts many parameters from non-Ptolemaic sources, including the *zīj al-Mumtaḥan* (literally "the Validated *Zīj*") which was based on contemporary observations by a team of Islamic scholars. Furthermore, his eclipse theory is influenced by the Indian tradition.

In this respect, Islamic scholars were heavily indebted to their Indian predecessors for iterative computational methods.[13] Fixed-point iteration techniques were used by Islamic mathematicians (as for the highly accurate computation of the Sine of 1°) using the Indian methods.[14] In the Islamic astronomical tradition, as was common in the Indian tradition, iterative approximations were usually presented alongside analytical solutions to the same problem. Ḥabash epitomizes this practice in his presentation of parallax theory. Rather than reconcile the disparate sources he draws from, he includes different approaches to the same problem alongside one another. For his computation of the longitudinal component of parallax, Ḥabash presents an alternative method to the standard analytical solution, namely, a fixed-point iterative algorithm for its determination.

Other critical techniques were imported and refined from India. Islamic scholars adopted the Indian sine function, using as their radius the Greek-inspired 60 which was originally the diameter in Ptolemy's chord function. They also adopted the Indian "shadow" or tangent functions tabulated with respect to various bases, including 12, 6.5, 6, 7, and 60, and occasionally 1. From a mastery of basic elements in trigonometry came critical insights into the computation of triangles on the sphere. This was inspired by the Greek tradition, and not the pure plane trigonometry of Indian mathematics. The Iranian scholar Abū'l-Wafā' (b. 940, d. 997/998) made advances in spherical trigonometrical techniques, particularly the Sine law for spherical triangles, which were to make many astronomical calculations theoretically more correct.

The work of Ḥabash's contemporaries also reveals similar circumstances; for instance, that of Yaḥyā ibn abī Manṣūr (d. 832) who worked alongside Ḥabash at the observatory in Damascus and Al-Khwārizmī (b. before 800, d. after 847). Yaḥyā and Al-Khwārizmī too were close collaborators and worked together at the Damascus observatory in 828. Al-Khwārizmī clearly reveals a reliance on Indian techniques and parameters; in particular he incorporates many elements from the "midnight system" of Brahmagupta's *Khaṇḍakhādyaka* (see section 5.6.5). From excerpts of Yaḥyā's work, in particular from a single manuscript (Escorial 927), elements of Indian influence can also be detected along with a close reliance on material from the work of al-Khwārizmī. On the whole, then, while on many occasions the resulting modifications made during this period did produce models that were in many

respects "improvements" over those expounded by their predecessors, there was still a glaring lack of command over the fundamental geometric apparatus and a marked diffidence about disturbing the fundamental aspects of the received imports.

However, such an overhaul did come before too long, and the developments that ensued represent a plateau in the history of astronomy in Islamic civilization. By the tenth century, the interest in maintaining Indian and (pre-Islamic) Iranian systems waned; although elements from these traditions were retained, the initial enthusiasm for their overall approach was lost and their inclusion in later works was more a result of their historical interest than anything else. For example, later *zīj*es tended to discard the approximate solutions resulting from the Indian-based iterative algorithms in favor of the more rigorous geometrical analytic solutions in the spirit of Ptolemy.[15] This and other such shifts seemed to secure the fate of the various models and methods circulating on the astronomical scene, resulting in the firm orientation toward the supremacy of the Ptolemaic geometrical model over these other schemes.

This first distinctly appears in the works of al-Battānī (b. before 858, d. 929/30). His *Zīj al-Ṣābi'* reveals a firm commitment to the models of Ptolemy, but as he states in the preface, he recognized that these models could be improved by observations, just as Ptolemy had requested. Al-Battānī was in an ideal position with respect to the tradition of observation; this activity had been up and running since the time of Caliph al-Ma'mūn (813–833). Al-Battānī had at his disposal those observations made by Yaḥyā included in the *Zīj al-Mumtaḥan*, as well as his own observations which he made at al-Rāqqa (located on the north bank of the Euphrates) over the four decades between 877/8 and 918/9. Following his lead, others adopted similar observational practices, including al-Khāzinī (fl. 1115–1130), Naṣir al-Dīn al-Ṭūsī (b. 1201, d. 1274) and al-Kāshī (d. 1429). The reexamination of other Greek works followed including those by Autolycus, Aristarchus, Hypsicles, Theodosius (as part of the so-called "Little Astronomy"), as well as other works of Ptolemy such as the *Planetary Hypotheses* and *Planispherum*. Rules and techniques were justified and studied by means of geometrical devices such as the analemma and stereographic projection.

Development in the astronomical scene would come now only from more philosophically or theologically motivated objections. Such careful scrutiny and scholarly dedication to Ptolemy's model invoked criticisms, such as those from ibn al-Haytham (ca. 1205) who wrote a work called *al-Shukūk 'ala Baṭlamyūs* ("Doubts Concerning Ptolemy").[16] Further analysis of the Ptolemaic model in this light eventually led to its restructuring, most radically by the replacement of the deferent by substitutes proposed by ibn al-Shāṭir (d. ca. 1375). Questions

about the extent of the legacy that was passed on to the Latin West in this respect and the extent to which the proposals of ibn al-Shāṭir may have influenced Western astronomers such as Copernicus, remain as yet unanswered.[17]

The Islamic scholars recognized their intellectual inheritance:

> the astronomers of ancient times, who established the principles of this science, have left nothing for their successors to do with the exception of improvements of expositions, arrangement of material so as to facilitate understanding, correction of certain mistakes on the basis of examples of procedure available in their own texts, or the rectification of errors which were introduced after their own time.
> (Ḥabash al-Ḥāsib *Damascus Zīj*, 144–145, as translated by Sayili (1960) p. 80)

This sentiment is also seen in the *Almagest* with Ptolemy's own explicit insistence on checking parameters derived from observations by means of further observations. This recommendation made such an impression on Islamic scholars that they deemed it a command and charged themselves with the responsibility of testing and checking received astronomical schemes.[18] The concept of "testing"[19] became a significant part of astronomical research programs and the notion of tests and comparisons, modeled on Greek practices of "trials" (*peira*) and "comparisons" (*synkrisis*),[20] further encouraged and organized efforts. Beyond this came the consideration of corrections of a more fundamental sort. The notion of "doubt" (*shakk*) in an investigative context had been passed down to Islamic scholars from Aristotle and his considerations of *aporia* as a philosophical tool. Such a theme was not limited to astronomy, but was applied to other fields of scholarship, such as cosmology, mathematics, optics, and medicine. Doubts raised against Ptolemy in an astronomical context were not so much against his basic assumptions, such as circularity and uniform motion, but rather against Ptolemaic propositions or hypotheses.[21] Furthermore, the intention of the Islamic scholars was not to dispense completely with the mathematical effects and structures of the Ptolemaic model, but rather to more thoroughly accommodate and harmonize his designs into a unified cosmology, to achieve a model that was ultimately representative of reality. This is what is explored by ibn Rushd, when he remarks:

> In our time, astronomy is not about something real; the [model of the] sphere existing in our time is a model conforming to calculation, not to reality.
> (Ibn Rushd *Tafsīr Mā ba'd al-ṭabī'a*, XII c. 45, as translated by Gerhard Endress, in Hogendijk and Sabra (2003) p. 122)

The search for "configuration" (*ha'ya*) was a call beyond that generated by discrepancies between theory and observation. For many, it was not an issue

of how well a model corresponded to empirically derived facts, but instead the harmony and consistency of the mathematically oriented schemes within the greater cosmological picture. The significance of the Arabic appellation for astronomy as "the knowledge of configuration" is noteworthy—astronomy becomes essentially a science of configuration characterized by a dual concern for the demands of both the mathematical and the natural, with neither one considered at the expense of the other. Islamic scholars took very seriously Ptolemy's admission that in some respects his proposals represented a "departure from theory" (*para ton logon*).[22] Those who noticed this admission interpreted it to have deep significance. Ibn al-Haytham reflects:

> this method has led him [Ptolemy] to assume what he confessed to be a departure from theory; and since he knew this, his excuse for having committed this [departure] on the grounds that it does not affect the [observed] movements of the planets can be accepted only with the admission that the true configuration is different from the one he assumed, and that he was not able to grasp its true nature.
> (ibn al-Haytham, as cited in Sabra [1998] p. 304)

The drive to uncover this configuration dominated all astronomical research henceforth in the Islamic Near East.

6.2 General Features of Eclipse Reckoning in the Islamic Near East

6.2.1 The Zīj

By far the majority of astronomical works in the Islamic Near East from the eighth century onward were composed in the format called the *zīj*. A *zīj* is an astronomical work that includes many tables with an accompanying text. The emphasis of this text is practical; it is intended to function as a guide to explaining how the tables included in the work should be applied, and it contains detailed descriptions of how to carry out the various procedures. It is not intended to be a complete theoretical exposition and in general little attention is given to revealing the origin and derivation of the various parameters and formulas presented therein.

Generally a *zīj* is applicable to a single geographical location, that is, particular terrestrial coordinates underlie all the tables. Although a number of *zīj*es were the product of original observational and computational efforts, many were simply refashioned versions of earlier *zīj*es with the occasional adjustment or elaboration. As a result, there are often inconsistencies between the text and the tables, or between different tables in the same work. Authors

may fail to recompute tables for their desired coordinates or dates, and commonly (and sometimes incompletely) borrow tables from other sources, especially from Ptolemy. They may borrow from multiple sources based on different fundamental assumptions and/or parameters.

Islamic astronomers adopted the Indian trigonometric functions, and every *zīj* had trigonometric tables containing (at least) the values of sine (*al-jayb*) and the tangent (*al-ẓill*), selecting most frequently R to be 60 (as in the Hellenistic tradition) or 1. Spherical trigonometric techniques were especially important to them, not only because of their emphasis on astronomical time-keeping but also other reasons (such as computing the *qibla*, or sacred direction).

*Zīj*es include several different tables for computing the circumstances with respect to eclipses.[23] Islamic scholars commonly adopted one of two approaches in the presentation of their tables of lunar and solar eclipses.[24] The first approach is the presentation of a table with the moon in various extremal circumstances; the situation when the moon is at epicyclic apogee and when it is at epicyclic perigee. Intermediate values of the lunar anomaly between these two positions are computed by means of interpolation. Features such as magnitude and duration are tabulated with respect to the latitude of the moon. The second approach adopts a double-argument format, in which magnitude and duration are tabulated as functions of lunar latitude and change in lunar longitude. Related tables (in no fixed sequence) include the following.

Tables of mean conjunctions and oppositions. These are tabulated with respect to the Hijra calendar, which, conveniently, is lunar. They give the time of the day (t) of the mean syzygy in question, along with the mean anomaly of the moon, the longitudes of the luminaries, and the longitude of the apogee of the sun.

Tables of solar and lunar rates at syzygy. These are tabulated with respect to mean solar distance from the apogee and lunar anomaly respectively.

Apparent radii of sun, moon, and shadow. These are tabulated as a function of mean solar longitude and anomaly of the moon, or the longitudes.

Parallax. Parallax is presented in several forms: tables of lunar and solar parallax on the altitude circle, tables for the "adjusted lunar parallax," which is the difference between the lunar and solar parallaxes, and tables for the latitudinal and longitudinal components of parallax (π_β and π_λ, respectively), tabulated for various latitudes or climes. In these tables, values are arranged using a double argument: the initial points of the zodiacal signs and the integer hours of conjunction before or after the local meridian.

Tables of lunar eclipses and of solar eclipses

Eclipse magnitudes. The unit of measurement is the "digit" (*aṣba'*, pl. *aṣābi'*), measured along the straight line through the diameter of the appropriate luminary's disk, which is divided into twelve digits.
In addition, following Hellenistic traditions, this "linear" magnitude could also be converted into "area digits" (*aṣābi' al-jirm*). That is, the area of the eclipsed body is divided into twelve and the eclipsed area is quantified by the twelfths of the whole. The conversion table between the two is usually reproduced directly from Ptolemy's *Almagest*.

Inclination (*inḥirāf*). This is the angle between the line running through the two centers of the eclipser and the eclipsed and the ecliptic, usually at first contact.

Notably, tables for the computation of parallax are generally found before any accounting of eclipses.

A major obstacle to a thorough understanding of eclipse theory in the Islamic Near East is the fact that almost all extant *zīj*es remain unedited and/or untranslated. Much of the following material is taken from studies that are based on partial surveys of the primary sources. For the most part, these consider relevant excerpts directly from the manuscripts, generally taken in isolation, and are not comprehensive interpretations of the text as a whole. Nevertheless, some general observations can be proposed. Initial works found inspiration in Indian methods, as evidenced particularly in the adoption of Indian techniques to calculate parallax. These procedures were sometimes included alongside methods inspired by or directly taken from Ptolemy; the presentation and exposition of Ptolemaic tables proved ideal for direct transfer into the *zīj* format. A preference for the Ptolemaic procedures was eventually established, but Indian elements were maintained, particularly mathematical innovations and spherical trigonometric techniques. Further modifications and developments involved the correction of underlying parameters based on observations, and a more comprehensive reckoning in tabular format, such as the adjustment of values for local geographical circumstances and the like.

With such a command over theory by this point and a developing interest in the physical makeup and configurations of the heavens, there arose further notable undertakings within astronomy at later stages. Eclipses are complicated astronomical events, with many sensitive cumulative components behind the features and circumstances of their occurrence. The Islamic scholars, largely as a result of Ptolemy, had within their grasp the technical apparatus to deal with this level of sophistication and account for each of these intricacies. But, on another level, practitioners realized that complex phenomena, with prudent

approximations and the right assumptions, could be simplified without significant loss of accuracy. The impetus to reduce calculational complexity sprang from the repetitive and numerous calculations demanded by the compilation of tables. As a result, in later stages, there arose proposals for various approximating arithmetical techniques, which consciously reduced the complexity of the problem with results that still preserved a sufficiently close connection to the phenomena. Earlier ambitions of modeling the problem at the deepest technical level were now sometimes trumped by a search for the best simplifying assumptions, emphasis on interpolation, and approximating techniques. Methods such as those that had been proposed by the earliest inquirers into eclipse phenomena were now being revisited, albeit with theoretical insight behind them.

6.2.2 Observation of Eclipses

Astronomers in the Islamic Near East had in mind several primary objectives when they observed eclipses.[25] The first, and most important, was to check and assess the accuracy of their theoretical expositions. As well as directly testing the accuracy of the eclipse theory itself, eclipses also are critical instants in which elements pertaining to a more general solar and lunar theory can be checked. A systematic program of observation was initiated by Aḥmad ibn Muḥammad al-Nihāwandī under the patronage of the family of Barmak during the reign of Hārūn al-Rashīd.[26] At this time, the primary motivation for the program was to address the sets of contradictory astronomical accounts present on the scene at the time; these included the Indian, the Iranian, and the Ptolemaic traditions. Despite the fact that these traditions all stemmed ultimately from Hellenistic astronomy, their parameters and methods were frequently significantly different. Another motive for observing eclipses was to establish or verify geographical longitudes by simultaneous observations at different locations.[27]

To some degree Islamic astronomers recognized the inherent limitations in their observational techniques, despite well-equipped observatories and instruments. For example, like (perhaps even instructed by) their Indian counterparts, they seemed aware of the differences caused by parallax[28] and the deficiencies of the human visual apparatus for registering certain celestial changes with precision. Thus, there were discrepancies between the times gained from their observations and those predicted using the tables from their *zījes* that were not just the result of imperfect parameter values or approximating mathematical techniques. The extent, then, to which these observations could be useful in suggesting constructive theoretical corrections is limited. Astronomers for whom there exist observational reports of eclipses almost

always incorporated these observations in their theoretical expositions[29] and included some sort of comparison between the two.

Those astronomers who extended the tables of Ptolemy to almost eight hundred years after their construction, generally cast poorer predictions. This is to be expected, as even very small errors inherent in those tables increased over a large span of time and the adjustments that needed to be made for the differing geographical circumstances were often completed imperfectly. The errors presented by these discrepancies compelled astronomers to attempt corrective measures to these parameters in their own tables. They both consciously articulated this objective and carried it out. At first, this ambition manifested itself in updates to parameter values and recomputation of tables, and more powerful mathematical techniques. But ultimately it resulted in an overhaul of the very model itself. Therefore, the discrepancies between empirical evidence and theoretical predictions did have an effect.

Al-Battānī attests to an instance where observations did provide the impetus for necessary correctives. In this example, he shows that there were some instances of eclipse phenomena that did undeniably suggest faults in theory, such as when it suggested a total eclipse and there was not:

> According to calculation from Ptolemy('s tables), the sun should have been totally eclipsed, and the (time of) the middle of the eclipse was later than the observed time by about two hours. Such a discrepancy is not acceptable.
> (al-Battānī *Al-zīj aṣ-Ṣabi'*, as translated by Said and Stephenson [1997] pp. 43–44)

Indeed, Islamic astronomers did achieve significant improvements in the accuracy of eclipse timing, pinpointing observed events to within ten minutes.[30] This was a significant accomplishment and their recording ability exceeded the achievements of all of their predecessors. On a theoretical level, the *zīj*es could produce the critical times of an eclipse with an accuracy of better than 20 minutes which also was a notable advance.

6.2.3 Parallax

An account of parallax, in Arabic *ikhtilāf al-manẓar* (literally "divergence of sight"), was considered essential and tables to correct for its effects are a standard inclusion in all *zīj*es. Commonly tabulated was the adjusted lunar parallax π_{adj}, which represents the difference between lunar and solar parallaxes, that is:

$$\pi_{\text{adj}} = \pi_m - \pi_s.$$

Other measures were commonly used for the computation of parallax. These included the (shortest) distance between the zenith and the ecliptic [$z(V)$], known by some Islamic scholars as "the latitude of the visible climate" (*arḍ iqlīm li'r-rū'ya*), perhaps because it is the complement of the angle between the ecliptic and the horizon, or the zenith distance of the nonagesimal. Another measure was the complement of the true altitude of the place of conjunction, or the zenith distance of the conjunction [$z(L)$]. Most frequently, parallax in altitude would be computed first and then separated into its longitudinal and latitudinal components. This conversion could be done either by means of plane triangles or by using more sophisticated (and technically more accurate) spherical trigonometry.

The basis for most parallax tables is *Almagest* V, 18 (see sections 4.5.1 and 4.5.2) which has tables of lunar and solar parallax measured along the altitude circle; these tables were copied and included in many *zīj*es. The practice of tabulating the difference between lunar and solar parallax—the adjusted lunar parallax—is original to Theon who then splits this into latitude and longitude components. He presents one table for each of the seven climes. Each of these tables has two arguments: the first the integer hours of conjunction before and after local meridian, and the second the initial points of each zodiacal sign in which conjunction occurs.

Frequently, despite the fact that parallax is highly sensitive to geographical coordinates, tables included in one *zīj* may be reproduced wholesale from other sources that were compiled with respect to different latitudes. It is less common that an author will recompute a table with the same underlying structure for the pertinent geographical latitude in question.

As well as a Greek influence, there is a clear Indian inspiration in procedures to compute parallax particularly in early texts. The techniques for computing longitudinal and latitudinal components are based on the same measures that Indian scholars used, such as the zenith distance of the nonagesimal, and the longitudinal difference between the sun and the nonagesimal. Iterative procedures for finding the correct time of conjunction also appear to be inspired by Indian methods.

6.3 Major Contributions to Islamic Eclipse Reckoning

6.3.1 al-Khwārizmī

Al-Khwārizmī (fl. 840) is perhaps best recognized for his contributions to mathematics, particularly his development of techniques that were to prove seminal for the growth of modern algebra as well as his introduction of the Indian numerals to mathematicians in the Islamic Near East. Indian inspiration also

affected much of al-Khwārizmī's work in astronomy. He wrote at least one work on astronomy, a *zīj* that was given the name *Zīj al-Sindhind*, which reveals its origins, as *Sindhind* is a corruption of the Sanskrit term *Siddhānta*, the technical term used to refer to Indian astronomical texts (see section 6.1.1 for previous works of the same title). Al-Khwārizmī's *zīj* was based on a Sanskrit work presented to Caliph al-Manṣūr as a gift from a visiting Indian delegation in around 770, which was closely related to Brahmagupta's *Brahmasphuṭasiddhānta*. Al-Khwārizmī's original text remains lost, but a later revision in the tenth century by al-Majrītī and its translation into Latin by Adelard of Bath ensured that details of the work have survived.

The importance of this *zīj* is that it reveals that the sources of inspiration for early *zīj*es were largely Indian.[31] This dependence is confirmed by al-Bīrūnī, who comments that for his account of eclipses, al-Khwārizmī used Brahmagupta's *Khaṇḍakhādyaka* (see section 5.6.5) and Vaṭeśvara's *Karaṇasāra* (see section 5.6.8)[32] for determining a number of important parameters including the diameters of the luminaries.

> the methods of the computations of the diameters of sun and moon, as given by the Hindu canones, such as the *Khaṇḍakhādyaka* and the *Karaṇasāra*, are the same as are found in the canon of al-Khwārizmī. Also the computation of the diameter of the shadow in the *Khaṇḍakhādyaka* is similar to that one given by al-Khwārizmī whilst the *Karaṇasāra* has the following method ... (*Alberuni's India*, chapter LV, as translated by Sachau [1914] p. 79)

As well as giving the source for al-Khwārizmī's methods, al-Bīrūnī's quotation reveals that Islamic scholars inherited an intellectual legacy from India which contained different methods to compute the same phenomena. This must have pressed upon them the notion that some techniques were better than or preferable to others. The value of the trigonometric radius that al-Khwārizmī uses to construct his trigonometric tables is $R = 150$, which is the value used by Brahmagupta (see section 5.6.5).

As well as those highlighted by al-Bīrūnī, other Indian techniques can be clearly detected in al-Khwārizmī's exposition of eclipse theory. Chapter 34 introduces the concept of parallax and explains how tables are used to apply it, but no account of the theory behind the numbers is offered. Each of the components for parallax is given in its own table. For parallax in latitude, one is to determine the "latitude of the visible climate" [$z(V)$], which is the complement of the angle the ecliptic makes with the local horizon (fig. 6.1), or the shortest distance between the zenith and the ecliptic, namely, the zenith distance of the nonagesimal. This forms the argument for the entries of parallax in latitude.

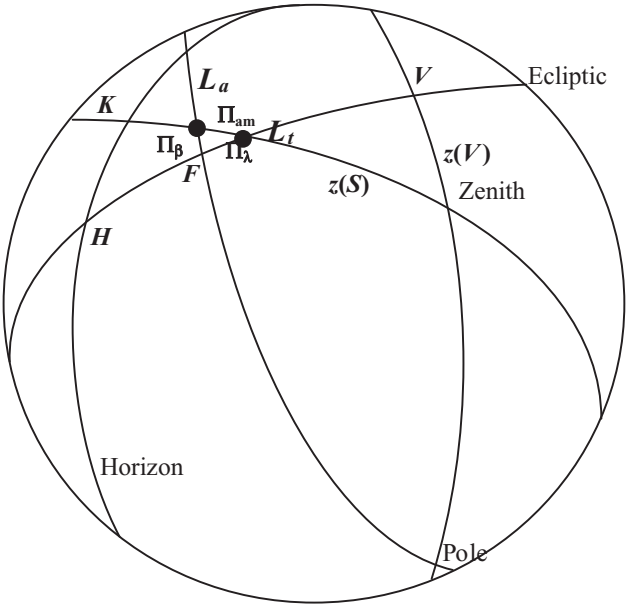


Figure 6.1 The determination of parallax in longitude.

As has been noted,[33] the Latin version of this text uses the term *medium caelum* ("midheaven") to designate one end of this measure, the arc between the zenith and the ecliptic, as measured along the prime meridian. Most frequently, the Latin term *medium caelum* refers to the point *M* (fig. 6.2), the meridian ecliptic point, which is the point of intersection of the prime meridian and the ecliptic. However, the shortest distance from the zenith to the ecliptic reaches the nonagesimal (*V*), the point 90° away from the ascendant (*H*). This detail was observed by later practitioners in this tradition, notably ibn al-Shāṭir.[34] But it may reflect a connection back to the Indian sources which often use one to approximate the other. The values that make up the parallax in latitude are directly computed from the relation

$$\pi_\beta = 0;48,45^\circ \sin[z(V)].$$

This expression is identical in form to most Indian approaches. It was standard Indian practice to use the zenith distance of the nonagesimal to calculate the latitudinal component of parallax, and also common to approximate the nonagesimal by the meridian ecliptic point. Furthermore, Brahmagupta's value of maximum parallax is 0;48,44, which is very close to the above figure of 0;48,45. Notably, in the *Sūryasiddhānta* (see section 5.6.2) the point of the ecliptic that crosses the meridian is explicitly mentioned in the Sanskrit, where

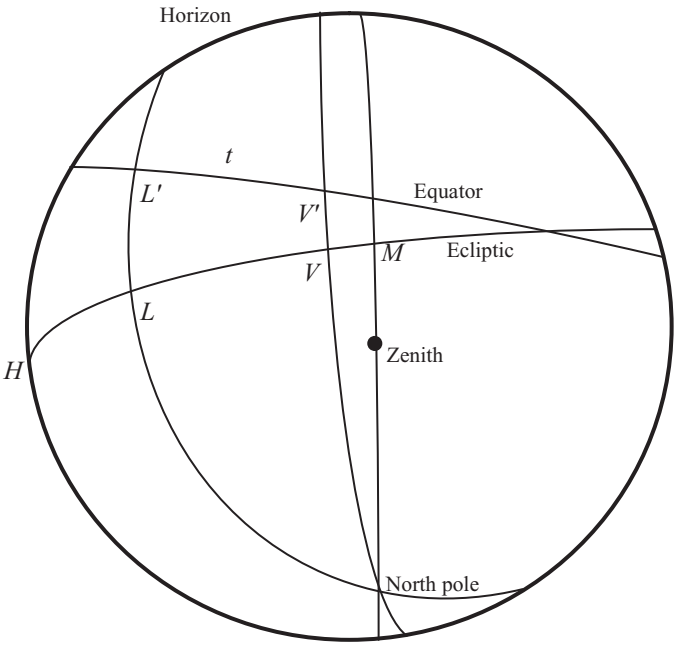


Figure 6.2 Parallax and the longitudinal difference between the conjunction and the nonagesimal.

it is (incorrectly) referenced as the direction of the vector of latitudinal parallax. Perhaps this was the source of the original confusion in the Arabic and later Latin sources.

The technique for longitudinal parallax also seems derived from Indian sources, possibly the *Sūryasiddhānta* as well. The method is based on the following procedure: take L to be the point of conjunction, and V the nonagesimal; then the projection of LV on the equator, that is, t, is the argument for the values of parallax in longitude (see fig. 6.2). Al-Khwārizmī gives his values of longitudinal parallax for terrestrial latitude, $\phi = 30°$. The computation of the various values of parallax in longitude seem to follow the relation[35]

$$\pi_\lambda = 1;36 \sin\theta(t),$$

where 1;36 may be 4×24 for values of θ between 0 and 150°.

This may be related to the iterative procedure described by Habaṣḥ (see section 6.3.3) and the Indian iterative procedures to determine conjunction, seen in the *Sūryasiddhānta* and more explicitly described in Vaṭeśvara's *Vaṭeśvarasiddhānta*. "4" represented the approximation by Indian astronomers of maximum parallax as 1/15 of the motion of the planet in a day or the motion

in four *nāḍī*s (see section 5.2.1; a *nāḍī* was a measure of time in the Indian tradition such that there were 60 in a 24-hour period) and the "24" converts from *nāḍī*s to minutes (1 *nāḍī* = 24 minutes).

Other important values can be inferred from the tables.[36] There exist two tables of lunar eclipses—the first with respect to the greatest lunar distance and the second with respect to the least lunar distance. The first is tabulated by half degrees through the range

$$180^\circ - 10;50^\circ \leq |\lambda_m - \lambda_n| \leq 180^\circ + 10;50^\circ$$

and the second

$$180^\circ - 13;17^\circ \leq |\lambda_m - \lambda_n| \leq 180^\circ + 13;17^\circ.$$

For these domains, the following phenomena are tabulated: eclipse magnitude, immersion, and duration of totality. In the same manner, there are two tables for solar eclipses, with limits

$$|\lambda_s - \lambda_n| \leq 6;37^\circ \quad \text{and} \quad |\lambda_s - \lambda_n| \leq 7;11^\circ$$

but there is no column for the duration of totality. The table from the *Almagest* that converts eclipse magnitude as a function of the diameter of the eclipsed body to the equivalent area obscured is included.

6.3.2 *Yaḥyā ibn abī Manṣūr*

Yaḥyā (d. 832) was a member of a Persian family of notable scientists. He rose to prominence, becoming the director of a team of scholars who worked at observatories in Baghdad and Damascus under Caliph al-Ma'mūn, with the intention of carrying out observations to improve existing astronomical parameters. The details of these observations were compiled into a *zīj* commissioned by al-Ma'mūn. The resulting work was called *al-Zīj al-Ma'mūnī lil-Mumtaḥin*.[37]

The table related to parallax from this *zīj* appears twice in separate places (f. 10 and f. 70). Its title is somewhat puzzling: it is called (literally) "the latitude of the sun" (*'arḍ al-shams*)[38] and is tabulated according to an argument called "the established latitude of the locality" (*arḍ al-balad al-muḥkam*) which runs from 1°, 2°, 3°, to 90°. There is some difficulty interpreting this table.[39] However, this established latitude of the locality may be the same as al-Khwārizmī's "latitude of the visible climate," as the function has a maximum of 0;48,32 and loosely resembles a sine function. At least eight values in this table can be identified as "primary" function values—intermediary values appear to have been generated by means of linear interpolation.[40] As with al-Khwārizmī, there

appears to be an influence from the *Sūryasiddhānta*'s treatment of parallax in latitude.

Further values of note can be extracted from other relevant tables. In particular, Yaḥyā's table of solar eclipses has two arguments. The first is the eclipse magnitude and runs 0;15, 0;30, 0;45, 1;20, 1;40, . . . , 12;20 digits as the maximum and the corresponding latitude is given in seconds. The second argument runs 0;27,30, 0;28,30, 0;29,30, . . . 0;33,30° per hour. These entries give the time of immersion in minutes. Yaḥyā's work, like that of his predecessor Al-Khwārizmī, shows in this early period the attempts of scholars to adapt Indian material to their own needs and circumstances and the various confusions that could result from this process.

6.3.3 Ḥabash al-Ḥāsib

Ḥabash al-Ḥāsib (fl. 835, d. 864 or 874) was a prolific astronomer and mathematician, working under the caliphs al-Ma'mūn and al-Mu'taṣim in Baghdad. Among the many works ascribed to Ḥabash, there are a reworking of the *Sindhind*, the *Mumtaḥan Zīj* (called *al-Qānūn* [or "Tables"] by Ibn Yūnus), the *Shāh Zīj*, the *Damascus Zīj*, and the *Ma'mūnī Zīj*. Two manuscripts of these tables of Ḥabash remain, one located in Istanbul and the other in Berlin, which have significant differences.[41] In addition to his astronomical activity, Ḥabash's contributions to mathematics were of fundamental importance, particularly in the field of trigonometry, the foundation for which he inherited from Indian achievements.

With respect to eclipse reckoning, it is valuable to consider the astronomical works of Ḥabash, as they contain contrasting methods for the determination of particular astronomical features and convey something of an astronomical tradition very much in transition. Especially noteworthy is Ḥabash's treatment of the components of latitudinal and longitudinal parallax. The contrasting techniques he presents in his works reveal the variety of sources the Islamic scholars were deriving inspiration from, as they attempted in these early stages both to combine and to discriminate between different approaches to the same problem.

Istanbul Manuscript

The first method presented in the Istanbul manuscript seems to represent a developing tradition poised between the mathematical techniques available to Ptolemy and the full-fledged spherical trigonometry used by later Islamic scholars. Ḥabash describes a rule that produces, according to him, the "first sine" (*al-jayb al-awwal*) (see fig. 6.1)[42] which he needs for computing the components of parallax. L_t is the true position of conjunction, V the nonagesimal,

K where the zenith distance circle through the point of conjunction intersects the horizon, and H the ascendant:

$$\text{“First sine”} = \frac{\sin L_t V \sin KL_t}{\sin HL_t \cos KL_t}.$$

Then one is instructed to compute the cosine of the arc of the first sine. The result is to be called the “second sine” (this is $\sin L_t$). Identification of the different sine components may be related to the Indian traditional five sines, which was first set out by Āryabhaṭa (see sections 5.2.1 and 5.6.3). Then, without further explanation, Ḥabash gives the rules of latitudinal and longitudinal parallax:

$$\sin \pi_\beta = (\text{first sine})(\sin \pi_{\text{adj}}),$$

$$\sin \pi_\lambda = \frac{(\sin \pi_{\text{adj}})(\text{first sine})}{\cos \pi_\beta},$$

where π_{adj} is the adjusted lunar parallax. Significantly, the triangles in question have been treated not as plane triangles but rather with spherical trigonometry techniques derived from Greek geometry.

This very same manuscript outlines a contrasting method for the determination of the components of parallax, which reveals that Ḥabash may have been influenced by Indian as well as Greek solutions to the same problem. As seen in the previous chapter, Indian practitioners very frequently employed iterative algorithms in mathematical astronomy, not only when there was no equivalent analytical solution available to them, but also even when one did exist.

Ḥabash commences his second approach to parallax by the presentation of a single table, titled simply “Parallax,” which tabulates the function $24 \sin \theta$. The accompanying text refers to it as the “Table of Lunar Parallax.”[43] The purpose of this table is made clear by Ḥabash’s following account of the computation of the longitudinal component of parallax. This procedure appears to be based on an iterative algorithm. One is first instructed to determine LV, the longitudinal difference between the conjunction and the nonagesimal (see fig. 6.2), and then enter Ḥabash’s parallax table. The resulting amount is the “first parallax.” Next, one is to add this to t and reenter the table with this sum. This is called the “second parallax,” and the procedure is repeated until one reaches the “fifth parallax.” This amount turns out to be a quarter of the parallax in longitude given in hours. The method also returns values close to those given by al-Khwārizmī.[44] This iterative technique bears similarities to that described in the *Sūryasiddhānta* (IX, 23) in which the apparent conjunction time is computed after repeated applications of parallax to the positions of the two luminaries, and more explicitly in Vaṭeśvara’s *Vaṭeśvarasiddhānta*

(beginning of the tenth century), who bases his method on techniques already familiar to Indian mathematicians for several centuries (see section 5.6.8).

The underlying approach of this method as it is described by Ḥabash[45] seems to invoke a fixed-point iterative algorithm for the determination of the longitudinal parallax in terms of time units measured in minutes. Total parallax is a function of the moon's zenith distance [$z(N)$] (where N_t and N_a are the true and apparent positions of the moon, respectively) and its component in longitude measured along the ecliptic is the arc VN_t, or $\lambda_{z(N)}$ (fig. 6.3). The time interval corresponding to this is given by the relation

$$t = \lambda_{z(N)} - 24 \sin(\lambda_{z(N)}).$$

The equation cannot be solved for $\lambda_{z(N)}$ directly. Thus, one takes an initial approximate $t = \lambda^0_{z(N)}$ and iterates the following:

$$\lambda^{(k+1)}_{z(N)} = t + 24 \sin(\lambda^{(k)}_{z(N)}).$$

This may confirm the necessity for the previous table of values of $24 \sin\theta$. Ḥabash, following a practice common among Arabic scholars, declares that this process is to be repeated five times.[46] The longitudinal component of parallax is then to be multiplied by four, i.e.,

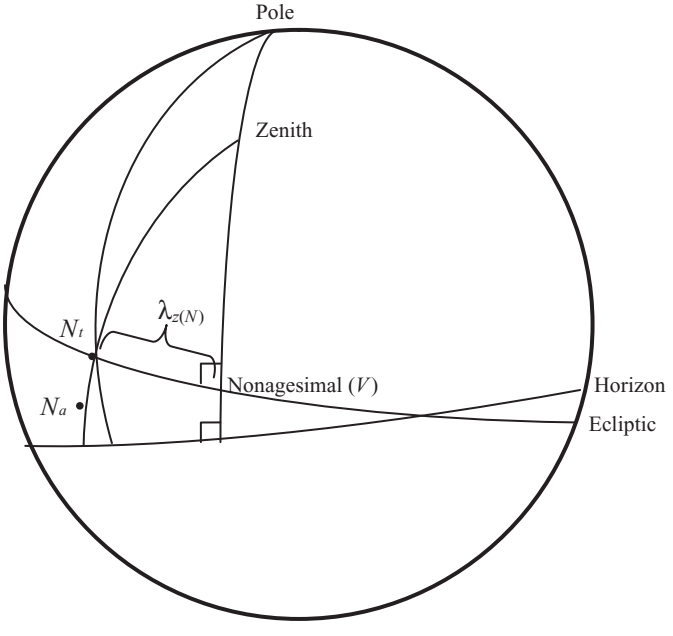


Figure 6.3 Lunar parallax.

$$\pi_\lambda = 24 \times 4 \sin(\lambda_{z(N)}).$$

This is identical to the iterative procedure given in many Indian texts, whose methods are based on techniques in which longitudinal parallax is given in *ghaṭikā*s (1 *ghaṭikā* = 24 minutes).[47]

Berlin Manuscript

The techniques related to eclipse computations in this work seem closer to Ptolemaic methods, but traces of Indian influence are still present. The parallax tables are those used in the *Almagest* for the parallax on the altitude circle and Ḥabash declares that he is using the methods of Ptolemy. There is another table (f. 153) entitled "Lunar Parallax for Visibility of the Crescent" (but this may more properly belong in another section).[48] The eclipse tables in this work are identical to those presented in the Istanbul manuscript.

The eclipse tables are tabulated for mean longitudes and mean motions, with the time intervals for mean syzygies reckoned in *Hijra* years. The functions tabulated are: longitude of mean conjunction or opposition, time with respect to days, the anomaly precise to minutes of arc, and the longitude of the node precise to minutes of arc. Eclipse magnitudes run from 0 to 21 digits, and for the moon at greatest and least distance Ḥabash gives the latitude of the moon, the times of immersion, and, where relevant, totality, and various appropriate interpolation schemes.

His table for solar eclipses runs to 12 digits. He includes tables of inclination, of immersion, and of totality. Importantly, it has been noted that next to these tables there is a long explanation of alternative methods to compute eclipses. Here, Ḥabash apparently describes techniques that use numerical and graphical methods as an alternative to relying on tables.[49]

6.3.4 *Al-Battānī*

By the tenth century, efforts in astronomical theory turned away from embracing many contrasting approaches to compute phenomena and toward resolving them into a single astronomical system. Increasingly, the focus shifted from the algorithms of the Indian procedures in favor of the more rigorous analytical solutions inspired by Ptolemy. Al-Battānī (b. 858) is representative of this new orientation, and epitomizes the most common approach to parallax in later authors. His influential *zīj*, the *Zīj al-Ṣabi'*, was composed in about 900 CE. He includes parallax tables for each of the seven climes which are reproduced directly from Theon. This is confirmed by ibn al-Shāṭir, who connects their origin back to what he calls the "*zīj*" of Theon.[50]

Even after was a single system was settled on, there were still challenges to be overcome by those that had settled on Ptolemaic techniques. Frequently, al-Battānī reproduces tables that are valid for one particular locality, but not for his own local circumstances. Despite the introduction of different mathematical procedures and different parameters (such as terrestrial latitudes and obliquity of the ecliptic) in the accompanying text, the overall resulting values in the tables remained unchanged, and have been simply copied wholesale and not recomputed for these slightly differing circumstances. This practice, where the explanatory text did not match the content of the tables, was frequent.

However, al-Battānī was devoted to observational astronomy, engaging for over forty years, beginning in 887 CE, in observations at his self-funded observatory in al-Raqqah. There are references to at least four eclipse observations included in the *Zīj al-Ṣabi'*, two of which were used to examine the accuracy of Ptolemy's tables, the other two to determine the difference in longitude between two cities.[51] Not only did the inherent inaccuracies and cumulative errors in Ptolemy's tables make it difficult for al-Battānī[52] to register meaningful discrepancies, but the low mean accuracy of al-Battānī's own observed times, about 0.31 hours, also prevented significant improvements. There are other circumstances in which inaccuracies in theoretical prediction and inaccuracies in observational practice coincidentally canceled each other out, thus leading scholars to *overestimate* the accuracy of their tables in predicting eclipses and become overconfident in their predictive abilities.[53]

6.3.5 al-Khāzinī

From humble origins, al-Khāzinī (fl. ca. 1115–ca. 1130) rose to intellectual accomplishments and practiced astronomy and mathematics under the patronage of the Seljuk court, located in Merv, an Iranian city which was at that time the capital of Khurāsān.[54] His work on astronomy is influenced by his predecessors, Thābit ibn Qurra and al-Battānī, and despite significant interest in Indian procedures and concepts and references to terms and techniques in his writing, his work stays firmly within the domain of the Islamic Ptolemaic tradition.

The *Wajīz* of al-Khāzinī

The *Wajīz* is an abbreviated version of al-Khāzinī's larger work, the *Zīj al-Sanjarī*[55] composed in 1130/1131. It represents the culmination of theoretical eclipse techniques in the Islamic scientific scene before the reforms of the thirteenth century; by the time of its completion, Ptolemaic methods and parameters were well understood and had been thoroughly assimilated into the Arabic astronomical tradition.

In the first part (*qism*) of his exposition on eclipses, al-Khāzinī sets out the means to determine the syzygies. He first instructs his reader to find the positions of the sun and the moon at noon on the day nearest the supposed day of the syzygy.[56]

Al-Khāzinī's intention is to find the exact point of syzygy when the two longitudes are identical. He does not discriminate between occasions when the difference between those two longitudes is 0° or 180°. He further specifies that, if the conjunction is in question, one can use the two values themselves to compare, but if an opposition is sought, one must first add six signs to the position of the moon and then compare. If they are the same, this is the point of syzygy, but, more often than not, if they are different, the elongation between the two must be sought. He then instructs the reader to find the two mean daily velocities (*al-buht*) of the sun and the moon. This term is a direct transliteration from the Sanskrit word *bhukti*, which, in an astronomical context, means daily velocity. He then defines a value that he calls *al-subq* (literally: "the precedence" or "priority"), which is the difference between the two velocities:

$$al{-}subq = v_m - v_s.$$

This expression is used in the next section, which gives the formula used to determine the time from or until the syzygy:

$$t = \frac{(\lambda_m - \lambda_s) \times 24}{(v_m - v_s)}.$$

In the following section, he converts this time interval found to the instant it occurs on the day in question. Al-Khāzinī uses the following phrases to indicate the respective positions of the sun and the moon: *kān al-bu'd li-šams* (literally "there is an elongation to the sun"), indicates that the sun is *ahead* of the moon (i.e., in the context of a lunar eclipse, that the opposition is yet to occur), and *kān al-bu'd al-qamar* (literally "there is an elongation of the moon") which indicates that the moon is *ahead* of the sun (i.e., that opposition has passed). In order to set up the time at which the syzygy will or did occur, he describes the following two scenarios.

In the first case, when there is "an elongation of the sun" at noon, the sun is ahead of the moon, so that the syzygy is yet to come. One adds the hours of half of the day, that is, the hours from sunrise until noon, to the hours of the elongation found in the previous section. If this sum is smaller than the hours of half of the day, then this sum is the number of hours *after* sunrise at which the syzygy will occur. If the sum is greater than the hours of the whole day (i.e., from sunrise to sunset), then one subtracts the hours of the day from

the hours of the elongation. The result is the number of hours *after* sunset at which the syzygy is going to occur.

In the second case, the moon is ahead of the sun at noon, so that syzygy has passed, and one needs to effectively calculate backward. Thus, rather than adding the values together, one subtracts them. Again, two values of the measure of half of the day and the hours of the elongation are compared; if the latter is smaller than the former, then one simply subtracts the latter from the former, which gives the number of hours *after* sunrise at which the syzygy occurred.

If the latter is greater than the former (which means that syzygy was further back in the past than sunrise), one subtracts the hours of half of the day from 24 and then subtracts the hours of the elongation from the result, and the amount obtained is the number of hours *after* sunset on the previous day at which the syzygy occurred.

Al-Khāzinī states in his section on how to obtain the "degree of the syzygy" the following two sets of two equations involving the solar and lunar longitudes λ_s and λ_m:
If $\lambda_s > \lambda_m$ then

$$\text{Solar longitude at syzygy} = \lambda_s + \tfrac{1}{12}(\lambda_m - \lambda_s),$$

$$\text{Lunar longitude at syzygy} = \lambda_m + \tfrac{13}{12}(\lambda_m - \lambda_s).$$

If $\lambda_m > \lambda_s$ then

$$\text{Solar longitude at syzygy} = \lambda_s - \tfrac{1}{12}(\lambda_m - \lambda_s),$$

$$\text{Lunar longitude at syzygy} = \lambda_m + \tfrac{13}{12}(\lambda_m - \lambda_s).$$

These equations are based on the very convenient approximations to the mean daily velocities of the sun and the moon as 1° and 12°, respectively. These are identical to the expressions for the longitudes at syzygy given by Ptolemy (*Almagest*, VI, 4, second half).

Next al-Khāzinī sets out his parameters for the possibility of the occurrence of a lunar eclipse. He states that one needs an opposition that occurs at night, or within two hours of the night. The latter condition is a rough approximation to include eclipses that are seen as the moon is rising or setting.

He gives eclipse limits in the form of maximum values for the moon-node elongation ω and the lunar latitude β:

$$\omega = 12;15^\circ,$$

$$\beta = 63 \text{ minutes}.$$

The value for latitude is not derived from Ptolemy's *Handy Tables* ($\beta = 64$ minutes), but rather could be a rounding down to the nearest minute of the value 1;3,36° given in the *Almagest*. The value for nodal elongation (ω) could be derived from an alternative value of the sine of the inclination of the lunar orbit, namely 11;40 instead of 11;30 (see section 4.5.1):

$$\begin{aligned}\omega &= 11;40 \times \beta \\ &= 11;40 \times 1;03 \\ &= 12;15.\end{aligned}$$

Al-Khāzinī does not give the diameters of the sun, the moon, and the shadow in this chapter but simply the standard formula, which he calls "the half of the two [lunar and solar] diameters," d_m and d_s. He uses this amount as a limit to compare against the latitude of the moon to be able to determine whether or not the luminaries are going to be in contact, as follows. If

$$\beta \geq \frac{1}{2}(d_m + d_s),$$

then there is no eclipse; otherwise, a lunar eclipse can occur. He defines the magnitude of the eclipse to be

$$\frac{1}{2}(d_m + d_s) - \beta.$$

Al-Khāzinī states the following three conditions for, respectively, an eclipse that is total at one instant, a total eclipse with a longer duration of totality, and a partial eclipse:

$$\begin{aligned}\tfrac{1}{2}(d_m + d_s) - \beta &= d_m, \\ \tfrac{1}{2}(d_m + d_s) - \beta &> d_m, \\ \tfrac{1}{2}(d_m + d_s) - \beta &< d_m.\end{aligned}$$

Following the Ptolemaic tradition, al-Khāzinī divides the diameter of the moon into twelve digits and uses these as a measure for recording the magnitude of the eclipse. He carefully states his level of precision as degrees and minutes, explaining the process by which the remainder of degrees may be converted into minutes.

In the next section, al-Khāzinī sets out how to determine the phases of the eclipse, using a technique that is clearly Indian in origin. He describes the following expression for the half-duration (Δt) given in hours:

$$\Delta t = \frac{\sqrt{\frac{1}{2}(d_m + d_s) - \beta} \times 24}{v_m}.$$

Using a procedure of computation employed frequently among the Indian commentators, al-Khāzinī then instructs one to put the "hours of opposition" (*sā'āt al-istiqbāl*)—that is, the instant of opposition—in three places. From the first one subtracts the above value, to the second one does nothing, and to the third one adds the above value. The three results represent the hours of the beginning, middle, and end of the eclipse, respectively. A similar process is used for the case when there is a total eclipse with a period of totality. In this situation, however, one is instructed to put the numbers down in five places, to account for the five phases of first contact, beginning of totality, mid-eclipse, end of totality, and release.

Al-Khāzinī then turns to the computation of solar eclipses. At the outset of this subject, he announces his intention to construct a "simple table" in order to facilitate the derivation of the values one will need in the computation of solar eclipses. He begins with a discussion about parallax, making reference to the tables of "Tāūn" a proper name transliterated from the Greek Θέων, obviously referring to Theon and the *Handy Tables* .

Parallax values for the desired terrestrial latitude are found by interpolation between values in the clime-based tables of parallax set out in the *Handy Tables*. Al-Khāzinī carefully sets out the simple linear interpolation procedure as follows: find the latitudinal and longitudinal parallax components for the nearest clime south of one's desired latitude ($\pi_{\beta c1}$ and $\pi_{\lambda c1}$) and for the nearest clime north of it ($\pi_{\beta c2}$ and $\pi_{\lambda c2}$). Then the difference $\phi - \phi_{c1}$ between the desired latitude and the latitude of the first clime, which he designates by the technical term *al-faḍala*,[57] is divided by the difference between the two climes $\phi_{c2} - \phi_{c1}$ (*al-tafḍal*). The parallax components for the desired latitude ϕ are found using this quotient as a scaling factor:

$$\text{Increment of } \pi_\lambda = (\pi_{\lambda c2} - \pi_{\lambda c1}) \times \frac{(\phi - \phi_{c1})}{(\phi_{c2} - \phi_{c1})},$$

$$\text{Increment of } \pi_\beta = (\pi_{\beta c2} - \pi_{\beta c1}) \times \frac{(\phi - \phi_{c1})}{(\phi_{c2} - \phi_{c1})}.$$

These values are then to be added to the parallax components for the first clime. The above computation is performed first using the parallax values for the beginning of the zodiacal sign in which the conjunction is taking place, and again using the values for the beginning of the next sign. A linear interpolation between the two results, with a scaling factor equal to the degrees in the sign where the conjunction occurs divided by the total length of the sign or 30°, produces the required values of parallax components. The final modification to them prescribed by al-Khāzinī is the correction due to the moon's position on its epicycle (see section 6.3.9).

To fully illustrate the procedure of computing parallax, al-Khāzinī gives a worked example. He does this by constructing his own "simple table" illustrating the results of the computations, given his parameters, for a specified example. The example he uses is a solar eclipse in Leo 25° observed at the latitude $\phi = 38°$. He interpolates as described above between parallax values calculated for Leo 0° and Virgo 0°, using the two latitudes $\phi_{c1} = 36°$ and $\phi_{c2} = 41°$, which are approximations to Ptolemy's clime IV (36°) and clime V (40;56°). Then, lastly, he corrects for the proper motion of the moon (*khiṣṣa*).

The Simple Table

The example of a solar eclipse at Leo 25° with $\phi = 38°$

				Beginning of the two zodiacal signs Leo and Virgo												
				For the beginning of Virgo						For the beginning of Leo						
khiṣṣa		Leo 25°		$\phi = 38°$		$\phi = 41°$		$\phi = 36°$		$\phi = 38°$		$\phi = 41°$		$\phi = 36°$		
β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	β	λ	Hours
																7
																6
																5
																4
																3
																2
																1
23	8	20	7	21	8	23	9	20	8	15	3	18	4	14	3	noon
28	4	24	3	25	2	27	0	24	2	17	6	20	6	16	8	1
33	14	28	12	30	11	31	9	29	12	21	17	24	15	20	17	2
36	23	32	19	34	18	36	16	13	19	25	24	28	22	24	24	3
42	22	36	24	38	23	39	21	37	24	29	30	32	27	28	31	4
47	31	40	27	41	26	42	24	40	27	34	33	35	30	33	35	5
49	33	42	28	43	27	44	27	42	28	38	32	38	31	37	34	6
50	34	43	28	44	28	45	28	43	28	40	31	42	28	40	33	7

Next al-Khāzinī gives the extreme latitudinal components at which parallax can still cause apparent contact of the moon and the sun for "our city" (presumably $\phi = 38°$ for Merv [Marw], latitude 37;36). These are

Maximum southward $\pi_\beta = 0;35°$,

Maximum northward $\pi_\beta =$ "bit more than 90."[58]

These figures are close to the Ptolemaic[59] figures.

Al-Khāzinī next lists the quantities needed to calculate the various parameters for a solar eclipse. These are the time of the conjunction, the degree of

the conjunction, the computed position of the node, the diameters of the sun and the moon and their respective motions in an hour, and the length in hours of half of the day. He then sets out further corrections to the time of the true conjunction. The rest, the calculation of duration, eclipse limits in terms of diameters, and so on, are all standard and are the same as those set out in the lunar eclipse section with the appropriate substitutions.

In addition to the text, al-Khāzinī presents several tables for the reckoning of eclipses for particular circumstances (see Appendix C). These include the velocities of the two luminaries and their diameters, a table for computing the circumstances of lunar eclipses, digit-to-area of the disk conversion for obscuration, the angle of inclination of the lunar orbit, parallax of the two luminaries, parallax for terrestrial latitude $\phi = 38^\circ$, parallax of the moon in longitude and latitude, parallax of the moon "according to Theon," and a table for computing the circumstances of solar eclipses. Analysis of these tables reveals a close dependence on Ptolemy with minor differences due to the use of different underlying parameters.

6.3.6 *Naṣīr al-Dīn al-Ṭūsī*

Al-Ṭūsī (b. 1201, d. 1274) is renowned as one of the most prominent figures in Islamic intellectual history. Well schooled in religion, logic, philosophy, mathematics, and astronomy, he made significant advances in all of the fields he applied himself to. His work in astronomy included the *Zīj-i Īlkhānī* in 1271, a recension of the *Almagest*. Even more importantly, al-Ṭūsī represents a new phase in astronomy. Having assimilated the models of Ptolemy, scholars became ever bolder about exploring its deficiencies. In particular, al-Ṭūsī authored a critique of Ptolemaic astronomy in a work called the *Tadhkira fī 'ilm al-hay'a* (literally "Memoir on Astronomy"). Although this work has been deemed the most thorough account of the inadequacies of the Ptolemaic model in medieval times,[60] it is a substantial theoretical treatise in its own right. Its description of eclipse reckoning is largely based on Ptolemaic methods, without extensive changes or criticisms, and it contains a thorough treatment of parallax.

In the *Tadhkira*, parallax is treated in chapter 12.[61] al-Ṭūsī gives a descriptive account, identifying the effects and the causes of parallax, illustrating his explanation with a diagram, and noting additional features such as the absence of parallax for a body at the zenith, and the redundancy of applying this effect to celestial objects very distant from the earth. He introduces the two components of parallax and the various celestial circumstances that cause these components to be maximized or minimized. He notes that one can determine the distance of the moon from the earth by means of the moon's parallax. Lastly, he reckons

the solar parallax to be small, not exceeding 3 minutes. Practically speaking, he comments, this is imperceptible.

Solar and lunar eclipses are discussed in the following chapter along with the variation of the moon's illumination. Al-Ṭūsī begins his discussion with an elaboration of the circumstances required for a lunar eclipse to occur, with an accompanying diagram, showing the alignment of bodies, the shadow cone, and the relevant orbits. He cites the eclipse limits to be 12° of elongation of the moon from the node. He also notes the ratio of the shadow-circle's diameter to that of the lunar disk to be 2⅗, which is the figure given in the *Almagest*. Al-Ṭūsī also explains the practice of dividing the diameters of the luminaries into twelve to measure magnitude, and then closes his discussion on lunar eclipses with the intervals between successive eclipse possibilities and the factors that contribute to determining them. Solar eclipses are also illustrated by means of a diagram. Al-Ṭūsī gives his values for the solar and lunar diameters at their two extremes:

$$d_s \text{ from 31 to 24 minutes,}$$

$$d_m \text{ from 29 to 36 minutes.}$$

The immediate effect of these values is that they allow for the lunar diameter to be smaller than the sun's during conjunction, a situation that is ruled out by Ptolemy's values. This consequence is not overlooked by al-Ṭūsī, and he relates the possibility of annular eclipses (*ḥalqat al-nūr* [literally "ring of light"].[62] This modification is possibly the result of observations affecting parameter values, as there are a number of observations reporting annular eclipses that postdate Ptolemy. Al-Ṭūsī gives solar eclipse limits, noting their sensitivity to changes in terrestrial latitude. He gives the limits for the fourth clime as follows:

> In the fourth clime, a solar eclipse is possible at a maximum distance of 18° after the node of the head or before the node of the tail, or at a maximum distance of 7° before the node of the head or after the node of a tail. (*Tadhkira*, as translated by Ragep [1993] p. 238)

These values appear rounded and may be traceable back to the procedures of Ptolemy. The practice of giving the solar eclipse limits with respect to the climes follows Ptolemy, who gives them for the boundaries of the seven climes: for Meroe (latitude = 16;27°) and Borysthenes (latitude = 48;32°) as 12;41° for the northern maximum and 8;22° for the southern. Al-Ṭūsī's eclipse limit values may possibly be reconstructed as follows.[63] The fourth clime is delimited by latitudes 33 5/8° and 39 9/10°. For the first case, when the moon is "after the node of the head or before the node of the tail," that is, north of the ecliptic, the maximum eclipse limit can be computed from assuming the following

extremes:[64] the latitude is 39 9/10°, the moon is at epicyclic perigee, and the sun is at perigee, conjunction occurs at Aries 0°, six hours before noon. Using the relevant tables from Ptolemy's *Almagest* (II, 13 and V, 18), the latitudinal and longitudinal components of parallax are 0;54° and 0;28°, respectively. Then the maximum distance of the moon from the node is

$$\begin{aligned}\text{Solar eclipse limit} &= (\pi_\beta + 0;35) \cdot 11;30 + \pi_\lambda \\ &= 17;32^\circ \\ &\approx 18^\circ,\end{aligned}$$

which fits nicely, given the general level of precision offered by al-Ṭūsī in this context. However, application of the same sorts of reasoning to the second case when the moon is "before the node of the head or after the node of the tail," that is, south of the ecliptic, results in an eclipse limit a little under 6°, which is notably low for al-Ṭūsī's 7°. This reconstruction is rather speculative, requiring multiple assumptions to obtain these figures.[65] The resulting discrepancy may bear witness to the modifications made by Islamic astronomers to refine Ptolemy's techniques and parameters as well as modern assumptions.

Al-Ṭūsī then comments on the intervals between successive solar eclipse possibilities, and wraps up his discussion on eclipses with a note about the parts of the luminaries that are eclipsed first (eastern part of moon in the case of a lunar eclipse, western part of the sun in the case of a solar eclipse), and those that reappear first. The inclusion of this rather trivial observation may link back to the importance of the direction of impact for astrological and divination purposes, included in many earlier accounts.

The *Tadhkira* was intended to give a theoretical overview of astronomical phenomena. More detailed parameters and tables are included in al-Ṭūsī's *Zīj-i Īlkhānī*, which was compiled after twelve years of observations and was designed as a working manual for those making astronomical predictions. The parallax tables included in this *zīj* are broken down into longitudinal and latitudinal components.[66] These are copied from Theon for the third, fourth, and fifth climes. An additional table for $\phi = 38^\circ$ (convenient for the location of al-Ṭūsī's observatory at Marāgha at latitude 37;20°) is included, which is identical to that given by al-Ṭūsī's contemporary, al-Khāzinī (see section 6.3.5), and was probably borrowed from his *zīj*.

6.3.7 Ibn al-Shāṭir

Ibn al-Shāṭir (b. ca. 1305, d. ca. 1375) has been considered to be the most distinguished astronomer in the Islamic Near East of the fourteenth century. Perhaps best renowned for his contribution to planetary theory, he also served as the

muwaqqit or astronomer in charge of timekeeping at the Umayyad Mosque in Damascus. His planetary theory reconfigured many aspects of Ptolemy's model to eliminate the inconsistencies for which it had been heavily criticized. He writes about the inspiration for his astronomical contributions:

> I came across the books of certain of my predecessors and found that most ... had adduced doubts concerning the well-known astronomy of the spheres according to Ptolemy. These doubts were indisputable and [concerned matters] incompatible with the geometrical and physical models that had been established [by Ptolemy]. These scholars took pains to make models that would adequately represent the longitudinal and latitudinal motions of the planets, and not introduce inconsistencies. They were not granted success however, and they admitted this in their writings.
> (*al-Zīj al-Jadīd*; as quoted by King [2008], vol. 12, p. 538)

Bearing in mind the fact that ibn al-Shāṭir's model was geocentric, his reconfigurations resulted in a model which was in some respects mathematically close to that of Copernicus.[67]

Among his researches in astronomy, he did important work in eclipse theory. His astronomical tables were all computed for his own particular geographical latitude, that of Damascus, which he derived in 1363 to be $\phi = 33;30$. Notably his work integrates material based on that of his predecessors with recomputations by him to reflect his own circumstances. In his *Zīj al-Jadīd*,[68] ibn al-Shāṭir includes a reproduction of Theon's tables for the longitudinal and latitudinal components of parallax. In addition, he includes a separate table for the latitude of Damascus of the adjusted lunar parallax, given to a precision of one sexagesimal place. He also gives a greater level of precision in his argument for this table—it is tabulated with respect to the initial point of each zodiacal sign, and then for every four minutes of daylight, calculated after sunrise.

He states five rules for the computation of the true position of conjunction L_t (see fig. 6.1):

$$\sin L_t = \frac{\sin z(L)}{\cos KL_t},$$

$$\cos L_t = \frac{\cos L_t H \cos z(L)}{\cos L_t K},$$

$$\cos L_t = \frac{\sin (L_t H - 90) \sin L_t K}{\cos (L_t H - 90) \cos L_t K},$$

$$\cos L_t = \cot L_t H \tan L_t K,$$

$$\cos L_t = \frac{\cos L_t H \sin L_t K}{\sin L_t H \cos L_t K}.$$

As can be quickly established by modern symbolic reasoning, these expressions are all equivalent, but of course in the original they were expressed rhetorically and the links could not be seen as clearly. For example, the direct equivalence of the third and fifth rules is established by the fact that an angle's sine equals the cosine of its complement.

Ibn al-Shāṭir then expresses the computation of the components of parallax as follows where π_{adj} is the adjusted lunar parallax:

$$\sin \pi_{\beta} = \sin \pi_{\text{adj}} \sin L_t,$$

$$\sin \pi_{\lambda} = \sin \pi_{\text{adj}} \cos L_t.$$

The latitudinal component is exact using the sine theorem,[69] however, the longitudinal component is based on a plane triangle, so that theoretically it is not as correct as in earlier methods, such as those set out by Ḥabash. Ibn al-Shāṭir also takes into the account the varying values of the lunar anomaly. This, he states (and confirmed by the original), is directly inspired by Theon.

Ibn al-Shāṭir also includes as part of his exposition on parallax a method to determine the parallax components for any locality based on the components for Damascus. For the component of parallax in latitude, his method is based on the assumption that latitudes increasingly south of Damascus will cause the syzygy to shift towards the zenith on the altitude circle. One is instructed to select the parallax in altitude for Damascus, take the difference in terrestrial latitudes between Damascus and the new locality, add this to the parallax in altitude, and then use this to find the parallax corresponding to the sum in the very same table. This procedure, it has been noted, may be related to techniques used by Yaḥyā ibn abī Manṣūr.[70]

For the parallax in longitude, ibn al-Shāṭir's conversion technique is a little more involved and is based on the following method. One is to assume that the sign of the conjunction is rising both for Damascus and for the new location, and then modify these figures for the actual position of the sign in which conjunction occurs (i.e., not on the horizon), by means of the ratios between them.[71] Supposing the longitudinal component of parallax for Damascus is $\pi_{\lambda\text{Damascus}}$ and that for the locality in question $\pi_{\lambda\text{local}}$. Then:

$$\pi_{\lambda\text{local}} = \pi_{\lambda\text{local(rising)}} \times \frac{\pi_{\lambda\text{Damascus}}}{\pi_{\lambda\text{Damascus(rising)}}}$$

Ibn al-Shāṭir comments:

> this method is handier than the tables of Theon, and easier, and this is a nice trick for ascertaining the moon's parallax in longitude and latitude at any

locality from a single table (of components), but God, be He exalted, knows better.
(*Zīj al-Jadīd* as translated by Kennedy [1956b] p. 46)

Indeed, the significance of this method lies in its deliberate roughness—ibn al-Shāṭir consciously reduces a nontrivial problem by means of crude simplifications, despite the fact that all the technical apparatus and geometrical insight needed are present to solve this problem directly. The adoption of a simple arithmetical method to model an involved problem harks back to techniques employed by earlier cultures, such as the ancient Near East, Greece, and India. The inclusion of this method as an alternative is tacit acknowledgment that more intricate models are, on the whole, too detailed and sensitive to be entirely useful, particularly when the level of accuracy they provide is not substantially greater.

The 80th chapter of ibn al-Shāṭir's *zīj* is called "On the knowledge of solar eclipses by another method." Here, ibn al-Shāṭir comments that the method in question was employed by many ancient scholars and that "there befell among them differences concerning it."[72] The tables and text which accompany this description are in fact identical to those included by al-Khwārizmī (see section 6.3.1). Ibn al-Shāṭir states, in particular, that with respect to the table for parallax in latitude one can use instead the zenith distance of the lunar orbit rather than that of the ecliptic as the argument. He prescribes subtracting the longitude of the nonagesimal from the ascending lunar node. With this difference one finds the corresponding lunar latitude and adds it to the zenith distance of the nonagesimal. With this modified amount, one can then enter the tables for parallax in latitude.[73] This is reminiscent of the technique set out by Indian scholars in the *Romakasiddhānta* (see section 5.5.2) which attempts to account for the fact that the moon is on the lunar orbit and not the ecliptic.

6.3.8 Ulugh Bēg

Ulugh Bēg (b. 1394, d. 1449), the "astronomer prince,"[74] was renowned for the fact that he pursued not warfare and conquest, but rather accomplishment in science, with an emphasis on astronomy. He established an institution of higher learning (*madrasa*) in Samarqand, and filled it with carefully selected experts. Soon after, he instigated the erection of an ambitious three-story observatory. Such industry resulted in the production of many astronomical works, including Ulugh Bēg's own *Zīj-i Gurgānī*, and the *Zīj-i Sulṭānī* produced by a group of practitioners from the Samarqand observatory. The *Zīj-i Sulṭānī* is noted to be mathematically rigorous and detailed in its computations.

The *Zīj-i Sulṭānī* contains parallax tables which are similar to those of the *Zīj al-Khāqānī* of al-Kāshī (see section 6.3.9), the differences being largely traceable to slightly different base parameters. The maximum adjusted lunar parallax, when the moon is at greatest distance, is 0;51,22° with equation 0;10,14. The maximum solar parallax (given to three sexagesimal places) computed for each degree of solar distance from apogee has a maximum of 0;2,29,44°.

The procedures for extracting the components of parallax in longitude and latitude are also similar to those described by al-Kāshī (see section 6.3.9)—they have been tabulated in separated tables for $\phi = 25,30,35,\ldots,50°$. In particular, the table for $\phi = 30°$ contains values that can be obtained by following al-Kāshī's method in his text. Curiously, however, the corresponding values in al-Kāshī's table are different from these!

6.3.9 *Jamshīd Ghīāth ul-Dīn al-Kāshī*

Al-Kāshī (d. 1429) worked in Samarquand with Ulugh Bēg, where he practised astronomy as well as overseeing the running and organization of the observatory there. His achievements and position commanded great respect among his peers and later generations deemed him "the second Ptolemy." Al-Kāshī is significant in many respects. But his work exemplifies a common failing among the writers of *zīj*es—that is, there is a disparity between explanations given in the texts and the actual values given in tables. Often, al-Kāshī's tables do not correspond to the values which can be reconstructed from his theoretical exposition. Frequently, the tables appear to have simply been copied without modification to allow for the different parameter values and circumstances al-Kāshī introduces in his accompanying text. Another important feature of his work is that he employs the geometric procedures of the Ptolemaic system but uses more recently developed and sophisticated mathematical methods to carry them out, such as spherical trigonometry of sines (retaining, however, the Ptolemaic value of $R = 1,0 = 60$ units).

Al-Kāshī's biggest work, the *Zīj al-Khāqānī*, is a revision of al-Ṭūsī's *Zīj-i Īlkhānī*. There are two extant copies of the *Zīj al-Khāqānī*: one from India and one from Istanbul. Both manuscripts include incomplete tables on parallax.[75]

The fundamental parameters that underlie al-Kāshī's computations for the lunar parallax π_m are the earth-moon distance (k_m) and earth-sun distance (k_s). It has been observed that these parameters are extracted from a separate work—one dealing with planetary distances—the *sillum as-samā'* (literally "the stairway of heaven").[76] The rule for the total parallax can be reconstructed as follows:

$$\pi_m = \sin^{-1}\left(\frac{\cos h_t}{\sqrt{d_m - \sin h_t + \cos^2 h_t}}\right),$$

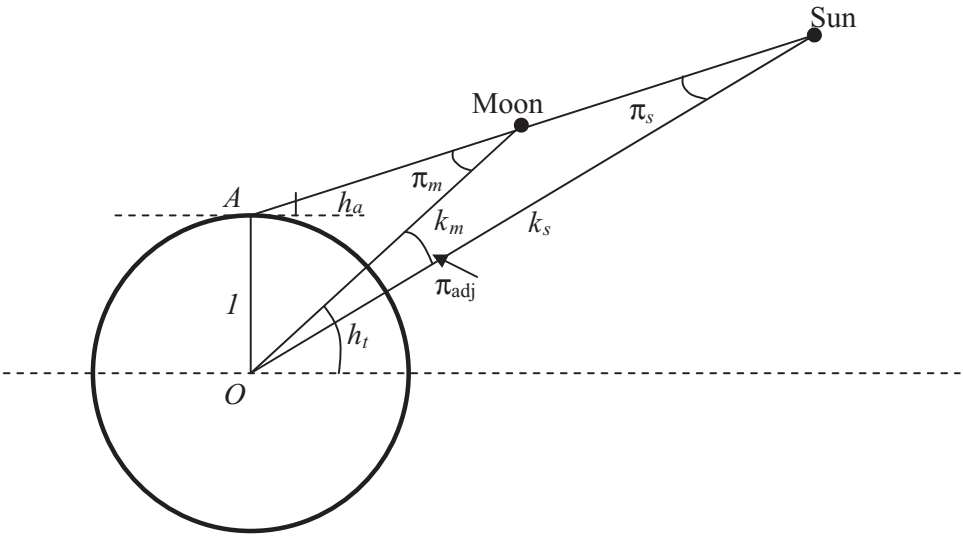


Figure 6.4 Adjusted lunar parallax.

where h_t is the true altitude of the moon (i.e., measured from the center of the earth) and k_m is the distance between the (center of) the earth and the moon (fig. 6.4). Al-Kāshī also gives the following formula for this same amount as a function of apparent altitude h_a:

$$\pi_m = \sin^{-1}\left(\frac{\cos h_a}{k_m}\right).$$

Al-Kāshī proposes further adjustments. Notably, lunar parallax has been computed for a zero elongation, confirming that this amount is for the purpose of computing eclipses. A zero elongation entails that the center of the moon's epicycle will be at its maximum distance from the center of the earth, but the actual position of the moon on that epicycle (its anomaly) is independent of elongation. This will have some effect on d_m, the earth-moon distance. Thus, the adjusted lunar parallax given as a function of zenith distance is presented with respect to a zero anomaly (i.e., on the epicyclic apogee). To find the adjusted lunar parallax for all intermediary positions for the moon as it moves from epicyclic apogee to epicyclic perigee, al-Kāshī proposes an interpolation scheme.[77]

Next, the components of latitude and longitude must be derived from the parallax in altitude so they can be applied directly to celestial coordinates. To introduce this procedure, Al-Kāshī poses and solves the following problem: given the celestial latitude (β) and longitude (λ) of an object, and the celestial longitude of the ascendant (H), compute the true altitude of that object (h_t)

(see fig. 6.1). He uses the "latitude of the visible climate," $z(V)$, which is the complement of the angle between the ecliptic and the horizon, or the shortest distance from the zenith to the ecliptic. This is found using the nonagesimal, a point 90° away from the ascendent on the horizon, therefore the highest point on the ecliptic. Al-Kāshī calls this point "the middle of the visible sky" (*wasaṭ-i samā'i rū'ya)*. The use of the nonagesimal in parallax computations is a clear link to the persistence of techniques developed in India. Al-Kāshī assumes that lunar latitude is zero during a solar eclipse, as the moon will be very close to the ecliptic. Another assumption, which is less negligible, is that the adjusted lunar parallax is assumed to be measured along the ecliptic. This is not quite true as the effect of solar parallax is to push the sun a little below the ecliptic.

Where Ptolemy used plane triangle approximations, al-Kāshī used spherical methods. However, as has been observed,[78] Ptolemy avoids the computation of lunar parallax except at syzygies, perhaps to obscure the fact that the moon comes much too close to the earth at quadrature. Al-Kāshī appears to be unconcerned by this, and computes parallax for any time. The result is a maximum lunar parallax of $1;43,33°$ when the moon is closest to the earth, which is significantly too large.

The discrepancies between al-Kāshī's tabulated values and those produced by his prescribed algorithms, as noted above, mirror similar inconsistencies in other *zījes*, which are largely an effect of the importation and imperfect assimilation of foreign material. It is absurd to suppose that al-Kāshī, who proved himself in other contexts to be an expert calculator, generated inaccurate tables merely through chronic computational errors. Rather, it is almost certain that his tables were simply copied from existing parallax tables derived with cruder procedures rather than by his own more sophisticated methods.

From the mid-eighth century onward, the Islamic Near East was one of the most prominent and industrious centers of scientific activity in the world. Investigations in the exact sciences—mathematics, astronomy, astrology, optics, geography, and related disciplines—were initially inspired by substantial influxes from other earlier cultures, Mesopotamian, Indian, Greek, and Persian among others, reaching the Islamic intellectual scene by various trajectories, some direct, some circuitous. Initial efforts were focused on recovery, preservation, and translation of ancient texts, but after a while scholars were motivated to develop new and improved theories and techniques in these areas; these disciplines by this time could be regarded as distinctly and originally Arabic. The factors which ensured the prominence and success of this scientific society were many. Key to their continued productivity was the fact that eventually a single language, Arabic, was the dominant medium of expression

for scholars. This ensured far-reaching access and continued communication amongst practitioners, as well as encouraging a conscious and mindful selection of technical vocabulary. In addition, Islamic religious requirements provided vital incentives for scholars to apply themselves to particular problems that enriched their understanding of the field beyond its immediate application to spiritual practice.

Generally, theories and techniques concerning eclipse reckoning were not substantially modified by Islamic scholars, but rather existing features and details of the imported Ptolemaic model were updated and improved upon. Of particular importance for the prediction of eclipses was the development of the description and application of parallax. Bolstered with improved mathematical techniques, including a richer array of trigonometric functions and a developed understanding of relations of angles and triangles on the sphere, Islamic practitioners made both computational and theoretical improvements. Comprehensive tables extended the application of theoretical parameters. Although parameters could be computed to a level of precision that could not be practically captured, the computational facility and accuracy of these scholars is remarkable.

Eventually, Arabic astronomical researches reached the Latin West, a tradition which was, at that time, in no way comparable in sophistication or power to that active in the Islamic Near East.[79] The Arabic influence on the ignition and growth of this area in the medieval west was therefore crucial. Arabic achievements and works stimulated astronomical research, both practical and theoretical. However, as is common within cross-cultural transmission, sources that reach an inheritor culture are rarely fully indicative of the actual achievements or state of affairs in the original culture from which these sources were retrieved. More often than not, aspects of the original culture are distorted as they are based on a selection of sources which may not be the most highly developed or most representative, but rather are transmitted because of geographical considerations, availability or popularity, or some curious historical accident. In this case, European scholars initially gained access to Arabic sources through Spanish connections. Ironically enough, as Spain was on the extreme periphery of the Islamic empire, it was not representative of the dynamic scientific activity of the major metropoles to the east, such as Damascus. Through these channels, European scholars gained access to texts that had earlier been popular in the east of the Islamic empire, such as those by al-Battānī and al-Khwārizmī, but by the time of transmission were no longer current, and had been outdated by more recent work.

In fact, evidence suggests that on the whole scholars of the European Renaissance had little access to the best and most current astronomical works of

Islamic scholars, but relied on earlier and somewhat out-of-date theories, by then several centuries old. By the time that these latest superior forms of astronomy were discovered by the "west," they were studied and analyzed more for historical interest than for their astronomical application or significance. Nonetheless, the preoccupations of the Arabic scholars, their theoretical insights, their emphasis on observation, their physical concerns, their computational achievements, and their mechanical and instrumental craft provided the critical platform from which scholars of the Latin West overhauled the entire field of astronomy.

After 1500, innovations by Islamic astronomers waned, but enthusiasm for the discipline did not diminish. Astronomical works, albeit those already written, were zealously studied and taught, and astronomical activity was maintained in its many applications throughout society.

CHAPTER SEVEN

Concluding Remarks

When Aristotle defined an eclipse as an alignment, his description represented a pivotal moment within the unfolding of a vast cultural transition. To characterize an eclipse as an alignment is simultaneously bold and subtle. From a modern perspective, it is so very natural, but for the ancients to suppose a geometry behind the changing images on the ceiling of the heavens was a major shift in perspective. Aristotle recognized that he could register the observational effects of these alignments, but not the geometrical circumstances that were responsible for the alignments, and by making such a connection, he appealed to a deeper reality beyond the appearances. The intellectual possibilities of this association were not only slow to be realized, but also gradual in their generation of further insights extending human understanding of the heavens.

Despite their visual prominence, eclipses presented early practitioners with many challenges. Aspects of eclipses that were easily detected—duration, magnitude, position, pattern—were, for the most part, beyond the ability of the earliest astronomers to predict quantitatively. Even the straightforward account of their periodic appearances proved formidably difficult. The eventual resolution of these difficulties inspired various developments in astronomy and mathematics and was refined over many millennia.

In light of this long and complex scientific maturation, the primary purpose of this study was to determine the ways in which knowledge about eclipses was originated, developed, preserved, and transmitted. As the preceding chapters have shown, there were significant variations not only in the ways that different cultures developed eclipse reckoning techniques, but in their approaches to preserving and documenting their discoveries and theories. This study has also attempted to penetrate the nature of the interaction between observations of these spectacular phenomena and theorizing about them. As has been seen, observation and careful measuring did not always precede or occur simultaneously with theorizing about eclipses.

Furthermore, perhaps because of their visually impressive nature, eclipses were also regarded as ominous events. Indeed, almost all cultures, ancient and modern, to some extent wrap human significance in whatever phenomena they take notice of. For those cultures without literacy, the foregrounding of human concerns completely pervades their modes of thought. Probably as a result, the earliest known records of attempts to explain eclipse phenomena were closely connected to the origination, systematization, and embellishment of elaborate divination schemes. Eclipse possibilities were cataloged in protases of omens, and simple empirical elements were assigned predictive meaning, such as date and time, frequency, color, and direction of impact. Consequently, in many instances, it was the application of eclipse characteristics to divination that provided the impetus for the practitioners to develop and improve upon computations for eclipse prediction. An investigation of these elements reveals how wonderfully transitional divination is. It embodies a blending of qualities of oral culture and theoretical ways of thinking. The mark of the former is the human significances and of the latter, the exhaustive listing of prognostications.

The practitioners in the ancient Near East, with no significant astronomical legacy, were compelled to determine basic parameters and periodicities from scratch. Thus, sustained observation played a significant role in their eclipse reckoning. The recent advent of writing sometime in the late fourth millennium BCE made the necessary record-keeping possible and allowed scholars to notice periodicities and details that could not have been detected otherwise. Arguably, one of the most fundamental empirical discoveries was the fact that lunar eclipses generally occurred six months apart; this six-month interval was recognized in Mesopotamia as an approximate eclipse period from very early on, at least from the late second millennium BCE, but possibly earlier. After establishing this basic pattern, it was noticed that there was an anomaly in this general periodicity. Occasionally successive lunar eclipses occurred at intervals that were sometimes one month *less* than a multiple of six months. A similar effect in solar eclipses would have been far less noticeable due to their greatly restricted visibility.

From this, Mesopotamian astrologer-scribes determined that the smallest interval between successive lunar eclipses was five months. What is significant about this discovery is that, during the centuries that these observations were made, as modern recomputations reveal, there was never a pair of eclipses separated by only five months where both eclipses were potentially visible to the observers. The eclipses that were actually seen were separated by longer intervals such as eleven months (rare), seventeen months, and so on. Therefore, despite the fact that their observations would have never suggested it, the astrologer-scribes nonetheless recognized the five-month interval based

entirely on inferences drawn from longer periods. This may have been because of their function: eclipses were ominous events and it was the duty of the astrologer-scribes to predict and report *all* eclipse possibilities to their king; his welfare and that of his country was at stake. It was more prudent to hypothesize the existence of the five-month interval on the basis of mathematical patterns, even if unsupported by any direct observation, than to run the risk of failing to predict an eclipse.

From the recognition of the interval between successive eclipses arose the speculation of larger eclipse-possibility periods, which distributed by careful arrangement the five-month intervals among the six-month intervals. The most common and the most accurate of these cycles was the Saros, an 18-year eclipse cycle: 223 synodic months contained 38 eclipse possibilities, 33 of which were separated from the preceding eclipse by six months and the remaining five by five months. This was used extensively to predict both solar and lunar eclipses, sometimes into the distant future. Furthermore, the fact that the Saros cycle was not perfect was recognized and the cumulative error of this cycle (each successive Saros differs from the last by about eight hours) seems to have been accounted for by some practitioners. The so-called Diaries, from the early first millennium BCE, which contain extensive records of various observations and span large amounts of time, also must have been critical for the recognition and formulation of periodicities. Subsequent dates and positions could be excerpted together for one particular phenomenon, and thus numerical parameters for longer periods could be derived and checked.

Later developments included a more numerical approach; the Astronomical Cuneiform Texts (ACT), which were a product of the late first millennium BCE, were texts in which complex motion was thought to be the product of simpler regular linear patterns such as step functions and zigzag functions. The primary aim of the Astronomical Cuneiform Texts was to compute the exact moment of syzygy, and thus determine the first day of the month. However, certain texts confirm that they were also used for the determination of eclipses. Although the results were more precise than other predictive schemes used by scholars in the ancient Near East, they were in fact *not* as accurate as the predictions that are found in other nonmathematical astronomical texts. In addition to this, the precision of these texts far exceeded the observational precision available to astronomers. Their measuring equipment was simply not sophisticated enough to support the precision of these calculations. The techniques used and developed were not adopted by mainstream astronomers in the ancient Near East.

This study also reveals that the techniques used and developed by practitioners in the ancient Near East were heavily arithmetical and were directed toward

an accountancy of the appearances. As far as the evidence reveals, there are no attempts to combine this numerical understanding of eclipse possibilities with any visualization of how they might depend upon the physical layout of the heavens and the shapes and configurations of the bodies within. Because of this, it may be contended that their investigations and successful predictions of eclipses did not constitute an articulation of theory. Although it has been noted that there was an increase in the success of eclipse prediction, the calculations of the Mesopotamian astronomers could not advance any further while a consideration of the geometrical dimension of these events was lacking. In fact, it may be observed that the techniques of the Mesopotamian astronomers perhaps indicate the limits to which astronomy can advance without any overt consideration of appropriate cosmographical elements. These scholars raised their command of eclipse phenomena to an increasingly sophisticated and refined accountancy of the appearances. But their near-exclusive focus on prediction and arithmetical conception meant that they never decisively shifted to a fully theoretical orientation of thought.

The focus on numerical patterns proved to be potent: in the fifth century BCE, ancient Near Eastern scholars were more advanced with respect to eclipse prediction techniques than their Greek contemporaries. However, the early Greek astronomers were considering precisely those elements that the Mesopotamian understanding lacked. What captured the attention of these Greek astronomers were the physical causes that brought about eclipses, conceived in terms of the shapes and dimensions of the heavenly bodies within the celestial sphere. Aspects such as the illumination of the moon, the relative configurations of the celestial bodies at the time of an eclipse, and the frequency of eclipses were all pondered from this perspective. The discovery of the spherical nature of the earth and the luminaries, and the modeling of the trajectories of the planets by means of circular, uniform, and regular motions meant that the astronomy of the Greeks had a vastly different nature to that of their Mesopotamian counterparts. For Greek thinkers, explanation emerged as a project of the mind increasingly attentive not so much to the appearances themselves but rather to the physical conditions that could be inferred from them. Astronomical alignments drew Greek thinkers beyond the outer appearances of celestial phenomena to a reflection of reality behind those configurations.

The conceptual change of course imposed on astronomy at the hands of the Greeks was brought about by the introduction of geometry into that science. The earliest investigations by the pre-Socratics beginning in the seventh century BCE reveal the ground-breaking character of these initial halting, yet potent theoretical preoccupations. Increasingly, a geometrical kinematic description of celestial motion became the primary objective of

Greek astronomy, and only when satisfactory models had been developed did the goal emerge of quantifying the models for practical predictive purposes. The planetary models of Apollonius in the third century BCE were able to represent the general patterns of the celestial bodies, but they had no predictive power. Hipparchus, a century later, seems to have been the first to assert the need to apply the appropriate numerical parameters to these models so that their timings and magnitudes reflected the actual motions and positions of the celestial bodies. It seems that he received this inspiration in part from contact with Mesopotamian arithmetical procedures and techniques. Now for the first time the arithmetical period relations could be accounted for by means of geometrical kinematic models. The fusion between the geometrical models of the Greeks and the arithmetical patterns appropriated from the ancient Near East meant that the aspiration to predict both solar and lunar eclipses with confidence, based on their physical properties, could at last be realized.

Simultaneous with this work in astronomy in Greece was the development of mathematical techniques which would be immediately useful to astronomy. The development of spherical mathematics with the definition and rigorisation of concepts such as the sphere, the poles, the great circles, and trigonometry for triangles on the sphere meant that astronomical fundamentals such as the ecliptic, the celestial equator, and celestial coordinates could finally be fully mathematized by means of the angles and arcs they contained. Similarly, more general mathematical developments concerning ratios, proportions, and geometrical techniques meant that problems such as the size of the earth and the sizes and distances of the celestial bodies could also be properly studied. The problem most crucial to a full understanding of eclipses is the determination of the relative sizes and distances of the luminaries, the earth, and the earth's shadow. Aristarchus, in the third century BCE, not only considered these relative sizes and distances, but demonstrated how they could be measured.

The consideration of the bodies as spherical and orbits as circular also led to the recognition of the displacement effect entailed by observing from the surface of the earth as opposed to its center. This effect, called parallax, was first noticed by the Greeks, and, once the lunar coordinates could be properly accounted for, was given theoretical expression. Notably, the effect of parallax is slight in most cases, and it was beyond the capabilities of the observational equipment available to the Greeks to measure it directly. This was an instance in which thinkers were able to theoretically deduce and describe an effect that they could not adequately discern empirically. Their theory was more advanced than their ability to test it. However, parallax left its observable traces through eclipse phenomena—its effects on the positions of the luminaries could make

an eclipse occur that was not otherwise predicted, or cause a predicted eclipse to pass by.

The compilation of Ptolemy's *Almagest* in the second century CE represented the culmination of eclipse prediction techniques, and indeed of geocentric mathematical astronomy. In the *Almagest*, a careful selection of observations from Ptolemy's time and from the older Babylonian records was applied to a further refined geometrical kinematic model in order to give the elements of the model appropriate size and velocity. The *Almagest* assigned to its model magnitudes that produced results which corresponded with the observed motions of the celestial bodies. From these parameters, Ptolemy derived accurate eclipse limits for both solar and lunar eclipses and thence created tables that could predict with confidence eclipse occurrences, as well as furnishing an account of the basic features that each particular eclipse would exhibit.

The success and accuracy of Ptolemy's *Almagest* meant that Greek astronomical works composed prior to its introduction into the scientific community were rendered out of date and ceased to be copied and circulated. Thus, early Greek astronomical works are available only to modern readers through the references and citations of later commentators. However, there is another, indirect source of evidence which reveals something of the nature of early Greek astronomy, namely, its transmission and incorporation into the Indian scientific tradition.

Early Indian astronomical activity was influenced by the theogonic cosmology which was familiar not only to the society as a whole but also in learned disciplines. The notion that all astronomical knowledge had been revealed by Gods and Sages and was thus indisputable had a significant impact on the development of Indian astronomy in general. Furthermore, the oral culture in which the Indian intellectual disciplines were embedded required that scientific material be memorable and metrical, and this requirement was often met to the detriment of the meaning of the text. What was esteemed above all was to preserve the tradition rather than to innovate and improve on it, especially if these innovations or improvements entailed fundamental changes. Indian astronomical works are thus highly repetitive. Instead of creating new methods or developing new theories which would better correspond with what could be observed, the Indian practitioners concentrated on manipulating the computational techniques themselves.

In the case of eclipse theory, these characteristics had some serious consequences. The need to preserve the tradition seemed to discourage astronomers from observation, or at least from recording and discussing details of observations. It may be that the discrepancy between what they could observe and the results the traditional methods furnished deterred them from using such

a comparison to verify their results at all. Furthermore, despite the fact that Indian astronomers would frequently assert the importance of achieving agreement between observations and computational results, it is clear that very few (if any at all) actual observations were influencing their reckoning of eclipses until at least the fourteenth century. The lack of observations meant that parameters and techniques which failed to successfully predict and model eclipse phenomena continued to be utilized.

In such a scientific environment and with these types of pressures, it is surprising that Indian astronomy did not remain entirely unchanged. In fact, it was to be curiosity about and inspiration from external scientific cultures that stimulated change and development. With the introduction of Greek astronomical concepts in the early Common Era, Indian practitioners devised a more theoretically structured model based on the Greek requirements that planetary motion be circular, uniform, and regular; they expanded the Greek epicycle concept to create a model that was distinctly Indian. However, the astronomers could not entirely dismiss the sacred and powerful presence of the older cosmology. Thus many Greek concepts had to be significantly modified in order to conform with the older traditional model.

The intellectual priorities of the Indian scholars did confer some advantages. Greek inquirers, who were committed to regular, uniform, and circular motions as fundamental to their geometrical kinematic designs, were hampered in their attempts to explain phenomena that could not be tackled and resolved easily (if at all) by such an orientation. Models built upon such assumptions were powerful, but were frequently deficient in flexibility and consistency, requiring very complex patches to account for their discrepancies, if they could be overcome at all. The Indian orientation, in contrast, was not overwhelmingly bound by any particular geometrical requirements, and scholars were able to reconsider, modify, and overhaul existing models creatively and without restriction to address their computational inconsistencies, in ways that would have not been permitted in other cultures. While they found initial inspiration in the geometrical structure of the Greek model, they were not bound to it, and modified it freely.

Material seen in early Sanskrit astronomical works suggest that there were at least two transmissions of Greek astronomical concepts to India. The first probably occurred in the second or third century CE and a later, fuller transmission was used by Indian astronomers in the fifth century CE. Both these transmissions were pre-Ptolemaic, but the second importation included ideas that were more sophisticated and employed spherical trigonometric techniques. The identification of parallax and the emergence of methods for computing it in India suggest that this later transmission was based on the concepts that

were circulating around the time of Hipparchus, if not a little earlier. Indian scholars applied their computational prowess to the calculation of parallax, devising approximations which eased the burden of computation as well as developing iterative procedures for achieving better precision.

With the transmission of scientific ideas, it is not always the most up-to-date or current theories and methods that are automatically selected. In fact, as has been observed throughout this study, many of the ideas most successfully assimilated in foreign traditions are out of date, second-rate, or simplified rule-of-thumb methods. Furthermore, the introduction of foreign ideas into a scientific climate distinct from their culture of origin had some dramatic effects. Definitions and parameters are more easily transmitted than the methodological assumptions and preferences of a scientific environment, which more often than not remain distinct in the originator and inheritor cultures. Scientific ideas removed from their original context may take on new identity. They may be altered, they may be misunderstood, they may be developed. But the fundamental displacement and modifications ensure that the concepts become, in a distinct way, an integral part of the inheritor culture. Furthermore, some aspects (such as the direction of impact in eclipse theory) that had been important to one culture, were taken on trust as important in the inheritor culture, even if it was essentially irrelevant to their own preoccupations and practices. In this way, meaningless procedures continued to be propagated.

The transmission of ideas from one culture to another was crucial for the foundation of the discipline of astronomy in the newly formed Islamic empire. Originating from multiple cultures throughout Eurasia, foreign sources began to penetrate Islamic intellectual culture as early as the eighth century. United by a single language of scientific expression, and motivated in part by an application of their efforts to address religious requirements, Islamic scholars translated and studied these sources with great attention. The result of the influx of many distinct imported astronomical ideas as a result of transmission was naturally a comparison between the different proposals. In turn, the resulting discrepancies, inconsistencies, and incompatibilities between these different models were immediately brought to light. These fundamental differences encouraged scholars to search for a compromise.

Eventually Islamic scholars settled on the Ptolemaic model, turning their attention to fixing the various inconsistencies and problems they detected in it. Mathematical refinements further improved the models. Al-Battānī in the tenth century represents the culmination of this quest for theoretical consistency; in his time, mathematical astronomy in the Islamic Near East attained a command over observation and theory that was to set the standard for the modern west. In the following centuries, deeper philosophical and

theological concerns prompted a revision of some of the basic fundamentals of the Ptolemaic model.

The astronomy carried out in the Islamic Near East was unprecedented in many ways. First, it drew upon the strengths and successes of the astronomical achievements in the cultures that preceded it. But, more profoundly, astronomy reconnected with certain intellectual fields, such as philosophy, physics, and cosmology, which placed new demands on it. It was no longer enough to adequately account numerically and geometrically for the phenomena. A theory that addressed the actual physical configuration of the heavens and its contents became a priority. In such an environment, Islamic scholars demanded a harmony of configuration comparable to that achieved in well-integrated mathematical theory. Their attempts to satisfy these demands anticipated the radical reforms of the Latin west with the beginnings of modern astronomy. Indeed, these demands were not fully met until at least Copernicus, or until Kepler, or possibly even until Newton. But they were consequential and perhaps even inspirational. They furthered the trajectory of the gradual development of eclipse science in the premodern world, from its very first beginnings in recently literate cultures toward a cosmology that embodied a thoroughly theoretical cognition of matters astronomical, including eclipses.

Thus, the path to successful eclipse prediction in the ancient and medieval world was strewn with profound scientific discoveries. Most importantly, this achievement was not attained by any one scientific culture on its own; it was cumulative, a collaboration in which each recipient would build on the results of its predecessors and contemporaries. It has been shown how the different cultures surveyed in this study furthered intellectual change in different and distinctive ways. It was the emphasis of the early Greek astronomers on precisely those elements the Mesopotamian model was lacking, and the subsequent synthesis between geometrical considerations with arithmetical patterns that saw the culmination of eclipse theory in the second century CE. In turn, the practitioners in the Islamic Near East critiqued, down to the smallest detail, the strengths and weaknesses of the Greek model, in a way that the Greeks could never have done. Iterative techniques and the expansion of mathematical methods by Indian scientists also contributed to the quantifying and predictive power of premodern astronomers to calculate the appropriate intervals and magnitudes.

Aspects in the story of eclipse reckoning illustrate many important themes through the history of astronomy and indeed of science in general: the exigencies of observation, the reliance on unifying mathematical patterns by early practitioners, the complex nexus of transmission and synthesis of ideas caused by cross-cultural contact, the role of wider societal concerns in encouraging

or distracting inquiry, the pursuit of ever more sophisticated mathematical expression and its impact on inquiry, the nuanced relationship between astronomy and divination and astrology, and the effects of literacy on intellection.

More generally, the unfolding of human command of eclipse phenomena is emblematic of a broader adumbration of theoretical modes of thought. Investigations in astronomy had the effect of inducing inquirers to reflect upon a deeper reality behind the appearances, which constantly drew them toward abstraction from appearance to theory. This process of building up an unseen but possibly very real framework backstage to explain the shadow-plays visible on the heavenly screen provided an intellectual paradigm for investigation in broader fields for early thinkers. Astronomical alignments of this sort were critical to this unfolding. Chasing shadows was to prove profoundly meaningful. The account of the increasing theoretical mastery of eclipse phenomena throughout early history is but a small trajectory in the gradual flowering of science, but one that captures all of its essential elements. Fundamental features of science proceeded from astronomy, and these aspects came into focus not instantly but in the slow convergence of knowledge through many languages and many millennia. The unfolding of eclipse science epitomizes this.

APPENDIX A

Abbreviations and Conventions

Abbreviations

A	*Āryabhaṭīya* of Āryabhaṭa
BSS	*Brāhmasphuṭasiddhānta* of Brahmagupta
GM	*Grahaṇamaṇḍana* of Parameśvara
GND	*Grahaṇyāyadīpikā* of Parameśvara
KKD	*Khaṇḍakhādyaka* of Brahmagupta
LB	*Laghubhāskarīya* of Bhāskara I
MB	*Mahābhāskarīya* of Bhāskara I
ŚDV	*Śiṣyadhīvṛddhidatantra* of Lalla
SS	*Siddhāntasundara* of Jñānarāja
SŚe	*Siddhāntaśekhara* of Śrīpati
SŚi	*Siddhāntaśiromaṇi* of Bhāskara II

Translation Conventions

( ) addition of necessary material to make grammatical sense of the text, or to aid to the flow of English, or to provide a word or phrase which is left unstated by the original author.
[ ] gloss by modern author to help explain the text.
Unless acknowledged, the translations are the author's own.

APPENDIX B

A Comparison of NMAT and MAT Eclipse Predictions

Solar Eclipses

−193 May 26

Syzygy (LT) = 0.9575 = 0:57
Contact 1 = 0.27 = 0:16
Sunset = 18.99 = 18:59

NMAT: 77° after sunset or 5;08 hours after sunset.
Sys. B: 41° after sunset or 2;44 hours after sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 18{:}59 + 5{;}08 = 0{:}07$$
$$\text{Difference} = 0{:}16 - 0{:}07$$
$$= 0{;}09 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 18{:}59 + 2{;}44 = 21{:}43$$
$$\text{Difference} = 0{:}57 - 21{:}43$$
$$= 3{;}14 \text{ hours}$$

−189 March 14

Syzygy (LT) = 9.4698 = 9:28
Contact 1 = 8.28 = 8:17
Sunrise = 6.10 = 6:10

NMAT: 30° after sunrise or 2;00 hours after sunrise.
Sys. B: 24° after sunrise or 1;36 hours after sunrise.

$$\text{NMAT} \Rightarrow \text{C1} = 6{:}10 + 2 \text{ hours} = 8{:}06$$
$$\text{Difference} = 8{:}17 - 8{:}06$$
$$= 1{;}22 \text{ hours}$$

$$\text{Sys. B} \Rightarrow \text{Syzygy} = 6{:}10 + 1{;}36 = 7{:}42$$
$$\text{Difference} = 9{:}28 - 7{:}42$$
$$= 1{;}46 \text{ hours}$$

−168 January 22

Syzygy (LT) = 22.7029 = 22:42
Contact 1 = 21.96 = 21:58
Sunset = 17.16 = 17:10

NMAT : 93° after sunset or 6;12 hours after sunset.
Sys. A : 71;13° before sunset or 4;45 hours before sunset.

Computing the ACT value as "before sunset" will give a very poor calculation of syzygy. Supposing the *šú* (before sunset) is a mistake for *du* (after sunset), the time is much better. However, that this value corresponded to such a date was only a tentative suggestion, as the tablet is badly damaged, and this discrepancy may indicate that the two do not go together.

$$\text{NMAT} \Rightarrow \text{C1} = 17{:}10 + 6{;}12 = 23{:}22$$
$$\text{Difference} = 23{:}22 - 21{:}58$$
$$= 1{;}24 \text{ hours}$$
$$\text{Sys. A} \Rightarrow \text{Syzygy} = 17{:}10 + 4{;}45 = 21{:}55$$
$$\text{Difference} = 22{:}42 - 21{:}55$$
$$= 0{;}47 \text{ hours}$$

−164 October 29

Syzygy (LT) = 22.8147 = 22:49
Contact 1 = 22.39 = 22:23
Sunset = 17.51 = 17:31

NMAT: 64° after sunset or 4;16 hours after sunset.
Sys. A: 309;59° before sunset or 50° after sunset (prev. day) = 3;20 hours after sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 17{:}31 + 4{;}16 = 21{:}47$$
$$\text{Difference} = 22{:}23 - 21{:}47$$
$$= 0{;}36 \text{ hours}$$
$$\text{Sys. A} \Rightarrow \text{Syzygy} = 17{:}31 + 3{;}20 = 20{:}51$$
$$\text{Difference} = 22{:}49 - 20{:}51$$
$$= 1{;}58 \text{ hours}$$

−102 July 8

Syzygy (LT) = 18.9238 = 18:55
Contact 1 = 18.51 = 18:31
Sunset = 19.13 = 19:07

NMAT: 30° before sunset or 2;00 hours before sunset.
Sys. A: 13;1° before sunset or 52 minutes before sunset.
Sys. B: 14° before sunset or 56 minutes before sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 19{:}07 - 2{;}00 = 17{:}07$$
$$\text{Difference} = 18{:}31 - 17{:}07$$
$$= 1{;}24 \text{ hours}$$
$$\text{Sys. A} \Rightarrow \text{Syzygy} = 19{:}07 - 0{;}52 = 18{:}15$$
$$\text{Difference} = 18{:}55 - 18{:}15$$
$$= 0{;}40 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 19{:}07 - 0{;}56 = 18{:}11$$
$$\text{Difference} = 18{:}55 - 18{:}11$$
$$= 0{;}44 \text{ hours}$$

Lunar Eclipses

−193 May 11

Mid-eclipse (LT) = 19.175 = 19:11
Contact 1 = 10.9 = 10:54
Sunrise = 5.19 = 5:11

NMAT: 94° after sunrise or 6;16 hours after sunrise.
Sys. B: 91° after sunrise or 6;04 hours after sunrise.

$$\text{NMAT} \Rightarrow \text{C1} = 5{:}11 + 6{;}16 = 11{:}27$$
$$\text{Difference} = 11{:}27 - 10{:}54$$
$$= 0{;}33 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 5{:}11 + 6{;}04 = 11{:}15$$
$$\text{Difference} = 12{:}17 - 11{;}15$$
$$= 1{;}02 \text{ hours}$$

−193 November 5

Mid-eclipse (LT) = 7.76 = 7:46
Contact 1 = 5.99 = 5:59
Sunset = 6.57 = 6:34

NMAT: 12° before sunrise or 0;48 hours before sunrise.
Sys. B: 15° before sunrise or 1;00 hour before sunrise.

$$\text{NMAT} \Rightarrow \text{C1} = 6{:}34 - 0{;}48 = 5{:}46$$
$$\text{Difference} = 5{:}59 - 5{:}46$$
$$= 0{;}13 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 6{:}34 - 1{;}0 = 5{:}34$$
$$\text{Difference} = 7{:}46 - 5{:}34$$
$$= 2{;}12 \text{ hours}$$

−191 April 19

Mid-eclipse (LT) = 19.175 = 19:11
Contact 1 = 17.85 = 17:51
Sunset = 18.50 = 18:30

NMAT: 2° after sunset or 0;08 hours after sunset.
Sys. B: 1° after sunset or 0;04 hours after sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 18{:}30 + 8 \text{ minutes} = 18{:}38$$
$$\text{Difference} = 18{:}38 - 17{;}51$$
$$= 0{;}47 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 18{:}30 + 4 \text{ minutes} = 18{:}34$$
$$\text{Difference} = 19{:}11 - 18{:}34$$
$$= 0{;}37 \text{ hours}$$

−189 February 28

Mid-eclipse (LT) = 5.645 = 5:39
Contact 1 = 3.78 = 3:47
Sunset = 6.33 = 6:20

NMAT: 30° before sunrise or 2;00 hours before sunrise.
Sys. B: 27° before sunrise or 1;48 hours before sunrise.

$$\text{NMAT} \Rightarrow \text{C1} = 6{:}20 - 2 = 4{:}20$$
$$\text{Difference} = 4{:}20 - 3{:}47$$
$$= 0{;}33 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 6{:}20 - 1{:}48 = 4{:}32$$
$$\text{Difference} = 5{:}39 - 4{:}32$$
$$= 1{;}07 \text{ hours}$$

−188 February 17

Mid-eclipse (LT) = 5.67 = 5:40
Contact 1 = 3.69 = 3:41
Sunrise = 6.51 = 6:31

NMAT: 34° before sunrise or 2;16 hours before sunrise.
Sys. B: 40° before sunrise or 2;40 hours before sunrise.

$$\text{NMAT} \Rightarrow \text{C1} = 6{:}31 - 2{;}16 = 4{:}15$$
$$\text{Difference} = 4{:}15 - 3{:}41$$
$$= 0{;}34 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 6{:}31 - 2{:}40 = 3{:}51$$
$$\text{Difference} = 5{:}40 - 3{:}51$$
$$= 1{;}49 \text{ hours}$$

−185 June 11

Mid-eclipse (LT) = 15.235 = 15:14
Contact 1 = 13.23 = 13:14
Sunset = 19.12 = 19:07

NMAT: 48° before sunset or 3;12 hours before sunset.
Sys. B: 78° before sunset or 5;12 hours before sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 19{:}07 - 3{;}12 = 15{:}55$$
$$\text{Difference} = 15{:}55 - 13{:}14$$
$$= 2{;}41 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 19{:}07 - 5{;}12 = 13{:}55$$
$$\text{Difference} = 15{:}14 - 13{:}55$$
$$= 1{;}19 \text{ hours}$$

−184 November 24

Mid-eclipse (LT) = 20.05 = 20:03
Contact 1 = 19.34 = 19:20
Sunset = 17.12 = 17:07

NMAT: 44° after sunset or 3;16 hours after sunset.
Sys. B: 40° after sunset or 2;40 hours after sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 17{:}07 + 3{;}16 = 20{:}23$$
$$\text{Difference} = 20{:}23 - 19{:}20$$
$$= 1{;}03 \text{ hours}$$

$$\text{Sys. B} \Rightarrow \text{Syzygy} = 17{:}07 + 2{;}40 = 19{:}47$$
$$\text{Difference} = 20{:}03 - 19{:}47$$
$$= 0{;}16 \text{ hours}$$

−131 January 17

Mid-eclipse (LT) = 22.245 = 22:15
Contact 1 = 20.56 = 20:34
Sunset = 17.42 = 17:25

NMAT: 60° after sunset or 4;00 hours after sunset.
Sys. B: 81° after sunset or 5;24 hours after sunset.

$$\text{NMAT} \Rightarrow \text{C1} = 17{:}25 + 4 = 21{:}25$$
$$\text{Difference} = 21{:}25 - 20{:}34$$
$$= 1{;}09 \text{ hours}$$
$$\text{Sys. B} \Rightarrow \text{Syzygy} = 17{:}25 + 5{;}24 = 22{:}49$$
$$\text{Difference} = 22{:}49 - 22{:}15$$
$$= 0{;}34 \text{ hours}$$

APPENDIX C

The Eclipse Tables in the *Wajīz* of al-Khāzinī

The tables relevant to eclipse computations in the *Wajīz* of al-Khāzinī are as follows.

Title	Istanbul (I)	Teheran (T)
Velocities of the two luminaries and their diameters	f. 66	p. 97
Lunar eclipse table	f. 66v	p. 98
digit to area conversion		
angle of inclination (*inḥirāf*)		
Parallax of the two luminaries	f. 68v	p. 101
Parallax for $\phi = 38^\circ$	f. 69	p. 102
Parallax of moon in longitude and latitude	f. 69v	—
Parallax of moon according to Theon	ff. 78v–79v	pp. 103–5
Solar eclipses	—	p. 106

Next table begins on facing page.

Table of the Mean Motions of the Two Luminaries and their Diameters and the Shadow

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Zodiacal sign		Sun				Moon							
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
1	0 0	0 0	56 55	2 23	31 28	0 0	12 6	30 18	30 3	78 6	0 0	65	60	0;0
	0 6	11 24	56 56	2 23	31 28	0 1	12 7	30 19	30 4	78 10	0 21	65	60	0;1
	0 12	11 18	56 57	2 23	31 29	0 2	12 8	30 21	30 6	78 15	0 52	65	60	0;2
	0 18	11 12	56 59	2 23	31 30	0 3	12 9	30 25	30 10	78 25	1 43	65	60	0;3
5	0 24	11 6	57 4	2 23	31 32	0 6	12 12	30 30	30 15	78 39	2 42	65	61	0;6
	1 0	11 0	57 11	2 23	31 36	0 9	12 15	30 36	30 21	78 57	4 1	65	61	0;9
	1 6	10 24	57 18	2 24	31 40	0 13	12 19	30 43	30 28	79 11	5 22	1	61	0;10
	1 12	10 18	57 27	2 24	31 44	0 17	12 23	30 53	30 37	79 36	7 13	1	62	0;17
	1 18	10 12	57 48	2 24	31 49	0 21	12 27	31 3	30 47	80 3	9 15	1	62	0;21
10	1 24	10 6	57 46	2 25	31 54	0 25	12 33	31 15	30 58	80 33	11 37	2	62	0;25
	2 0	10 0	57 56	2 25	31 59	0 29	12 39	31 27	31 11	81 3	14 0	2	63	0;29
	2 6	9 24	58 7	2 25	32 6	0 33	12 45	31 41	31 25	81 38	16 47	2	63	0;33
	2 12	9 18	58 19	2 26	32 14	0 38	12 52	31 54	31 38	82 12	19 35	2	64	0;38
	2 18	9 12	58 33	2 26	32 21	0 42	12 59	32 9	31 52	82 50	22 36	3	64	0;42
15	2 24	9 6	58 47	2 27	32 29	0 46	13 6	32 23	32 6	83 28	25 35	3	65	0;46
	3 0	9 0	59 7	2 27	32 38	0 50	13 13	32 41	32 24	84 14	28 43	4	65	0;50
	3 6	8 24	59 18	2 28	32 47	0 54	13 21	32 59	32 41	85 0	31 48	4	66	0;54
	3 12	8 18	59 33	2 28	32 55	0 58	13 28	33 18	33 0	85 48	34 54	5	67	0;58
	3 18	8 12	59 49	2 29	33 3	1 2	13 36	33 36	33 19	86 37	38 0	5	68	1;2
20	3 24	8 6	60 3	2 29	33 10	1 6	13 43	34 54	33 37	87 20	41 7	6	69	1;6
	4 0	8 0	60 17	2 30	33 18	1 10	13 50	34 12	33 55	88 12	44 0	7	69	1;10
	4 6	7 24	60 30	2 30	33 24	1 14	13 57	34 30	34 12	88 57	46 55	8	69	1;14
	4 12	7 18	60 47	2 31	33 30	1 18	14 3	34 48	34 30	89 42	49 30	9	70	1;18
	4 18	7 12	60 51	2 31	33 36	1 22	14 8	35 6	34 47	90 27	51 35	9	70	1;22

(continued)

Table of the Mean Motions of the Two Luminaries and their Diameters and the Shadow *(continued)*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Zodiacal sign		Sun				Moon							
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
25	4 24	7 6	61 0	2 32	33 41	1 26	14 13	35 22	35 4	91 10	53 48	9	70	1;27
	5 0	7 0	61 9	2 32	33 46	1 30	14 17	35 37	35 20	19 45	55 37	70	71	1;30
	5 6	6 24	61 17	2 33	33 51	1 33	14 21	35 49	35 31	92 20	57 16	70	71	1;33
	5 12	6 18	61 23	2 33	33 54	1 36	14 25	35 59	35 40	92 40	58 17	70	71	1;36
	5 18	6 12	61 28	2 33	33 56	1 38	14 26	36 3	35 45	92 56	59 21	1	72	1;38
30	5 24	6 6	61 30	2 33	33 57	1 39	14 27	36 4	35 46	93 0	59 41	1	72	1;39
	6 0	6 0	61 31	2 33	33 58	1 40	14 28	36 5	35 47	93 2	60 0	72	72	1;40

App. Crit.

col 1:14 T 8; col 3:16 I 2; col 3:23 I 42; col 4:25,26 I 2;33; col 4:31 I 2;34; col 5:30 I 49; col 7:11 I 29; col 10:5 I 29; col 10:5 I 54; col 10:5 I 14; col 11:8 I 18; col 11:9 I 45; col 11:20 I 6; col 11:24 I 18; col 11:26 I 30; col 11:30 I 11; col 12 not in T; col 13 I has 60, 60, 60, 61, 62, 63, 64, 66, 68, 69, 71, 73, 74, 76, 78, 79, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 92, 92, 92.; col 14 not in T.

Contents

Columns 1 and 2 Argument in 6° increments of a zodiacal sign.

Column 3 Daily velocities (*al-buhṭ*) of the sun in minutes and seconds; Min = 0; 56, 55; Max = 1; 01, 31.

Column 4 Velocity of the sun in one hour, in minutes and seconds (divide column 3 by 24). T has the title *marakahā fī sa'aha*. I has the title *masairhā fī sa'aha*.

Column 5 Diameter of the sun, in minutes and seconds. Min = 0; 31, 28; Max = 0; 33, 58.

Column 6 Equation of the diameter of the shadow, in minutes and seconds. Min = 0; 0; Max = 0; 1, 40.

Column 7 Daily velocity of the moon, in degrees and minutes (even though both copies give "minutes" and "seconds"). Min = 12; 6; Max = 14; 28.

Column 8 Velocity of the moon in 1 hour. Min 0;30,18; Max 0;36,5. Note: this is *not* the result of column 7 divided by 24.

Column 9 Diameter of the moon, in minutes and seconds. Min = 0; 30, 3; Max = 0; 35, 47.

Column 10 Diameter of the shadow, in minutes and seconds. Min = 1; 18, 6; Max = 1; 33, 2.

Column 11 Minutes and seconds of equation (*al-taqwīm*). Min = 0; 0; Max = 1; 00 (interpolation column).

Column 12 (Not in T.) Minutes of the variation (*ikhṭilāf*) of the epicycle (*al-tadwīr*).

Column 13 The variation of the epicycle (*ikhṭilāf al-tadwīr*) T 60–72, I 60–92.

Column 14 (Not in T.) Equation of the diameter of the shadow, in minutes and seconds 0;0 –1;40.

Table for a Lunar Eclipse

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	21 30	0 4	1 53	0 3	0 49	0 3	33	8 6	2 16	1 28	0 9
	1	21 6	0 8	1 53	0 3	0 49	0 3	34	7 42	2 20	1 26	0 9
	2	20 42	0 12	1 53	0 3	0 49	0 3	35	7 18	2 24	1 24	0 10
	3	20 18	0 16	1 53	0 3	0 48	0 4	36	6 53	2 28	1 22	0 11
5	4	19 54	0 20	1 53	0 3	0 48	0 4	37	6 29	2 32	1 20	0 12
	5	19 29	0 24	1 53	0 4	0 47	0 5	38	6 5	2 36	1 17	0 13
	6	19 5	0 28	1 52	0 4	0 47	0 5	39	5 40	2 40	1 15	0 14
	7	18 41	0 32	1 52	0 4	0 46	0 5	40	5 16	2 44	1 13	0 15
	8	18 16	0 36	1 51	0 4	0 46	0 6	41	4 52	2 48	1 11	0 16
10	9	17 38	0 40	1 51	0 5	0 45	0 6	42	4 28	2 52	1 9	0 17
	10	17 28	0 44	1 51	0 5	0 45	0 6	43	4 3	2 56	1 7	0 19
	11	17 3	0 48	1 51	0 5	0 44	0 7	44	3 39	3 0	1 4	0 21
	12	16 39	0 52	1 49	0 5	0 43	0 7	45	3 14	3 4	1 2	0 24
	13	16 44	0 56	1 49	0 5	0 42	0 7	46	2 50	3 8	0 58	0 27
15	14	15 49	1 0	1 48	0 5	0 41	0 7	47	2 26	3 12	0 53	0 30
	15	15 25	1 4	1 48	0 5	0 40	0 7	48	2 2	3 16	0 48	0 34
	16	15 1	1 8	1 47	0 5	0 38	0 8	49	1 37	3 20	0 43	0 38
	17	14 36	1 12	1 47	0 5	0 35	0 8	50	1 13	3 24	0 34	0 43
	18	14 12	1 16	1 46	0 5	0 32	0 9	51	0 28	3 28	0 26	0 47
20	19	13 48	1 20	1 46	0 6	0 29	0 20	52	0 24	3 32	0 16	0 50
	20	13 23	1 24	1 45	0 6	0 25	0 11	53	0 0	3 36	0	1 1
	21	12 59	1 28	1 45	0 6	0 21	0 12	54	0 0	3 15	0	0 58
	22	12 35	1 32	1 44	0 6	0 0	0 31	55		2 55	0	0 55
	23	12 10	1 36	1 44	0 6	0 0	0 29	56		2 32	0	0 51
25	24	11 46	1 40	1 43	0 6	0 0	0 27	57		2 14	0	0 47
	25	11 22	1 44	1 42	0 7	0 0	0 22	58		1 54	0	0 44
	26	10 58	1 48	1 41	0 7	0 0	0 0	59		1 33	0	0 40
	27	10 33	1 52	1 40	0 7	0 0	0 0	60		1 13	0	0 36
	28	10 8	1 56	1 38	0 7	0 0	0 0	61		0 53	0	0 33
30	29	9 44	2 0	1 36	0 8	0 0	0 0	62		0 32	0	0 27
	30	9 19	2 4	1 34	0 8	0 0	0 0	63		0 16	0	0 18
	31	8 55	2 8	1 32	0 8	0 0	0 0	64		0 0	0	0 0
	32	8 31	2 12	1 30	0 8	0 0	0 0					

App. Crit.

col 2:10 T 52; col 2:14 T 14; col 4:11, 12 T 50; col 5:20 T 5; col 5:30 T 7; col 7:20 T 10; col 9:2 T 41; col 9:20 T 27; col 10:11ff. T 3 0, 3 4, 3 8, 3 12, 3 16, 3 20,3 24, 3 28, 3 32, 3 36, 3 15, 2 55, 2 34, 2 14, 1 54, 1 33, 1 13, 0 54, 0 32, 0 12, 0 0.; col 12:22 I 18; col 12:30 I 24; col 12:31 I 8.

Contents

Columns 1 and 8 Latitude of the moon in minutes; the argument is from 0 to 64.

Columns 2 and 9 Digits and sixtieths of the eclipse.

Columns 3 and 10 Equation in hours and minutes.

Columns 4 and 11 Hours of the half-duration in hours and minutes.

Columns 5 and 12 Equation in hours and minutes.

Column 6 Hours of half-duration of totality in hours and minutes.

Column 7 Equation in hours and minutes.

In contrast to both the *Handy Tables* and the *Almagest*, the table above begins from minimum latitude (i.e., $\beta = 0$) and increases up to 64 minutes, the maximum inferior eclipse limit, so that the maximum possible eclipse is presented first. The table is tabulated using the actual latitude as an argument, not the nodal elongation. This is useful because it simultaneously gives the northern and the southern latitudes. Totality finishes between 21 and 22 minutes of latitude. The maximum eclipse is 1 hour 53 minutes $\times$ 2 = 3 hours 46 minutes. Unlike in the *Almagest*, two separate columns for greatest and least distance are not given, but rather, for each column, its equation is given, which gives for maxima:

$$21;30 \pm 0;4 = 21;34 \text{ or } 21;26.$$

Compare these figures with those in the *Almagest*: 21;36 and 21;31, respectively. The sum of the diameters can extracted from the table:

$$r_u + r_m = 1;04^\circ$$

Compare this with the 63 minutes given before in the text. Using information from the previous table for maximum lunar and shadow diameter,

$$0;35,47 + 1;33,2 = 2;08,49 \Rightarrow \Sigma r = 0;64,24^\circ,$$

which closely corresponds to the above value.

Column 11 is reproduced from *Almagest* VI, 8 "The Table of Correction": column 3 "the sixtieths," for interpolating between greatest and least distances. However, because al-Khāzinī only gives one distance in his lunar eclipse table, then the inclusion of this table is somewhat superfluous. There are frequent mistakes in copying, but the maximum and minimum are correct.

On the same folio, there is a table for the conversion from digits to area magnitude for a solar and lunar eclipse:

	1	2	3
	Digits	Moon	Sun
1	1	0;30	0;20
	2	1;10	1;0
	3	2;0	1;50
	4	3;10	2;40
5	5	4;20	3;30
	6	5;30	4;40
	7	6;45	5;50
	8	8;0	7;0
	9	9;10	8;20

(continued)

	1	2	3
	Digits	Moon	Sun
10	10	10;20	9;40
	11	11;20	10;50
	12	12;0	12;0

App. Crit.
col 2:1 I 0;7; col 3:3–7 T 2;50, 3;40, 4;30, 5:40, 6;50. In the margin, right next to lines 8–10, are the numbers 8,9,11,12 in a traced rectangle, and below outside the rectangle is 50. The significance of this is unclear.

In this table, with respect to the column for the sun, I and T differ substantially. I is closest to the *Almagest*. There are only three values that differ from the *Almagest*: col. 2:3 *Alm.* = 2;4 col. 3:3 *Alm.* = 1;45, col. 3:5 *Alm.* = 3;40.

Also on the same leaf is one more table entitled "Table for the Inclination of the Obscuration":

	1	2	3
	Digits	I	II
1	0	90;0	
	1	72;30	
	2	65;10	
	3	59;27	
5	4	54;30	
	5	50;14	
	6	46;15	
	7	45;30	
	8	39;2	
10	9	35;42	
	10	32;30	
	11	29;23	
	12	26;28	90;0
	13	23;20	63;37
15	14	20;36	52;21
	15	17;48	43;26
	16	15;1	30;41
	17	12;48	28;40
	18	9;36	22;1

(continued)

	1	2	3
	Digits	I	II
20			
	19	6;55	15;43
	20	20;15	9;43
	21	13;6	3;36
25	21;30	0;0	0;0

Digits column = beginning of eclipse and end
Column I = Inclination (*al-inhirāf*)
Column II = Immersion and clearing

I am unable to find a corresponding table in T.

This is in fact the Arabic equivalent of Ptolemy's *prosneusis*, but it seems to have been misunderstood, or at best, only partially copied. As seen previously, Ptolemy's table of inclinations has four columns of numbers: the argument, the sun: beginning of eclipse and end of emersion, the moon: beginning of eclipse and end of emersion, and the moon: end of partial phase and beginning of emersion. This Arabic version only has the angles of inclinations (*inhirāf*) for the moon.

App. Crit. (from the Almagest *[A]):*
col 2:5 A 54;34; col 2:8 A 42;31; col 2:11 A 32;29; col 2:13 A 26;23; col 2:14 A 23;28; col 2:18 A 12;18; col 3:17 A 35;41; col 3:18 A 28;38; col 3:21 A 9;36; col 3:22 A 3;35; A does not have 21;30 0 0 row

Several differences between the *Almagest* and manuscript I, which are within one minute of each other, might suggest that they had been recomputed, and different roundings used. Similarly, differences such as the reading 48 for 18, or 28 for 23, indicate that this was copied from an earlier Arabic manuscript, because with respect to Arabic numerals there is often confusion between the numbers 3 and 8, and 40 and 10, as they look very similar.

Here, the value of 0;21,30 (which is not in the *Almagest* in this particular table, but is in VI, 8) is the mean value of radius of the shadow (the *Almagest* here has d_u= 0;40 and 0;46 at greatest and least distance respectively).

Parallax Tables

Expressions for parallax are found in several forms:

- parallax for particular climes
- adjusted parallax
- lunar parallax in longitude and latitude
- parallax of two luminaries

The following is a table of parallax tabulated as a function of the zenith distance (ZD) along the altitude circle:

	1	2	3	4	5	6	7	8
	ZD	Solar parallax	Lunar parallax	Equation	ZD	Solar parallax	Lunar parallax	Equation
1	0	0;0	00	0;0	46	2;4	38;57	7;35
	2	0;7	1;54	0;13	48	2;8	4(!)9;54	7;49
	4	0;13	3;48	0;45	50	2;12	41;28	8;3
	6	0;19	5;41	1;7	52	2;16	42;39	8;16
5	8	0;25	7;34	1;30	54	2;20	43;48	8;30
	10	0;31	9;26	1;51	56	2;	44;48	8;43
	12	0;32	11;50	2;12	58	2;23	45;48	8;53
	14	0;42	13;10	2;33	60	2;26	46;47	9;3
	16	0;48	15;0	2;54	62	2;29	47;40	9;13
10	18	0;53	16;50	3;15	64	2;32	48;30	9;22
	20	0;58	18;36	3;36	66	2;35	49;15	9;31
	22	1;3	20;22	3;57	68	2;37	49;57	9;39
	24	1;9	22;6	4;18	70	2;39	50;36	9;46
	26	1;14	23;49	4;40	72	2;40	51;15	9;53
15	28	1;20	25;30	4;59	74	2;42	51;44	9;59
	30	1;25	26;9	5;18	76	2;44	52;15	10;4
	32	1;30	28;46	5;37	78	2;46	52;33	10;8
	34	1;35	30;20	5;55	80	2;47	52;53	10;11
	36	1;40	31;54	6;13	82	2;48	53;9	10;14
20	38	1;45	33;25	6;30	84	2;49	53;20	10;15
	40	1;49	34;11	6;47	86	2;50	53;24	10;16
	42	1;54	36;15	7;4	88	2;51	53;34	10;17
	44	1;58	37;34	7;20	90	2;51	53;35	10;17
	0	0	00	00	0	0	0	0

App Crit.
col 2:7 A 0;37; col 2:12 A 1;4; col 2:20 A 1;44; col 3:6 A 9;27; col 3:7 A 11;19, T 11;20; col 3:10 A 16;49; col 3:16 A 27;9; col 3:18 A 30;21; col 3:20 A 33;24; col 3:21 A 34;51; col 3:22 A 36;14; col 3:23 37;37; col 4:2 A 0;23; col 4:5 A 1;29; col 4:14 A 4:39; col 6:1 A 2;3; col 6:6 A 2;23 (in T and I it is blank; from thereon, T and I are out of synchronization with A by one entry, but in the last three entries are identical to A.; col 7:2 A 40;14; col 7:5 A 43;45; col 7:8 A 46;46; col 7:14 A T 51;11; col 7:16 A 52;12; col 7:17 A 52;34; col 7:20 A 53;21; col 7:21 A 53;29; col 7:22 A 53;33; col 7:23 A 53;34; col 8:5 A 8;29; col 8:6 A 8;42; col 8:20 A 10;16

There are two places in which T has values identical to the *Almagest* (or only 1 minute off) and is different from I; this suggests T may be closer to the original. Notably, both T and I have 53;35 for maximum lunar parallax (cf. *Almagest*: 53;34). The solar parallax column and the first column of the lunar parallax have been reproduced from the *Almagest*, and the next column, labeled "equation," is the fourth column of the *Almagest*, namely,

"the moon's difference at second limit." The remaining five columns, which account for the positions of the moon with variable anomaly and elongation and contain the corrections, have been left out. It seems the compiler of this table has not understood its original function very well.

The argument of the table of parallax is the zenith distance (which goes from 0 to 90°). The argument is in 2° increments—the *Almagest* also has its argument in steps of 2°. The maximum solar parallax is 2;51°, as in the *Almagest*. Column 2 is the moon's parallax at the first limit—it is taken straight from the *Almagest* with a few modifications (i.e., maximum parallax is 53;35 not 53;34 as in the *Almagest*).

Table of Parallaxes for $\phi = 38°$

	1	2 3	4 5	6 7	8 9	10 11	12 13	14 15
	Hours	Cancer 7 20	Leo 7 2	Virgo 6 37	Libra 6 5	Scorpius 5 23	Sagittarius 4 58	Capricorn 4 40
		π_λ π_β	π_λ π_β	π_λ π_β	π_λ π_β	π_λ π_β	π_λ π_β	π_λ π_β
1	8	38 37	44 25					
	7	39 33	45 24	49 15				
	6	41 28	45 19	48 13	49 12	49 15		
	5	39 24	44 16	46 12	48 13	48 46	44 25	38 34
5	4	35 20	39 14	42 12	45 14	45 19	41 59	34 37
	3	29 17	32 13	36 13	38 16	33 23	36 33	28 40
	2	20 14	27 12	28 15	31 20	33 27	28 36	20 43
	1	11 13	14 13	18 18	22 24	23 32	19 39	11 44
	0	0 12	3 15	8 21	13 29	14 36	9 42	0 45
10	1	11 13	7 17	2 25	3 33	4 40	1 44	11 44
	2	20 14	17 21	11 30	5 37	6 43	10 45	20 43
	3	29 17	24 25	18 34	12 41	44 44	19 44	28 40
	4	35 20	30 29	23 38	19 43	20 45	26 43	34 37
	5	39 24	33 34	26 41	24 44	25 45	31 41	38 34
15	6	41 28	32 38	27 43	26 46	27 44		
	7	39 33	31 40	28 44				
	8	38 37	30 40					

App. Crit.
col 3:1 I 34; col 3:17 I 34; col 4:7 I 24; col 9:11 I 34; col 10:12 I 14; col 10:6 I 39; col 11:4 I 16

It has been noted* that the *Īlkhānī Zīj* of Naṣīr al-Dīn al-Ṭūsī (fl. 1250) gives al-Khāzinī's table for $\phi = 38°$, as well as those for climes II, IV and V. He probably correctly relates that this was because Naṣīr al-Dīn's observatory was located in Marāgha, which has a latitude of $\phi = 37;20°$. In addition, the *zīj* of Shams al-Munajjim (fl. 1320), reproduces al-Khāzinī's table for $\phi = 38°$, as well as the *Handy Tables* parallax tables for all climes, and states that it can be used in Tabriz. He has another table for use at Marāgha, which has slightly different entries.

*Kennedy (1956), p. 46.

Table of Parallaxes according to Theon

I includes the following additional parallax tables (not reproduced here):

f. 78v Parallax for first clime ($\phi = 16°$)
f. 78v Parallax for second clime ($\phi = 24;15°$)
f. 79r Parallax for sixth clime ($\phi = 45;22°$)
f. 79v Parallax for seventh clime ($\phi = 48;23$ (or 33?)$°$)
f. 79v Parallax for fifth clime ($\phi = 44;14°$)
f. 79v Parallax for fourth clime ($\phi = 36;22°$)

Table for Solar Eclipses

This table is only found in T. The table is tabulated for minutes of latitude ranging from 0 to 34 minutes, implying that the solar eclipse limit (presumably pre parallax) is $\beta = 0;34°$. Taking Ptolemy's maximum lunar radius and maximum solar radius

$$17;40 + 15;40 = 33;20,$$

from which the 34 may come, rounded up to the nearest minute.

	1	2	3	4	5
	β_m	Digits of the eclipse	Equation	Duration (hours)	Equation
1	0	11 23	0 55	1 6	0 0
	1	11 1	0 55	1 6	0 0
	2	10 39	0 55	1 6	0 0
	3	10 17	0 55	1 5	0 0
5	4	9 54	0 55	1 5	0 0
	5	9 32	0 56	1 5	0 0
	6	9 10	0 56	1 4	0 0
	7	8 48	0 57	1 4	0 0
	8	8 26	0 57	1 4	0 0
10	9	8 4	0 58	1 3	0 0
	10	7 42	0 58	1 3	0 0
	11	7 20	0 59	1 2	0 0
	12	6 58	0 59	1 1	0 0
	13	6 36	1 0	1 0	0 0
15	14	6 14	1 0	0 58	0 0
	15	5 52	1 0	0 56	0 0
	16	5 30	1 0	0 54	0 0
	17	5 8	1 1	0 52	0 0
	18	4 46	1 1	0 50	0 1

(continued)

	1	2	3	4	5
	β_m	Digits of the eclipse	Equation	Duration (hours)	Equation
20	19	4 24	1 1	0 48	0 1
	20	4 2	1 1	0 46	0 1
	21	3 40	1 1	0 44	0 1
	22	3 18	1 2	0 42	0 2
	23	2 56	1 2	0 40	0 2
25	24	2 34	1 2	0 38	0 3
	25	2 12	1 2	0 36	0 4
	26	1 50	1 3	0 34	0 5
	27	1 28	1 3	0 32	0 6
	28	1 6	1 3	0 29	0 8
30	29	0 44	1 4	0 25	0 10
	30	0 32	1 4	0 19	0 15
	31	0 0	1 4	0 0	0 31
	32		43	0 0	0 27
	33		21	0 0	0 22
35	34		0	0	0

As is immediately obvious from the first entry, the values deny the possibility of a total solar eclipse. Again, as in the lunar eclipse table, it is tabulated with respect to the latitude of the moon.

Notes

CHAPTER ONE Heavenly Hide and Seek

1. *Posterior Analytics* II 2 90a15–16.
2. See Crowe (1990) pp. 198–220, esp. 211–13.
3. See Steele (2000a) p. 8 and n. 15, in which he describes his three basic heritages of astronomy, of which those societies included in this study form his western heritage.
4. For details on Chinese astronomy, see, for example, Sivin (1969) and Needham (1974). For Mesoamerican astronomy, see, for example, Aveni (1980).
5. Pingree (1990) p. 129.
6. See Bowen and Goldstein (1988).
7. Pingree (1990) p. 133.
8. *Republic* 529a and 616b and *Timaeus* 39b and 40a.
9. This was translated from Naṣīr al-Dīn's Arabic version by Indian scholar Jagannātha at Jayapura in 1732. See Pingree (1981) p. 31.
10. For a thorough overview of this genre, see Pingree (1981).
11. Pingree (1989).
12. Pingree (1992).
13. Pingree (1973).
14. Pingree (1976).
15. Pingree (1971).
16. Pingree (1978a) esp. pp. 362–63.
17. Ibid., p. 361.
18. For a detailed account, see Pingree (1973).
19. See Pingree (1970) and (1968), respectively.
20. Gutas (1999).
21. Pingree (1978a) p. 361.
22. See, for example, Plofker (2005) and Minkowski (2002).
23. Lloyd (2004) pp. 82ff.
24. In the context of eclipses, see Steele (2000) pp. 21–84.
25. Steele (2000) esp. p. 58.
26. See Ptolemy's discussion of this with respect to the problem of determining the apparent and true angular diameters of the planets in *Planetary Hypotheses* I 5 and 7.

27. See Lloyd (1982) pp. 146-47.

28. Lloyd (1982) esp. p. 130, n. 3.

29. Lloyd (1982).

30. Bowen and Todd (2004).

31. For further discussion of this mode of evidence, see Bowen and Todd (2004) pp. 10–15.

32. *Republic* VII 529: "astronomy compels the soul to look upward and leads us from this world to another."

CHAPTER TWO The Rudiments of Alignment

1. Literally, "syzygy" means yoked together, which seems only to refer to conjunction. However, the yoking could be imagined to be when they are in a straight line, whether the moon is adjacent to the sun or 180° from it.

2. There exist numerous accounts of the mechanisms behind eclipses. See, for example, Steele (2000), pp. 9–15, Evans (1998) pp. 45–46, Harrington (1997) pp. 1–18, and McEvoy (1999).

3. One cannot observe a "mean" motion, only calculate it using the relationship between the number of repetitions and a known time interval.

4. Neugebauer (1975) p. 991 n. 8.

5. See Steele (2000a) p. 14. He notes that when the sun is near the horizon the westward rotation of the earth has only a small effect on the speed of the eclipse shadow, but in contrast if the sun is overhead at the equator, it can slow the shadow's passage by about 50%.

6. See also Aaboe (2001) p. 16, Neugebauer (1975) p. 129, and Steele (2000a) p. 11.

7. *Meteorology* 2.5,362a32.

8. See his *Geography* or *Almagest* II, 13 for a fuller account.

9. For an excellent survey of the emergence and development of trigonometry, see van Brummelen (2009).

10. See Toomer (1973).

11. See Plofker (2008) pp. 49–52.

CHAPTER THREE An Accountancy of Appearances

1. This refers to the format in which the omens were expressed. The protasis statement ("If . . .") contains the sign that was connected to a prediction ("then . . .") statement, known as the apodosis.

2. Horowitz (1998) p. xii.

3. See in particular KAR 307 (= VAT 8917) in Horowitz (1998) pp. 3–19.

4. Ibid., p. 330, KAR 307 plate 1 ln. 34.

5. Ibid., p. 318.

6. Ibid., p. 334.

7. See ibid., p. 334, for further examples.

8. Such communications, or "signs" were referred to by the term *ittu* (Akkadian) or GISKIM (Sumerian).

9. Literally "its loosening," that is, the loosening of the evil.

10. See, for example, Parpola (1993) p. 347.

11. Parpola (1983) pp. xxii–xxxii.

12. For an excellent and thorough account of the sources, see Steele (2000a) pp. 21–34.

13. Steele (2000a).

14. For general overviews, see Rochberg (2004) pp. 66–78, and Hunger and Pingree (1999) pp. 6–22, for example.

15. "Plausible" here is meant to convey that modern scholars are uncertain as to what phenomena are being described. It may be that the references are to real observable phenomena, but modern scholars can only guess what they might be.

16. Rochberg-Halton (1988).

17. Van Soldt (1995).

18. And none (with perhaps the arguable exception of Tablet 63) are intended to record actual datable occurrences.

19. Recall that the length of the lunar (synodic) month is determined by successive conjunctions and hence its usefulness in eclipse prediction–solar eclipses occur at conjunction and lunar eclipses at opposition, which are the beginning and the middle of this interval respectively.

20. In fact date distributions can be seen, as set out by Rochberg-Halton (1988) p. 38.

21. Perhaps these extra days (19–21) were significant in the hemerological (day-by-day) texts for a different reason and their significance was transferred to *EAE*.

22. See Rochberg-Halton (1988) pp. 36–63.

23. Magnitude in this context always refers to the "angular magnitude," that is, the size of the angle against the backdrop of fixed stars that the moon covers while eclipsed, and not the brightness, as it is commonly intended to refer to in modern astronomy.

24. This refers to the traditional division of the sky into three general bands, introduced and commonly invoked in the ancient Near East. Anu, Enlil, and Ea are deities of the Sky, Earth, and Ocean respectively. The band of Anu is the vertical band of the eastern horizon, those of Enlil and Ea lie to the north and south, respectively.

25. For the transmission of these aspects to the Hellenistic world, see Williams (2008).

26. A similar connection also seems to be referred to in the Assyrian Letters and Reports. See the next section for details.

27. Rochberg-Halton (1988) p. 283.

28. This term can also refer to the amount of time after the eclipse in which the bad effects of the eclipse are still effective.

29. See Steele (2000b) pp. 425–428 and Hunger and Pingree (1999) pp. 116–123. The Letters and Reports are documents sent by the astrological scholars to the Assyrian kings of the eighth and seventh century BCE, that is, during the reigns of Esarhaddon (681–669 BCE) and Assurbanipal (669–648 BCE). The Letters (Parpola [1993]) are the answers of the scholars to questions from the king about omens, including celestial ones. The Reports (Hunger [1992]), in contrast, are generally unsolicited descriptions of future significant astronomical events. For further information on the scribes themselves, see Oppenheim (1969), Hunger (1992), and Parpola (1993) esp. p. xxvi.

30. See, for example, for lunar VIII:4, VIII:46, VIII:250, VIII:279 (16th day), VIII: 300 VIII:336, VIII:387, VIII:447, VIII:535, X:137, X:147, X:149, X:168, X:351 and for solar VIII:3, VIII:47, VIII:305, VIII:384, X:45, X:159, X:363, X:377 (mentions keeping watch on 28th, 29th, and 30th). Consider also X:45 with the astrologer watching on the 26th for a solar eclipse.

31. Although the exact wording of the apodosis is not the same, the protasis (month III from the 1st to the 30th day) is found in *EAE* (see Rochberg-Halton [1988] pp. 114 and 134, omen §VI.11) as well as the following protasis (if an eclipse occurs not at its calculated time) (*ibid.*, §VI.12).

32. See also Steele (2000b) p. 426.

33. Steele (2000a) p. 77.

34. See Parpola (1983) p. 51, who claims "in addition to the simple rule-of-thumb that the moon may be eclipsed every sixth synodic month, the scholars of this period certainly recognized the 47 month period and probably also the 18 year Saros."

35. Steele (2000a) p. 77, is justifiably more cautious, due to the lack of any direct evidence, concluding "it may well be that these eclipse periods were in use, but we do not have the evidence to be certain."

36. See also Schnabel (1924a).

37. Although Parpola read 4, Steele comments ([2000b] p. 427 and n. 19) that it would make more sense to read this as 5, and Walker has confirmed that this reading is indeed possible.

38. See Aaboe et al. (1991) Table 5 and n. 17; Steele (2000a); Britton, Private Notes; Huber (2004).

39. The first potential pair separated by five months is E.P 16/17 SC 21. The first of the pair had a magnitude of only 0.1 d, while the second was only marginally visible, if at all, before sunrise.

40. This eclipse observation is also referred to in two other reports: VIII:300 and VIII:336. Hunger and de Meis (1998) pp. 54–55 comment on the date proposed by Parpola (1983) −677 May 22 that the details are not in complete agreement with the actual event and suggest that −652 July 13th is also a plausible candidate.

41. This interval is about five hours—more than the longest possible eclipse.

42. See VIII:250, VIII:251, VIII:320, VIII:388, VIII:487, VIII:502, X:26, X:71, X:78, X:358, X:377.

43. Within the career of Nabû-ahhe-eriba (Parpola [1983] p. 68 estimates 680–663) three eclipses occurred within the morning watch and Parpola, discounting two, gives October 15 667 as a possible date. However, these dates must be treated with caution as Hunger and De Meis (1998) point out.

44. Dated by Parpola as 669-II-29.

45. See VIII:46, VIII:94, VIII:321, VIII:344, VIII:409, VIII:447, X:158, X:358.

46. When the moon and the sun are *both* above the horizon, the moon is not yet 180° away.

47. (2000a) p. 78.

48. See Pannekoek (1917) and Hunger and Pingree (1999) p. 185.

49. A connection that may have its antecedents in *EAE*; see the previous section.

50. In both reports, the day of disappearance is denoted by the Sumerogram UD.NÁ.ÀM (*ūm bubbuli*) or *biblum* + *abālu*. *Biblu* is a technical term in Akkadian for the day of the disappearance of the moon.

51. For a thorough description of these see, for example, Steele (2000b) p. 430.

52. Steele-Stephenson (1997) p. 119.

53. Neugebauer proposes (Neugebauer [1975] pp. 319–322 and Toomer [1978] p. 216) that Hipparchus also arranged these lists of eclipses observed by his Mesopotamian predecessors into a useful order; however, this is speculative.

54. See Hunger and Pingree (1999) pp. 181-82.

55. See Sachs (1955) and Hunger and Pingree (1999) p. 181.

56. The tablet breaks off before the list ends, and the last fragmentary reference is to Antigonus year 2, i.e., -314/3.

57. For convenience, modern scholars denote these simply by the numbers 7 and 8 and combine them together in strings, such that the expression 7-8-7 denotes six 6-month EPs followed by one 5-month EP, then seven 6-month EPs followed by one 5-month EP, then six 6-month EPs followed by one 5-month EP.

58. See Aaboe et al. (1991) n. 17; the only possible candidate is one after -375 but it was small, probably not even visible.

59. See Britton (1989) for the initial list.

60. See Hunger and Pingree (1999) pp. 183–188 and Brack-Bernsen and Steele (2005).

61. See Aaboe (1972), first published by Neugebauer.

62. Aaboe (1972) p. 111; Aaboe appears to conclude that it was such sawtooth functions that were the basis for the layouts of the Saros schemes, either the ones they were using in the Diaries, or the mathematical ones of the Saros Canon that Steele has reconstructed. Steele clearly shows that the layout in question will simply be arrived at by distributing the five-month intervals in the simplest way possible, that is evenly, in an 8-7-8-7-8 pattern (which is a result of Steele's theoretical layout, as opposed to Aaboe's 8-8-7-8-7).

63. Aaboe (1972) also gives two other examples of similar schemes. One, from China, explicitly states the relation 23 EPs in 135 months and describes a method that seems to imitate the Babylonian scheme. The text also states that it deals with oppositions, and hence applies to lunar eclipses. However, Aaboe notes that later in the text it is mentioned that the rule could also be applied to conjunctions. Another relevant text is a Demotic Papyrus (P. Carlsberg 31), which includes a scheme that is based on 22 EPs in 129 months. In fact the text says "years" not months, but Aaboe takes this to be a mistake, even though it is done repeatedly. This is one eclipse and six months shorter than the 23 135 cycle.

64. Aaboe et al. (1991) pp. 43–71 esp. p. 49.

65. For problems with this analysis, see Hunger and Pingree (1999) p. 195.

66. Hunger and Pingree (1999) pp. 184ff.; for the 47-month eclipse period, see Parpola (1983) p. 51 n. 87; see also van der Waerden (1968) pp. 122ff.

67. See also Beaulieu-Britton (1994). Huber and De Meis (2004) p. 188 list it as a so-called noneclipse, magnitude = −.60.

68. Bealieu and Britton suggest the prediction was made on the basis of the "original Saros cycle."

69. Cited from Hunger and Pingree (1999) p. 186.

70. According to Huber and de Meis (2004) p. 211.

71. Aaboe et al. (1991) pp. 25–31 esp. 28–29.

72. See Steele (2000a) p. 443, and n. 49 for comments on this correction.

73. Aaboe and Sachs (1969) Texts B, C, and D pp. 11ff.

74. Britton (1989).

75. Britton (1989) and Steele (2000b) pp. 442–443. As a parallel example, sources from China support the analogous treatment of solar eclipse theory. See Aaboe (1972) p. 113 and Sivin (1962) p. 46.

76. See Pannekoek (1917) for the initial observations of this; his ideas are set out in Hunger and Pingree (1999) p. 185.

77. And there is reason to suppose that the tradition began almost a century before this. See Steele (2000a) p. 429.

78. Hunger, Sachs, and Steele (2001) p. 390.

79. This is Steele's term to represent texts that compile lunar eclipse possibilities in Saros cycles, typically those that use alternative schemes for predicting eclipses. Rather than being the articulation of "theory," they too, like the texts in the other categories are simply the mathematical extension and accountancy of patterns, albeit sophisticated, to predict eclipses. They do not offer illumination beyond the appearances of the phenomena, nor have explanatory intent.

80. Goal-year texts are texts which compile material that was used to make predictions for a specific "Goal" year.

81. A category thus named by Steele (2000b) p. 430.

82. See Sachs and Hunger (1988) p. 23 and also Steele (2000b) p. 430 and n. 34.

83. *šamáš* and *sin* refer to the sun and the moon, respectively and $AN.KU_{10}$ is a Sumerian word which can function as either a noun meaning "eclipse" or a verb meaning "is eclipsed."

84. Steele (2001/2).

85. and sometimes even BE DIB (ibid., p. 108).

86. Steele also points to the similarities in terminology of BE and BAR used in the Diaries and as they appear in the ACT texts (see below).

87. Steele and Stephenson (1997) and Steele (1997).

88. Steele (2000a) p. 28.

89. Walker (1997).

90. Steele (2000a) p. 80.

91. See the two texts discussed by Aaboe et al. (1991).

92. Steele (2000a) p. 80 and also section 2.7.

93. These have been compiled and edited by Neugebauer (1955) in three volumes.

94. Neugebauer (1955) pp. 41–43.

95. This literally means the tabulated positions of a phenomenon day to day, but the unit of tabulation was not restricted to days; it could be different intervals as well, such as month by month.

96. Some tablets contain up to 18 columns (e.g., ACT No. 122).

97. For a complete list of the columns and their functions see Neugebauer (1955) p. 43 or Hunger and Pingree (1999) p. 222.

98. See also Neugebauer (1955) p. 39 for a comprehensive list of units of measurements and their conversions.

99. For a thorough treatment of column E, see Aaboe and Henderson (1975) pp. 196ff.

100. The modern term "function" will be invoked here to refer to the sequence of numbers presented in the ephemerides, tabulated with respect to the line number.

101. For details, see Neugebauer (1983) pp. 47–49.

102. In order to mathematically unravel the inherent structure of column E, Aaboe and Henderson (1975) found it useful to consider E in terms of a "center function," E_c so that E is able to be defined more precisely in terms of nodal elongation η. Their function, E_c, has extrema of $\pm 6,0$ŠE $(=5°)$ and meant that E could be viewed as a continuous, piecewise function of η.

103. The following four logograms are representations of the following verbs in Akkadian along with their literal meanings: LAL *maṭû* to become less, to lessen, to diminish or *šaqû* to be high, be(come) elevated, to rise; U (=BUR_3) *palašû* to perforate to bore, drill; Sachs (1967) p. 203 mentions the identification of BUR_3 as "low"; NIM *elû* to go up, ascend; SIG *šaplû* to be(come) deep, to be(come) low. There is a general trend for LAL and BUR_3 to be used in ephemerides and NIM and SIG in procedure texts.

104. See Neugebauer (1975) pp. 190–193.

105. In fact such a procedure is outlined in BM 41004 (Text E).

106. Hunger and Pingree (1989) Tablet II i 1 (p. 70): DIŠ KASKAL ^{d}*Sin* DU-*ku* ^{d}UTU DU-*ak* (The sun travels the [same] path the moon travels).

107. ACT 200 section 9 obv.11, p. 194.

108. It was previously thought that the value seen in Atypical Text E of 1;40 was a crude approximation for the nodal motion derived from the Saros (see Aaboe and Henderson [1975] p. 200 n. 16) based on the assumption that the node exactly completes one 360° rotation in 18 years of 12 "months": 360/18/12 = 1;40° per month. However, Brack-Bernsen has since shown that this cannot be understood in this way (private communication). For further study on the lunar nodes, see Neugebauer (1975) p. 549.

109. The nodal elongation η for conjunctions can be derived as follows:

$$\eta = B(n) - \lambda_N(n)$$

where $B(n)$ is the longitude of the moon at syzygy and $\lambda_N(n)$ is the longitude of the node, and on the assumption that the nodes recede (go backward) uniformly the following expression holds:

$$\lambda_N(n) = \lambda_N(0) - n \times 1;33,55,30.$$

The two differences of column E can be derived in the following way (see Aaboe and Henderson [1975] pp. 199–200). The monthly change in column B (a step function to

compute solar longitude) is

$$\Delta B = \begin{cases} 30 & \text{fast arc} \\ 28;7,30 & \text{slow arc} \end{cases}$$

and as $\Delta\eta = \Delta B + \Delta\lambda$ (where $\Delta\lambda$ is the mean monthly progress of the moon),

$$\Delta\eta = \left\{ \begin{matrix} 30 \\ 28;7,30 \end{matrix} \right\} + 1;33,55,30 = \begin{cases} 31;33,55,30 & \text{on fast arc} \\ 29;41,25,30 & \text{on slow arc,} \end{cases}$$

$$\Delta E = 4\Delta\eta = \begin{cases} 2,6;15,42 & \text{on fast arc} \\ 1,58;45,42 & \text{on slow arc.} \end{cases}$$

110. Private communication.

111. Compare this with the value given in Neugebauer (1975) p. 517: 29;6,19,0, 57, . . . which is clearly a simple arithmetical error.

112. ψ is a particularly fertile function, as it has provided much material for many related offspring, variously represented by historians as ψ',ψ'', $\psi*$, $\Delta\psi$, etc . . . , as they seek to understand the underlying structures that make it up.

113. As noted previously this indicates a change in orientation from south to north of the sun's path, or vice versa.

114. Neugebauer (1955).

115. See BM 41004 (Text E) line 11.

116. There has been a lot of discussion as to the reading and meaning of HAB-rat, based on its use in other sources. The sign for HAB is equivalent to the Sumerian word *nigín* (LABAT 483), which stands for, among other things, the Akkadian verb and its derivatives *sahāru* (to go round). It has been argued that HAB-rat should be read as hap-rat = hapratu, from the verb *hapāru* (to surround) and not as an ideogram. However the evidence against this interpretation is that simply HAB occurs without -rat (for example, often in ACT procedure texts). It could be argued, as in many instances in the ACT material, that it is just a single sign abbreviating a word, on many occasions however. *nigín* is seen most frequently as *sihirtu* (circumference). HAB can be read *sihirat* (*sihirtu* in the construct state) in which the -rat serves as the phonetic complement.

117. That HAB looks like a celestial body itself (as much as can be achieved by straight lined wedges) may be significant.

118. In this context, it means "excluded."

119. For verification, see the table in Neugebauer (1955) p. 163 (compare ACT 151 and ACT 152 for such usage, although only BE and HAB appear). Unfortunately the range of values of ψ in this tablet (from 1,6 to 30) is distinct from any other instance of ψ, and thus it is difficult to draw confident conclusions from it.

120. Bearing in mind that there are only two surviving references of BE in the Diaries in this context. See Steele (2001/2) Table 4 p. 111.

121. Neugebauer (1955) p. 277.

122. Steele (2000a).

123. See Steele (2000a) tables 2.4, 2.5, 2.7, and 2.8.

124. Steele (2000a) p. 69 and p. 73; see also section 2.8.

125. ACT 15 obv. VI 4: M = *sig* 28 13,1 *šú*.

126. ACT 122 obv. XII 17: M = 28 14 *šú*.

127. See Steele (2000a) table 2.8.

128. ACT 100 obv. VII 16: M=28 1,7 *šú*.

129. ACT 101 obv. VIII 3: M = 29 41 *du*.

130. ACT 130 rev. II 2: M = 8 *kur*; although this may not be related to this date which is on the obverse.

131. ACT 130 rev. II 5: M = 1,19 *šú* (This date is incorrectly printed in Steele [2000a]).

132. ACT 3b obv. III 3: M = *zíz* 28 1,11,13 *šú*-tentative.

133. ACT 5 rev. VI 1: M = 29 5,9,59 *šú*.

134. ACT 120 obv. -IV 6: this is concluded from ψ; M = 28 23 *me nim*.

135. ACT 120 obv. IV 11: M = 28 1 *šú*.

136. ACT 11 VI 12: M = *zíz* 29 3,30,23 *šú*.

137. ACT VI 17: M = [*šú* x] 49,54 *šú*.

138. See Steele (2000a) table 2.7.

139. ACT 135 obv. VI 11: M= 22 *šú*.

140. ACT 135 obv. VI 12: M = 1,17 *du*.

141. ACT 135 obv. VI 14: M = 1,31 *nim*.

142. ACT 135 obv. VI 18: M = 20 X; Neugebauer computes 1 *du* which would fit better.

143. ACT 135 rev. VI 6: M = 1,18 *šú*; Neugebauer computes 1,19 *šú*.

144. ACT 120 rev. IX 10: M= 14 1,21 ge_6.

145. ACT 102 rev. VIII 6: M = [28] 24 *nim*.

146. ACT 4 obv. III 10: not predicted by column E; instead there is an EP for the previous month XII.

147. ACT 135 obv. VI 15: M = 15 *kur*.

148. ACT 135 obv. VI 24: M = 27 *kur*; Neugebauer computes 35; compare ACT 102 rev. VIII 17 which also has 35.

149. ACT 135 obv. VI 26: M = 40 *kur*.

150. ACT 135 rev. VI 9: M = 40 *du*; Neugebauer computes it as 15 *du*.

151. Using Huber's computer programs which recompute the circumstances of lunar and solar eclipses taking into account the Δt function.

CHAPTER FOUR Beyond the Appearances

1. Iliad 8.485–486.

2. Although the explanation that this simply refers to sunset is more likely.

3. See Aristotle, *de Caelo* 2.14. This observation, as Neugebauer points out in (1975) pp. 1093–94, is quite inconclusive.

4. *Almagest* I 1 7, as translated by Toomer (1984).

5. See, for example, Heath (1932) pp. xviii–xix, Neugebauer (1975) p. 604, Panchenko (1994), and Stephenson and Fatoohi (1997).

6. Diels (1929) pp. 349 and 360.

7. Hicks (1950) I, 24.

8. Diels (1929) p. 559: Hippol. *Refut.* i 6, 3.

9. Diels (1929) p. 354 (Aet. ii.24 3).

10. A related concept, that of the mock-sun (*prati-sūrya*), appears in Indian texts. See Pingree (1981) p. 71 as cited from the *Meghamālā*.

11. Aet. ii 29 4.

12. σελήνης αὐτὸν ὑπερχομένης. See Aet. ii 24, 6.

13. *Almagest*, I.1 6: 17–21.

14. See Heath (1981).

15. See Heath (1981) pp. 412–14.

16. For details, see, for example, Evans (1998) pp. 205ff.

17. *Almagest*, IX.

18. See Toomer (1978).

19. For details on his life and work see Toomer (1978) pp. 207–224 and Neugebauer (1975) pp. 274ff.

20. See Toomer (1978) p. 208 and n. 8 and van Brummelen (2009) pp. 34ff.

21. For objections to this, see Toomer (1978) p. 209, and for a partial vindication see Duke (2005).

22. See van Brummelen (2009) p. 43.

23. The idea of measuring the diameter of the moon with respect to the size of its own orbit was taken on in a slightly different format by the Indian astronomers. They gave formulas and rules for the apparent diameters by employing the inverse proportionality between velocity and diameter. See chapter 5.

24. Opera II p. 153, 18–20; see Neugebauer (1975) p. 313.

25. These ratios are also found in Tamil eclipse computations. See Neugebauer (1952) p. 272 (3) and also (1975) p. 313.

26. *Almagest* V, 7.

27. See, for example, BM 41004 (Text E) and also the ACT procedure texts. The knowledge of latitudinal deviation is seen in phrases such as "the path of the moon is 12°," which suggests a method in which it is measured against a background of fixed stars.

28. Kugler (1900).

29. See Toomer (1978) pp. 218–9.

30. A term invoked by classical authors. In fact, the Chaldeans (from the Babylonian *kaldû*) were a nomadic tribe who inhabited the marshes of southern Iraq. See Robson (2008) pp. 269–70.

31. It has been suggested that he used works connected with these individuals for the computations of lunar eclipses (see Neugebauer [1975] p. 263), but this is disputed.

32. For details see Neugebauer (1975) pp. 610–612. For more references see Pingree (1997).

33. *Anonymi fragmentum de revolutionibus lunae* from Paris gr. 2841. For an edition, English translation, and commentary see Jones (1990).

34. See Sidoli (2004).

35. See Toomer (1978) p. 210 and Sidoli (2004).

36. See van Brummelen (2009) pp. 166–172.

37. Commentary on Aratus II,4 as cited in Toomer (1978) p. 210. For discussion of this phrase, see Sidoli (2004) p. 73–75.

38. See *Zīj al-Sanjarī* by al-Khāzinī (unpublished).

39. For details on Hipparchus and his work on the distances of the sun and moon, see Toomer (1974).

40. Neugebauer (1975) pp. 322–325.

41. *Almagest* V, 5. An example of application of parallax in longitude *Almagest* V,5 from observation of the moon (-126 May 2). The correction of the observed longitude is:

$$\pi_\lambda = 21;40 - 21;27,30 = 0;12,30,$$

which Neugebauer (1975) p. 92 has shown to be close to that of Ptolemy (= 0;15,53) assuming the simple lunar theory.

42. See Neugebauer (1975) pp. 323–24.

43. This is distinct from the value found in Ptolemy (63;48°) by two degrees. Rome (1931) p. 152 n. 2 notes that this value is given in all the manuscripts.

44. This is reminiscent of Menelaus' naming of a spherical triangle as *trípleuron* in his formal definition—although of course it is not at all certain that this is the word that Hipparchus would have used.

45. Rome (1931) p. 154 lines 9–10.

46. It has been suggested by Toomer (1978) pp. 215–16 and others that there might be some resonance from Hipparchus' procedure for the computation of parallax affecting those set out in Indian texts. However, there are some fundamental differences between Hipparchus' approach (as far as it can be reconstructed) and the Indian one. One vital element for the Indian computation of parallax is the nonagesimal. It is the distance of the luminary from this point that determines the longitudinal parallax, and not the distance from the zenith, as seems to be the assumption of Hipparchus. See the next section.

47. Pliny *Nat. Hist.* II, 57.

48. As has been seen, the Babylonians were experimenting with the placement of the five-month interval, and in ACT texts, would highlight the occurrence of such an interval in their lists of numbers with the phrase "5 itu."

49. For discussion on his date, see Bowen and Todd (2004) pp. 1– 4.

50. The eclipse is March 14 189 BCE. See Neugebauer (1975) p. 316 n. 9. This eclipse was recorded in Mesopotamian sources: the Diaries (see Steele [2000a] p. 62) record it as occurring 30° after sunrise, with 15° from first contact to maximum phase, and 15° from maximum phase until fourth contact. ACT 102 (rev. VIII, 6) predicts mid-eclipse to occur 24° after sunrise.

51. See, for example, Ptolemy's demonstration of the first simple anomaly of the moon using a pair of eclipse triples, ancient and modern (*Almagest* IV, 6).

52. Pliny *Nat. Hist.* II, 57.

53. See Aaboe (1955).

54. He also notes that they tripled this relationship to get a period with an integer amount of days, and says they called this the "exeligmos."

55. Toomer (1984) p. 176 n. 10; Neugebauer (1955) p. 73; Aaboe (1955) p. 124. This period relation gives the ratio 1,30,58/1,38,43 = 0;55,17,22,8, . . . so that the length of a draconitic month is

$$29;31,50,8,20 \cdot 0;55,17,22,8 = 27;12,43,52,27,\ldots^{d}.$$

56. At the end of this chapter, he gives calculations based on his own observations with the instrument for the maximum inclination of the lunar orbit with respect to the ecliptic (i). Ptolemy uses measurements of the moon at the summer solstice (Cancer 0°) when it was, according to him, near its northern limit. Cleverly, this situation corresponded to the moon being almost at the zenith (with the zenith distance $\zeta =$ 2;7,30 being very close to zero), so that parallax could be ignored. As Neugebauer notes (1975) p. 101, using the instrument to make a "reading" of 2 1/8 is fairly contentious. The reading on the instrument corresponding to crdζ with $\zeta =$ 2 1/8 would have given the value of 2;13,20, a value which is unlikely to have been made by observation. More likely could have been crd$\zeta = 2;15$, giving a value $\zeta = 2;9$, which is nearer to 2 1/6. Probably this figure was meant to be a convenient number to give the desired result $\beta_{\max} = 5°$, although use of 2 1/6 gives 4;57 which is also acceptable to round up to 5°. Factoring in the terrestrial latitude (ϕ), the obliquity of the ecliptic (ϵ), and the zenith distance (ζ), he computes the inclination of the lunar orbit:

$$\begin{aligned}\beta_{\max} &= \phi - \epsilon - \zeta \\ &= 30;58 - 23;51 - 2;7,30 \\ &= 4;59,30 \\ &\approx 5°.\end{aligned}$$

57. See Neugebauer (1975) p. 100ff.

58. He sets out four columns (c_3, c_4, c_5, c_6) and three additional columns (c_7, c_8, c_9) for interpolation. For further details, see Neugebauer (1975) pp. 112–115.

59. The organization of the mean syzygies into 25-year periods is distinct from the organization of his solar and lunar mean motion tables, which are tabulated in 18-year increments. However, Ptolemy has selected this arrangement purposefully. A 25-year increment, a year here being equal to an "Egyptian" year of 365 days, has almost certainly been selected, as Neugebauer observes ([1975] p. 119) because

$$25 \text{ years} \approx 309 \text{ synodic months}.$$

60. From the *Almagest* VI, 2–3

$$\begin{aligned}309 \text{ synodic months} &= 309 \times 29;31,50,8,20 \\ &= 2,32,4;57,12,55^{d} \\ &= 2,32,5^{d} - 0;2,47,5^{d} \\ &= 25^{y} - 0;2,47,5^{d}.\end{aligned}$$

Thus the differences between sequential entries in the 25-year table in the second column "Days of Thoth" are only 0;2,47,5. Neugebauer (1975) p. 119 gives on two occasions the value 2,57,5—most probably an editorial error.

61. Compare with Aristarchus' proposition 8.

62. Neugebauer (1975) p. 104 and n. 4.

63. Ptolemy gives as his first pair the following eclipse with date, magnitude (maximum being 12), mean anomaly $\overline{\alpha}$, and the true argument of latitude of the moon ω'.

Eclipse	Date (year/month/day)	Magnitude (digits)	$\overline{\alpha}$	ω'
1	127 Nabonassar III 27/28	3	340;7	80;40
2	225 Nabonassar VII 17/18	6	28;5	262;12

64. The second pair he gives as follows:

Eclipse	Date (year/month/day)	Magnitude (digits)	$\overline{\alpha}$	ω'
3	574 Nabonassar VII 27/28	7	163;40	98;20
4	607 Nabonassar V 2/3	3	178;46	280;36

65. For mathematical validation of a modern sort this can be derived by treating the pair of relationships like simultaneous equations, subtracting the second from the first:

$$\begin{aligned} (d_u + d_m) - b_1 &= \mu_1, \\ (d_u + d_m) - b_2 &= \mu_2, \\ b_2 - b_1 &= \mu_1 - \mu_2. \end{aligned}$$

66. In another section in fact he further rounds this figure to 12. See *Almagest* VI, 10 (Toomer [1984] p. 311).

67. As Neugebauer shows, we can find the time t it takes to travel from the mean position to the true: $t = (d + c_m)/v_m = (d - c_s)/v_s d = (c_s v_m + c_m v_s)/(v_m - v_s)$. Substituting in the approximations $v_m = 13$ and $v_s = 1$, we find d to $d = (2;23.13 + 5;1.1)/(12) = 3°$. See Neugebauer (1975) pp.125–26.

68. Climes, derived from the Greek *klima* (region). In early Hellenistic times the known world was divided into seven regions, or climes, each delimited by a parallel of terrestrial latitude. See *Almagest* II, 13, for example, for the details of the seven different climes.

69. See Neugebauer (1975) pp. 126–28 for the details.

70. See Neugebauer (1975) pp. 129-34 for details.

71. See Neugebauer (1975) p. 134.

72. From Archimedes *Measurement of the Circle* in Heath (1912) pp. 91–98.

73. As will be discussed in the next chapter, this concept is also preserved in the Indian tradition (see below), which names the angle the *valana* and, as Ptolemy does, splits it up into two separate components, the sum or difference of which determines the total angle. The Indian version of these two angles is quite distinct from the two as set out by Ptolemy.

74. Neugebauer (1975) p. 143. See also Pappus and his commentary to *Almagest* VI: Rome (1931) p. 309.

75. See Tihon and Chabás (1993) p. 123.

76. For issues of authorship, see Neugebauer (1975) p. 969 and Tihon (1999) particularly pp. 357-58.

77. Namely, the epoch of Philip (−323).

78. The most reliable copy of the *Handy Tables* extant is Vat. Gr. 1291 [V]. This contains tables specifically calculated for use in Byzantium—most notably in this context, its tables of parallaxes. There also exists a somewhat careless edition by Halma [H]. A large comprehensive edition is in preparation by A. Tihon.

79. For example, the second entry would than be 5;30/11;30 = 28;41; cf. 29;20.

80. For example, as Ptolemy gives in VI 7, three digits = 0;23,30 (3/12 of the sun's diameter is 7;50; cf. *Handy Tables* [HT] which give 0;24).

81. Greatest 0, 12;32, 17;19, 20;43, 23;27, 25;38, 27;8, 28;29, 29;32, 30;20, 30;54, 31;13, 31;20 and least 0, 12;57, 17;54, 21;28, 24;14, 26;27, 28;16, 29;45, 30;55, 31;51, 32;33, 33;1, 33;16, 33;20 + 2,0.

82. Rounded up correctly in col 5:10 (34,53) but in all other second numbers the 0;0,30 is dropped off and rounded down.

83. For comments, see Neugebauer (1975) pp. 990ff.

84. See Kennedy (1956) p. 35 for term used in Islamic astronomy.

85. In order to extract the lunar parallax from the combined value, one must increase the value by 1/20 of the amount, which is Ptolemy's value of about 0;3° for the maximum of the solar parallax against about 1° for the moon. For further details, see Ptolemy's introduction to the *Handy Tables*, Heiberg (1903), II p. 175, 19–21 and Theon's commentary Halma (1822) HT I p. 68.

86. Neugebauer (1975) p. 991.

87. Recent studies, such as that of Tihon and Chabás (1993) pp. 126–27, have recomputed the entries following the methods outlined by Ptolemy and Theon and concluded that they are related.

88. Tihon and Chabás (1993) p. 125.

89. Ibid.

90. Kennedy and Kennedy (1987) pp. 93–94 have values for the latitude of Byzantium (under Constantinople) ranging from 43;00 to 45;00. Two manuscripts do give 43;05 Halma (1822) and Geography of Ptolemy (2000). Berggren and Jones (2000) pp. 29 and 171 give $\phi = 43;30$ and longest day = 15;15 equinoctial hours.

91. Tihon and Chabás (1993).

92. See Tihon in Swerdlow (1999) pp. 357ff.

93. Tihon comes to some critical conclusions in her assessment of Theon's commentaries (ibid., p. 357). She argues that he did not introduce any changes into the *Handy Tables* nor did he have any extra information concerning the structure of the tables.

94. For a detailed discussion see Tihon (1985) pp. 118–42 and Neugebauer (1975) pp. 994–96.

95. Largely because of the following statement found in Lydus *De Ostentis* 9: Ὁ δέ γε Ῥωμαῖος Καμπέστριος ταῖς Πετοσιριακαῖς ἀκολουθῶν παραδόσεσι . . . [The Roman Campestrius who follows the Petosirian traditions . . .].

96. Because of the many similarities, this chapter of Hephaestio's text is to be paired with a fragment published by Boll in *CCAG*, henceforth called *Excerpt* text.

97. [month] stands for month name, which can represent any of the thirteen possibilities (twelve months + one intercalary month).

98. See also *Apot.* I 25 13 and Lydus 9d p. 27 ln. 7ff.

99. See, for example, *EAE* 20 I B n. 9, 20, text e 14 and n.3, g 10, and n. 9.

100. This may represent Egyptian influence. See Parker (1959).

101. Because this interval is expressed as "3 months and 10 days" (ITI.3.KAM UD.10.KAM), the interval of three months is easily extracted by dropping off the ten days.

102. *Excerpt* text Virgo, p. 136.

103. *Excerpt* text Libra, p. 138.

104. *Excerpt* text Libra, p. 138.

105. *Excerpt* text Pisces, p. 146.

106. Compare also the system given in the Vienna Demotic Papyrus which contains similar schemes for lunar and solar omen indications. One of the schemes begins in IV *Akhet*, the fourth month of one of the three seasons. This also seems to be an odd starting point until one correlates it to its Babylonian counterpart, which is also Nisannu.

107. Rochberg-Halton (1988) p. 83, argues that *EAE* 16 included an additional section at the start that dealt with the occurrence of eclipses with various meteorological phenomena (clouds, rain, and thunder).

108. See, for example, VIII:502 p. 278 and X:159 p. 120.

109. This may betray its Egyptian passage. Section A of the Vienna Demotic Papyrus deals with lunar and solar eclipse phenomena simultaneously. See Parker (1959).

110. For further discussion on this transmission, see Williams (2008).

CHAPTER FIVE Recipient and Remodeler

1. The Myrobalan is an astringent plumlike fruit produced from a tropical tree.

2. That is, the pursuit of the science from ancient times up until the nineteenth century when European astronomy replaced the traditional science.

3. See Pingree (1981) for a thorough treatment of the discipline. The Indian traditional division of *jyotiḥśāstra* was threefold: *gaṇita* (astronomy), *horā* (astrology), and *saṃhitā* (omens).

4. Pingree (1978) p. 629.

5. For discussion concerning this date, see Pingree (1981) pp. 9–10.

6. For details on the date see Pingree (1978a) p. 555, Pingree (1967–8), and Morrissey and Pingree (1989).

7. For his date and life, see Neugebauer and Pingree (1970) p. 7. The commentator Āmarāja, in his commentary on Brahmagupta's *Khaṇḍakhādyaka*, writes confidently however (KKD, p. 109, lines 23–24) *navādhikapañcaśatasaṅkhyaśake varāhamihirācāryo*

divaṃ gataḥ [The teacher Varāhamihira went to heaven in the Śaka year measured by 509]. Śaka 509 = 587/8 CE. We know Varāhamihira was writing in the sixth century (Brahmagupta was familiar with his works), so this date could be plausible. Further support is provided by his mention of Aulikara, a king who ruled in ca. 550. See Salomon (1989).

8. The *Pañcasiddhāntikā*, as its name implies, was a compilation of five different astronomical systems that were obviously influential in the previous centuries. The names of two of the five *siddhāntas*, the *Romakasiddhānta* (*siddhānta* of the Romans) and the *Pauliśasiddhānta* (*siddhānta* of Paulus?) bear witness to the importance already of Hellenistic influences on techniques and parameters which had been worked into an Indian setting. Babylonian aspects can be detected in the *Vasiṣṭhasiddhānta* and the *Pauliśasiddhānta*, particularly in lunar theory. In fact, all the *siddhānta*s contain, in some part, the influence of a complex mixture of Greek and Babylonian elements. In addition, another one of the five *siddhānta*s, the *Sūryasiddhanta*, is the earliest record in existence of this particular *siddhānta*; it exists independently as a work only in a much later, revised text. Furthermore, upon a systematic study of the contents of the *Pañcasiddhāntikā*, it has been remarked that it would not have been possible to carry out the required computations with only the information presented in the texts. This suggests that these texts were summaries of presumably more complete original works. That, then, these different systems were condensed versions of earlier more complete works suggests that the astronomical tradition had been flourishing for some time already.

9. For further details on number representation, see Plofker (2008), pp. 73–77.

10. Spiritual seekers, seers, i.e., those on earth that had the closest relationships with the divine.

11. Pingree (1978a) pp. 533–34 with a minor adjustment in light of new material. For eighteenth-century European influence, e.g., the use of de La Hire at Jayasiṃha's court, see Pingree (1997).

12. See work by Pingree (1990), Minkowski (2004), and Plofker (2005), on *avirodha* texts.

13. According to the philosophical school of the Vedānta. The Nyāya philosophical tradition allows only the first four of the above, the Sāṃkhya philosophical tradition only three: *pratyakṣa*, *anumāna* and *śabda*. There were other philosophical schools that added three more, to a total of nine: *sambhava*, equivalence, *aitihya*, tradition or fallible testimony, and *ceṣṭā*, gesture. See Monier-Williams (1982) *pramāṇa* p. 685; Dāsa and Dāsa (1995) esp. pp. 18–22.

14. The southern *ayana* in this context is the half year between the autumnal equinox and the vernal equinox.

15. The *bīja* (literally "seed") corrections were conversional constants to convert the calculations of one astronomical system to another.

16. This work is a hypercommentary on the Indian commentator Govindasvāmin's (fl. ca. 800–850) commentary on Bhāskara's (fl. 629) *Mahābhāskarīya*. See Sastri (1957).

17. See Pingree (1978a) p. 534; Plofker (2008) pp. 69–71.

18. In contrast to the opinion of Billard (1971) p. 30.

19. See Pingree (1978a) p. 629.

20. Indian planetary theory was based on a double-epicycle model. The so-called anomaly accounts for the displacement of the true position of a planet from its mean and is dependent on its displacement from its apogee. In the Indian system, the sun and the moon have a single anomaly, known as the *manda* (literally "slow") epicycle. This is equivalent to the fact that the moon's orbit is elliptical. The planets Mercury, Mars, Venus, Jupiter, and Saturn have an additional anomaly, known as the *śīghra* (literally: "fast"). This accounts for the relative orbital motion of the planets with respect to the earth, which at times causes them to appear to move backward.

21. For further general discussion see Plofker (2008) pp. 249–51. For details on the relations between Nīlakaṇṭha's proposals and those of Tycho, see Ramasubramanian et al. (1994) and Sarma (2004). For further proposals of Nīlakaṇṭha, as well as a refutation of the idea that his planetary proposals were consciously heliocentric, see Pingree (2001).

22. The earliest instance of the term *graha* appearing with *rāhu* may be the *Atharvaveda*. See Yano (2004) p. 332.

23. See, for example, the *Bhāgavatapurāṇa* 8.9.25ff and *Mahābhārata* 1.19.

24. Many early Indian astronomers treat these points as planets. It may be possible to determine when Rāhu and Ketu become considered as planets. They do not occur in Sphujidvaja's *Yavanajātaka* (269 CE.) as such, but there is one reference to them by the foreign ruler (*yanvanādhirāja*) Mīnarāja (ca. 300/325) in his work the *Vṛddhayavanajātaka*, who enumerates them among the planets.

25. For example, the large omen text, the *Gargasaṃhitā* written in the first century BCE or CE. Many of the omens contained in this work are paralleled in cuneiform texts. See Pingree (1981) pp. 69–71.

26. For an excellent edition and translation of this chapter and commentary, see Ikeyama (2003) pp. S1–S305.

27. This is a serious criticism. The foundation of learning in the Sanskrit tradition consisted of the four Vedic *saṃhitās* (collections of hymns) and other works, such as the Upaniṣads. These texts were known as *śruti* (literally "that which is heard"), because they derived from direct divine revelation. However, in the first millennium BCE, this foundation of knowledge broadened to include six other derivative disciplines as well. These disciplines, referred to as "limbs" (*Vedāṅga*), are known as *smṛti* (literally "that which is remembered") because they issue from human sources, and thus are secondary to the *śruti* tradition.

28. For details on Lalla's life, see Pingree (1978) pp. 582–83.

29. For more details on this genre see, for example, Pingree (1978) pp. 533ff, Pingree (1981), or Plofker (2008) pp. 66–72.

30. According to Brahmagupta, BSS V, 1 (see section 5.6.4).

31. For an overview of this mathematical idea and its transmission, see Sarma (2002) pp. 133ff, esp. p. 141.

32. See the Appendix in *Corpus Inscrption Indicarum* written by S.B. Dikshit, 161–76, Salomon pp. 197–98, which is based on the period of the sidereal revolution of Jupiter in approximately twelve solar years. These twelve years are given names corresponding to those of the ordinary lunar months. There exist other cycles such as the 60-year cycle. See Dikshit 163 table VIII for a complete list.

33. See the Sarkho Plates, for example, in the next section.

34. See Monier-Williams (1994) p. 850. This division of the night is reminiscent of the watches in Mesopotamia.

35. For a thorough survey of Indian Divination, see Pingree (1981), chapter IV. For the transmission of Mesopotamian omen texts into India, see ibid. pp.67–70 and Knudsen (2008).

36. See Pingree (1981) p. 67.

37. See Pingree (1992) esp. p. 379.

38. Pingree, (1992) p. 376. For details of the terrestrial omens and their comparison with *Šumma ālu* see pp. 376–78.

39. For further details see Pingree (1998) p. 131 and n. 85. In a similar but later story, the *Śārdūlakarṇāvadāna*, a section is included among the omens on the significance of a lunar eclipse in each *nakṣatra*.

40. See Pingree (1981) p. 75.

41. The moment at which the sun enters a zodiacal sign, or a solar month.

42. It seems likely that here the success of the propritiation was based on the prediction's being wrong—Pingree (1978) p. 362 notes that "it was the practice to issue plates in advance, but to date them on the day of the eclipse. In a large number of cases, no eclipse was in fact visible in the locality of the grant on the given date. This, of course, was no argument against the accuracy of the prediction made by the court astronomer, but rather evidence of the efficacy of the donation of land to Brāhmaṇas, which averted not only the evil potentially indicated by an eclipse, but the eclipse itself."

43. There exists another inscription that refers to this very eclipse, but gives less information. See *CII* VI 6, p. 28.

44. Lines 23–24, *CII* IV p. 427.

45. *Bṛhatsaṃhitā*, II, 4.

46. See Pingree and Neugebauer (1970).

47. Chapters VI and VII are attributed to the *Pauliśa*, a *siddhānta* that is demonstrated to have been based on an Hellenistic source (see Pingree and Neugebauer [1970] p. 12). As they point out, Pauliśa may represent the Greek name Παῦλος. See also Pingree [1978] p. 545. Although it is not explicitly stated, the attribution of both chapters to *Pauliśa* is almost certainly correct since the material in each is closely related. Furthermore, the parameters that occur in these chapters are elsewhere in the *Pañcasiddhāntikā* associated directly with the *Pauliśa*; see, for example, I, 11–13.

48. Compare Brahmagupta's fourteen which he enunciates at the beginning of chapter 4, "On Lunar Eclipses" (IV, 1–2).

49. Neugebauer and Pingree (1970) II p. 51.

50. *Bṛhatsaṃhitā*, V, 43–52.

51. This interpretation was set out by Neugebauer and Pingree (1970) II pp. 52ff.

52. As has been assumed in Pingree (1978a) p. 549, but alternative suggestions have been made. See, for example, the translations of Neugebauer and Pingree (1970) I p. 77 and Thibaut and Dvivedi (1997) pp. 43–44, both of which say the difference of the *moon* and the node. For a possible derivation of this rule, see Neugebauer and Pingree (1970) II p. 53.

53. Textual difficulties noted by Neugebauer and Pingree (1970) II: vi, 8, p. 54 suggest that it could simply be manuscript corruption that the verse for the *ayanavalana* has been lost.

54. See Rochberg-Halton (1988) pp. 55–57.

55. As observed by Neugebauer and Pingree (1970) p. 55.

56. This is reminiscent of Ptolemy's *epiparallax*.

57. See Neugebauer and Pingree (1970) pp. 63–64.

58. See, for example, Plato *Timaeus* 38C–39D; Aristotle *de Caelo* II, 289a11–291b23.

59. See Pingree (1978a) p. 555.

60. Pingree (1963) pp. 237–40; see also Pingree (1978a) p. 555.

61. Pingree (1978a) p. 555; (1978b) p. 361; (1976) pp. 109–123.

62. See Pingree (1967–68) pp. 472–510. The astronomical section does however exist as an independent work; see Pingree (1981) p. 17 and n. 29, and Pingree (1990) pp. 276–79.

63. Pingree (1978a), p. 561.

64. Pingree (1967–68) p. 504.

65. This is one of the sides of a right-angled triangle.

66. This is another of the sides of a right-angled triangle.

67. Except for a passage on astronomical instruments. See Shukla (1976).

68. Neugebauer and Pingree (1970) p. 73.

69. See also Neugebauer and Pingree (1970) II pp. 74–76.

70. This can be verified by noting that $\mathrm{Sin}[\delta(M)] = \frac{\mathrm{Sin}\phi \times \mathrm{Sin}\eta}{R} = \frac{\mathrm{Sin}\epsilon \times \mathrm{Sin}\lambda(M)}{R}$.

71. For further details, see Neugebauer and Pingree (1971) pp. 75–6 and also X, 1–2.

72. See Neugebauer and Pingree (1970) II p. 77 for the derivation of this formula by using material from IX, 15–16.

73. See Pingree (1978a) pp. 590–602 for details of this *pakṣa*.

74. The exact date of this work is unclear although in it he mentions the fact that in the year 499, he was 23 years old (tentatively making his date of birth 476).

75. These may be derived ultimately from Babylonian sources.

76. Āryabhaṭa gives the diameter of the earth as 1050 *yojanas* and the circumference as 3,375 *yojanas*. Thus the value of pi here must be 3 3/14, despite the fact that elsewhere $\pi = \sqrt{10}$ and also pi = 216000/6876.

77. See *Gītikā* 4 for this relationship.

78. Compare BSS XXI,32: "the . . . earth, the sun and the moon consisting of soil, fire, and water."

79. Compare with that given in the *Paitāmahasiddhānta* V, 3.

80. For detailed information see Pingree (1981), pp. 21ff.

81. See Plofker (2008) p. 319.

82. See Pingree (1978a) p. 416.

83. In fact, there are 25 chapters; the 25th is a karaṇa using a contemporary epoch of 21 March 628.

84. Note that he does not include this phase in the above list of significant elements.

85. This parameter is identical with that given in the *Paitāmahasiddhānta* (III, 18) and is also repeated (using $R = 150$) in the *Khaṇḍakhādyaka* (see below). Lalla (*Śiṣyadhīvṛddhidatantra* V, 11) uses 270/3438, which he simplifies down to 15/191 (see below). Vaṭeśvara (V 20) gives the following approximations to 270/3438 (see below):

$$\frac{135}{6876/4} = \frac{100}{1273} = \frac{4}{51}.$$

86. See, for example, the ratio 1:2;30 given by Hipparchus and as cited in *Almagest* IV 9. Ptolemy gives a slightly different ratio 1:2⅔ or 1:2;36, which he observes is slightly too small (see section 4.5.1). Compare also to Eudoxus who gives the ratio as 2:1—see Heath (1981) p. 353, proposition 5.

87. Compare *uktaṃ* as given in Dvivedin's edition (1902) p. 91.

88. The sun is 6500 *yojanas* in the *Paitāmahasiddhānta*. See III,8.

89. Pingree (1978a) p. 605, (587 Śaka).

90. KKD, p. 126 lines 10–11.

91. Cf. BSS XXI, 32.

92. Āmarāja asserted earlier (KKD p. 126 lines 5ff) that the daily motion of the moon in *yojanas* according to Brahmagupta is 11 858;45.

93. However, this edition it gives Indra instead of Indu. Clearly, given the inclusion of Śaśi, Indra is the correct reading.

94. See KKD p. 150 line 6 to p. 151 end.

95. If $d_m = 31{,}30$, then $56{,}23 - 15{,}45 = 40{,}38$ so that the ratio is about 1:2;30. Compare this ratio with Hipparchus' (see section 4.4.1).

96. See KKD pp. 154 line 16–156 line 26.

97. MB V, 47–67; in 74a he complains that "some astronomers do not advocate a parallax correction in a lunar eclipse" (MB commentary p. 15).

98. Cf. BSS IV, 6.

99. For more details, see Pingree (1978a) pp. 582–83.

100. Compare Āryabhaṭa Daśagītikā 6, Bhāskara I MB vii 20.

101. Lalla's approximation for π is $3927/1250 = 3.141600$. This value is borrowed from Āryabhata's sine tables. Bhāskara II gives the same method in SSI I v (commentary). The particular value of π affects the radial distance value.

102. This rule is also seen in the *Paitāmahasiddhānta* V, 2, MB v 5, LB iv 5, BSS xxi 34, sse v 6 and SSI I v 7.

103. See Plofker (2008) pp. 97–102.

104. For more information see Chatterjee (1981) vol. 2 pp. xxiii–xxvi. Mallikārjuna's date is unknown, but he uses the epochs 1100 *Śaka* and 1107 *Śaka* to illustrate rules.

105. The mid-eleventh century astronomer Śrīpati also follows this layout and gives *cālana*s for both intervals (SŚe VII, 3–4); however, his values are slightly different.

106. See Pingree (1978a) pp. 590 and 599–600.

107. For the larger context, see Plofker (2002) pp. 176–79.

108. See Plofker (2002) p. 177.

109. For more on astronomy in Kerala, see Sarma (1972) and (1990).

110. For more details on his life, see Pingree (1978) vol. X, pp. 313–14.

111. See verses 1 and 84.

112. See, for example, Plofker (2008) pp. 217–53 and (2001).

113. For a similar approximation, see Gupta (1967) and (1986).

114. These have been computed using Peter Huber's SOLEC and LUNEC programs, which were circulated privately.

115. This eclipse in Nilā was of magnitude 0.160.

116. See also *Grahaṇamaṇḍana* (95) and *Grahaṇanyāyadīpikā* (80) for the same limit.

117. In fact, on many astrolabes there are two units of measurement on the shadow square: the first, in feet, generally contains seven divisions, and the second, digits, contains twelve divisions–twelve being the standard height of a gnomon.

CHAPTER SIX A Science of Configuration

1. See, for example, King (1999).

2. Rashed (1996) p. x characterizes it as "international."

3. There is no direct word for "science" in Arabic. The word *'ilm* literally means "knowledge," and the various branches of inquiry invoke this term: mathematics is captured by the phrase *'ilm al-hisāb* (literally "knowledge of reckoning"), theology by *'ilm al-kalam* (literally "knowledge of all"), and so on.

4. King (1996) p. 143.

5. For a thorough analysis of this key movement, see Gutas (1998).

6. Pingree (1973a) p. 36.

7. For details, see Pingree (1968a) pp. 97–98.

8. For further details on this scholar, see Pingree (1970).

9. Or at least some text from the *ārdharātrikapakṣa*.

10. Pingree (1973a) p. 38.

11. Saliba (1994) p. 143, n. 1.

12. For more details, see Pingree (1973a) pp. 38–41.

13. See Plofker (2002).

14. Plofker (2002) notes that there is no Indian antecedent for these procedures being used in mathematical contexts, but closer links can be established in astronomy.

15. See Pingree (1973a).

16. Sabra (1998) translates this title as "Aporias Against Ptolemy," noting the specific emphasis behind the use of the Arabic preposition *'ala* (literally "upon," "at," "against") instead of the more common *fī* (literally "in", "on", "about", "regarding"). Sabra argues this indicates an "aggressive, critical intent," purposefully invoked by ibn al-Haytham.

17. See, for example, Kennedy (1966), Roberts and Kennedy (1959), and Saliba (1987).

18. This point was raised and developed by Sabra (1998) pp. 288ff.

19. Expressed in Arabic variously, including *minha and tajriba i'tibar*.

20. Sabra (1998) p. 289.

21. This point is strongly upheld by Sabra (1998) p. 290.

22. Ptolemy in Toomer (1984) p. 422, as cited and discussed by Sabra (1998) p. 295ff.

23. Kennedy (1956b) has produced a comprehensive survey of about 100 *zīj*es; more detailed and recent summaries, such as those by Benno van Dalen are still in progress.

24. Kennedy (1956b) p. 143.

25. For a thorough account of observational practices of eclipses in the Islamic Near East, see Steele (2000a) pp. 107–23.

26. See Pingree (1973a) pp. 38–39.

27. For a description of this method, practised since early antiquity, see, for example, Steele (2000a) pp. 110-11.

28. For example, reflections on this by Al-Bīrūnī and Ibn Yūnus are cited by Steele (2000) p. 111.

29. Steele (2000a) p. 110.

30. Steele (2000a) p. 111.

31. As also observed by Kennedy (1956b) p. 48.

32. The original Sanskrit version of this work is unknown and is preserved only in some quotations by al-Bīrūnī.

33. Kennedy (1956b) pp. 48–49, who observes that Suter took the meaning as such, but also remarks on the correction.

34. See Kennedy (1956b) p. 49, who notes that Ibn al-Shāṭir warns his readers not to use the point M, but reassures them that the "mistake" was a common one.

35. Kennedy (1956b) proposes a possible reconstruction (p. 50), but with some outstanding problems, for example the relation between t and θ is not at all clear. The author has no access to the original to consider this further.

36. These have been extracted from Kennedy (1956).

37. For details see Kennedy (1956) p. 44. This work is in Escorial (Madrid) Arabe 927. Vernet (2008) vol. 14, pp. 357–58, notes that this manuscript is poorly bound and disorganized and contains many folia that are obviously not Yaḥyā's original text.

38. Of course, the sun travels *on* the ecliptic, the reference from which celestial latitude is measured!

39. Kennedy (1956b) p. 44–45.

40. As observed by Kennedy (1956b) p. 44. The author has not had access to the original.

41. The first, in Istanbul, is Yeni Cami (no. 784) and the second, in Berlin, no. 5750.

42. First explained by Kennedy (1956b) p. 42. He notes that this is in fact $\cos L_t$.

43. Kennedy (1956) p. 154 observes that approximately half the values given by al-Khwārizmī in his table of π_β agree to within one significant figure with the values in this table.

44. For more details see Kennedy (1956b) p. 51–52.

45. For further details, consult the description and contextualization of this iterative algorithm as described in Plofker (2002), esp. pp. 177–79.

46. See Plofker (2002) p. 179 n. 13, where she observes that Islamic practitioners commonly prescribe limits to their iterations, unlike Indian practitioners and Ptolemy, who simply direct the reader to carry out the process repeatedly.

47. While Kennedy (1956b) believes this parameter 24 to be an approximation to the obliquity of the ecliptic (ϵ), Plofker (2002) p. 179 n. 14 states that it is rather a conversion factor between the time units (1 *ghaṭikā* = 24 minutes).

48. Kennedy (1956) observes that this table presents three functions (p. 152). He identifies the first as some sort of interpolation scheme for variation due to lunar anomaly, the second as $0;6 \sin\theta$, and the third as $23;35 \sin\theta$ (which must be a rough declination table). Perhaps these belong to parallax, but more likely, he supposes, they belong to the section on visibility problems.

49. Kennedy (1956a) notes that these probably are Indian in origin, but need to be studied more.

50. As cited by Kennedy (1956a) p. 41.

51. For details, see Steele (2000a) p. 114–16.

52. Steele (2000a) p. 116 suggests that the true accuracy is about 1.16 hours.

53. This was the case, for example, with the Banū Amājūr, a group of astronomers active in Baghdad between 885 and 933. See Steele (2000a) p. 116–17.

54. For details of his life, see Hall (2008) vol. 7, p. 335-351.

55. For an account of the complex manuscript tradition of this work, see Pingree (1997b). The author has had direct access to both manuscripts of this work (from Teheran and Istanbul) and is preparing a critical edition and translation for upcoming publication of the whole work.

56. One must be wary of al-Khāzinī 's use of the terms for "day" in Arabic. He uses *al-nahār* and *al-yaum*, both of which mean "day" in Arabic. The former most usually refers to the interval between sunrise and sunset, and is often found in phrases such as *niṣf al-nahār* (half of the day = noon) or *sā'āt al-nahār* (period of daylight). The latter term is most often combined with *bi-lailatihi* to indicate a nychthemeron (*al-yaum bi-lailatahi*).

57. From the root *faḍl* meaning surplus, remainder.

58. Manuscript T has the reading 70 minutes.

59. See Neugebauer (1975) vol. 1 p. 126; Ptolemy's figures are $\beta \approx 1;31°$ northward and $\beta \approx 0;41$ southward.

60. For further details, see Nasr (2008) vol. 13, p. 511ff.

61. See Ragep (1993) pp. 223–39.

62. For more see Ragep (1993) p. 461.

63. Ragep (1993) vol II, pp. 461–63.

64. Following Ragep (1993) p. 462.

65. Ragep (1989) rightly cautions the reader about the dangers of this reconstruction.

66. See Kennedy (1956a).

67. See Roberts and Kennedy (1959) and Saliba (1987), for example.

68. Summarized from the exposition given in Kennedy (1956), pp. 43–44.

69. Kennedy (1956b) p. 43.

70. Kennedy (1956) p. 44.

71. This is reconstructed by Kennedy (1956b) p. 45.

72. As quoted from Kennedy (1956b).

73. Kennedy (1956b) p. 49 considers this method as reasonably good.

74. This name was a title granted to him in place of his birth name, Muḥammad Taragay.

75. Kennedy (1956b) pp. 36ff. With respect to the Indian manuscript, f. 164v includes a table for parallax for $\phi = 30°$. On the upper half of this same folio is the ruling

for an identical table, but it has been left empty except for the title: $\phi = 20°$. Likewise, on the following f. 165r, two more such tables with $\phi = 40°$ and $\phi = 50°$ in their titles appear but have been left empty. This is also seen in the Istanbul manuscript.

76. Kennedy (1956b) p. 37.
77. As constructed by Kennedy (1956b) p. 37.
78. Kennedy (1956) p. 39.
79. See Rashed (1996) p. 284 and Pedersen (1975).

Glossary

ahargaṇa the number of accumulated days since the beginning of some designated epoch.

altitude one of the pair of orthogonal coordinates in the horizon coordinate system (the other is the azimuth). It is the angular distance from the horizon to a given point on the celestial sphere.

analemma a diagram that represents the celestial sphere projected onto the two-dimensional plane and captures the relations between great circles, points, angles, and arcs.

aṅgula the unit of linear measure equivalent to a digit.

anomaly ($\overline{\alpha}$) the angular distance between the apogee and the position of the mean celestial body.

apogee (*A*) the point at which a celestial body is farthest from the earth on its orbit.

ascendant (*H*) the point of the ecliptic which is rising on the eastern horizon.

astrolabe an instrument used by astronomers that captures the celestial sphere as projected stereographically onto the plane of the equator. Its moving parts allow it to act as a quantitative working model of the heavens. It can be used to observe celestial phenomena and then compute various astronomical details, such as rising times, altitudes, positions, intervals, and times of various celestial phenomena.

ayana the Indian term used to refer to a half-year interval—the northern *ayana* is when the sun travels north (from the winter solstice to the summer solstice) and the southern *ayana* when the sun travels south (between the summer solstice to the winter solstice).

azimuth one of the pair of coordinates in the horizon coordinate system (the other is the altitude). It is the angular distance along the horizon from the north point to any given point on the horizon.

bīja a correction that converts the parameters of one astronomical system to another.

celestial sphere a giant sphere enclosing the observer, who is positioned at its center, on which all celestial bodies are imagined to be projected.

celestial equator the great circle on the celestial sphere which falls in the plane of the terrestrial equator.

chord a line segment that joins two points on the circumference of a circle.

clime a geographical region created as a result of dividing up the terrestrial sphere into distinct regions, usually using latitude circles parallel to the terrestrial equator.

concentric can refer to a circle or a sphere whose center is located at the same place as the earth.

conjunction the instant at which two celestial bodies, usually the sun and the moon, have the same longitude and hence are at the same (or very nearly the same) position in the sky.

cosmography a description or a representation of the structure of the universe.

cosmology an accounting for the structure of the universe.

day circle a small circle on the celestial sphere parallel to the celestial equator, which represents the apparent course of a celestial body (usually the sun) during one daily rotation of the celestial sphere.

declination (δ) one of the pair of orthogonal coordinates in the equatorial coordinate system (the other is the right ascension). It is the angular distance from a given point on the celestial sphere to the celestial equator.

deferent a circle whose center coincides with the center of the earth. A smaller circle, or epicycle, moves on its circumference.

eccentric a circle whose center is distinct from the center of the earth. The distance between these two centers is called the eccentricity (e).

eclipse, annular a type of solar eclipse in which the apparent size of the moon is smaller than the apparent size of the sun, which appears to cast a ring or annulus around the disk of the moon, outlining its shape.

eclipse, lunar a celestial alignment that occurs when the moon passes through the shadow created when the earth blocks light from the sun. Because this alignment occurs only when the sun, the earth, and the moon are on the same line (or very nearly so) and in that order, lunar eclipses can only occur at opposition.

eclipse, solar a celestial alignment that occurs when the moon passes across the face of the sun, hence depriving certain locations on the earth wholly or partially of its light. Because this alignment occurs only when the sun, the moon, and the earth are on the same line (or very nearly so) and in that order, solar eclipses can occur only at conjunction and their appearance is highly dependent on the location of the observer on the earth.

ecliptic the great circle which delimits the apparent path of the sun in a year.

elongation (η) the difference in longitude between two celestial bodies.

emersion the reappearance of an eclipsed body after it had been obscured by an eclipsing body.

epicycle a small rotating circle whose center is located on the circumference of the deferent circle.

epoch a point in time selected to be the initial instant from which all phenomena are computed.

equation the angular distance between a celestial body's mean and true longitude.

equinox either of the two points at which the ecliptic intersects the celestial equator. The vernal equinox (Υ 0°) is the point that the sun appears to pass through in

springtime, and the autumnal equinox the point through which the sun appears to pass in the fall.

gaṇita a Sanskrit term used to represent the Indian discipline of mathematics, particularly as it relates to astronomy.

geocentric a cosmological hypothesis considering the earth the central point of the universe (in contrast to heliocentric).

ghaṭikā an Indian division of time—there are 60 *ghaṭikās* in one 24-hour period, so that one *ghaṭikā* is the equivalent of 24 minutes. The term *nāḍī* can be alternatively used.

gnomon a small straight rod which casts a shadow from the sun, from which various astronomical features can be computed, such as time, direction, and place. The standard gnomon in Indian astronomy is often 12 units in length.

gola a Sanskrit term for a sphere or the mathematical branch of spherics. An alternative definition of *gola* is the half year.

half-duration the time interval between the beginning of the eclipse and mid-eclipse.

half-duration of totality the time interval between the beginning of totality, or total immersion of the disk of the moon and mid-eclipse.

ha'ya the Arabic term that means "configuration." Used to represent the field of astronomy.

heliocentric a cosmological hypothesis considering the earth the central point of the universe (in contrast to geocentric).

horizon a great circle related to the location of an observer on the surface of the earth. This circle is demarcated by where the plane tangent to the surface of the earth at the observer's location bisects the celestial sphere.

hour-angle the angle between the plane of the prime meridian (the plane given by the poles of rotation of the earth and the local zenith) and the plane through the poles of rotation of the earth and the position of the celestial body.

immersion the first instance of the disappearance of the body being eclipsed by an eclipsing body.

impact the instant at which a body is first obscured by an eclipsing body.

interpolation any mathematical technique used to construct an intermediary value between two tabulated values.

jyā (Sanskrit, literally "the bowstring") the sine (or chord).

jyotiṣa the term used to represent the field of astronomy and astrology in the Indian tradition.

Kaliyuga a common epoch used in Indian texts, which began on 18th February 3102 BCE.

karaṇa a type of Indian astronomical handbook, usually with an epoch contemporary with its composition.

kaṭapayādi a way of expressing numerals by means of the association of letters of the Sanskrit alphabet with numbers.

latitude, celestial (β) one of the pair of orthogonal coordinates in the ecliptic coordinate system (the other is the celestial longitude). It is the angular distance from the ecliptic to a given point on the celestial sphere.

latitude, terrestrial (ϕ) the angular measurement north or south of the equator in degrees. The latitude at the equator is set at 0° and increases northwards or southwards until reaching a maximum at the north and south poles of 90° N and 90° S respectively. The complement of latitude is the colatitude ($\bar{\phi}$).

longitude, celestial (λ) one of the pair of orthogonal coordinates in the ecliptic coordinate system (the other is the celestial latitude). It is the angular distance along the ecliptic from the venal equinox to a given point on the celestial sphere.

lunar orbit a circle which represents the apparent path of the moon during a month.

lunar orbit, inclination (ι) the angle between the planes of the lunar orbit and the ecliptic, that is, the amount by which the lunar orbit is inclined to the ecliptic. This amount is usually approximated to be around 5°.

magnitude (μ) a measure of the amount of obscuration during an eclipse. Commonly it is a measure along the diameter of the eclipsed body, and less frequently, an area measure.

mean motion ($\bar{\nu}$) the average angular velocity of an orbiting celestial body.

meridian any one of the great circles which pass through the zenith and the point opposite it (the nadir), intersecting the horizon.

month, anomalistic the interval it takes for the moon to travel from its perigee back to its perigee, a little over 27½ days.

month, draconitic the interval it takes for the moon to travel from its node to the same node, a little under 27⅓ days.

month, full or hollow given that the length of a synodic month is not a whole number of days, but a little over 29½, the visibility of the new crescent is sometimes after 29 days and sometimes after 30. Accordingly, a month that is 30 days long is called "full," and one that has 29 days, "hollow."

month, sidereal the interval it takes for the moon to return to the same position on its circuit with respect to the background of fixed stars, about 27⅓ days.

month, synodic the interval between successive conjunctions of the moon with the sun, a little over 29½ days.

nāḍī (see *ghaṭikā*).

nakṣatra an Indian term for one of the 27 or 28 constellations roughly equally spaced along the orbit of the moon.

nodes, lunar the two points at which the ecliptic and the lunar orbit intersect. The point at which the moon crosses the ecliptic from north to south is known as the descending node, and the point at which it crosses from south to north is known as the ascending node.

nonagesimal (V) the point on the ecliptic 90° from the ascendent above the horizon.

north pole (N) the point at which the axis of the earth's rotation (and hence the celestial equator) intersects the celestial sphere.

nychthemeron the 24-hour interval, invoked to prevent the ambiguity associated with the word "day." It is a compound deriving from the ancient Greek terms for night (*nukt*) and day (*heméra*).

oblique ascension the arc of the celestial equator which rises at the same time with a given arc of the ecliptic, above the horizon for an observer.

obliquity (ϵ) the angle between the planes of the ecliptic and the celestial equator, that is, the amount by which the ecliptic is inclined to the celestial equator. It is a little under 24°.

opposition the instant at which two celestial bodies, usually the sun and the moon, have a longitudinal difference of 180° and hence are directly opposite one another.

padabhā (Sanskrit, literally "foot-shadow") the shadow of a gnomon that measures in feet.

pakṣa a "school" or tradition in Indian astronomy, the works of which all use the same base parameters and assumptions.

parallax (π) the angular difference between a celestial body's true position, which has been calculated with respect to the center of the earth, and its apparent position, taken with respect to an observer's position on the surface of the earth. The effect of parallax is to make objects appear closer to the horizon for the vantage point of the observer, than they would be seen from the center of the earth.

partial eclipse an eclipse in which the eclipsed body is only partially obscured.

penumbra the fanlike region of shadow that is less dark than the umbra in which light is not completely cut off by the eclipsing body.

period relations ratios of large integers representing numbers of planetary periods between successive conjunctions of synodic phenomena.

planets in early times were considered those celestial bodies that changed their positions with respect to the backdrop of fixed stars. They included the sun and the moon as well as the five planets visible to the naked eye: Mercury, Venus, Mars, Jupiter, and Saturn.

pole, ecliptic (P) the point at which the axis of the ecliptic intersects the celestial sphere.

precession the effect of the motion of the equinoxes with respect to the ecliptic against a backdrop of fixed stars. It is about 1° every 72 years.

prime meridian the great circle that passes through the zenith and the north and south points of the horizon.

prime vertical the great circle that passes through the zenith and the east and west point of the horizon.

qibla the sacred direction of prayer in the Islamic religious tradition.

rāśi the Indian term to refer to the 30° zodiacal intervals of the ecliptic.

release the instant at which an eclipsed body completely clears the eclipsing body.

retrograde the motion of a celestial body when it appears to travel backwards, that is from east to west, against the background of fixed stars.

right ascension one of the pair of orthogonal coordinates in the equatorial coordinate system (the other is the declination). It is the angular distance along the celestial equator from the vernal equinox to the given point. It can be expressed in units of arc, or of time.

Saros the 18-year eclipse cycle in which successive eclipses from cycle to cycle share duration, magnitude, and spacing.

śāstra Sanskrit term for an intellectual discipline, field of inquiry, or branch of knowledge.

sexagesimal a system of numeration with sixty as its base.

siddhānta Sanskrit term for a detailed expository astronomical treatise.

solstice either of the two points which are 90° away from the equinoxes, along the ecliptic. The summer solstice is the point which the sun appears to pass through in the middle of summer, and the winter solstice the point through which the sun appears to pass in the middle of winter.

sphaera obliqua the situation in which an observer is located anywhere on the surface of the earth, with some latitude ϕ.

sphaera recta the situation in which an observer is located at the equator, and hence their latitude, ϕ, is 0. Here, the north pole, N, is situated on the horizon, and the celestial equator is perpendicular to the horizon.

step function a (usually periodic) mathematical function which remains constant at each of its values for finite intervals.

stereographic projection a mathematical technique of projecting the two-dimensional sphere onto a plane, at least in an astrolabe. It projects every point on the sphere onto the intersection of the plane with a line passing through that point and a given reference point on the sphere.

synodic an adjective used to describe a time interval or motion with respect to the sun. The synodic month, for example, is the interval from the conjunction of the sun and the moon, back to this same position.

syzygy the alignment of three (or more) astronomical bodies along the same plane, usually the sun, the moon, and the earth. It comprises conjunctions and oppositions.

tithi Sanskrit term for the interval at which the difference in the eastwards motions of the sun and the moon accumulates to 12°, and for this reason is often called a lunar day. There are 30 *tithis* in a synodic month.

totality, eclipse the period during which the eclipsed body is completely obscured by the eclipsing body.

trairāśika (Sanskrit, literally "rule of three") a mathematical proportion based on the equality of two ratios.

umbra the conelike region of the shadow in which light is completely blocked by the eclipsing body.

valana the angle of inclination related to eclipses.

yojana a measure of distance, used in the Indian tradition. Its exact amount is disputed, but is likely in the order of about 10 kilometers.

zenith the point directly overhead for the observer at any given location.

zenith distance the angular distance of a celestial body from the zenith point, along any meridian.

zigzag function a (usually periodic) mathematical function alternating between linear increase to a maximum and linear decrease to a minimum, in which successive values of the function differ by a constant amount.

zīj a genre of astronomical works in Islamic astronomy which consist of tables and accompanying explanation.

zodiac the partition of the ecliptic into twelve sections of 30° each. Each section derives its name from the configurations of stars that roughly occupy the same space.

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